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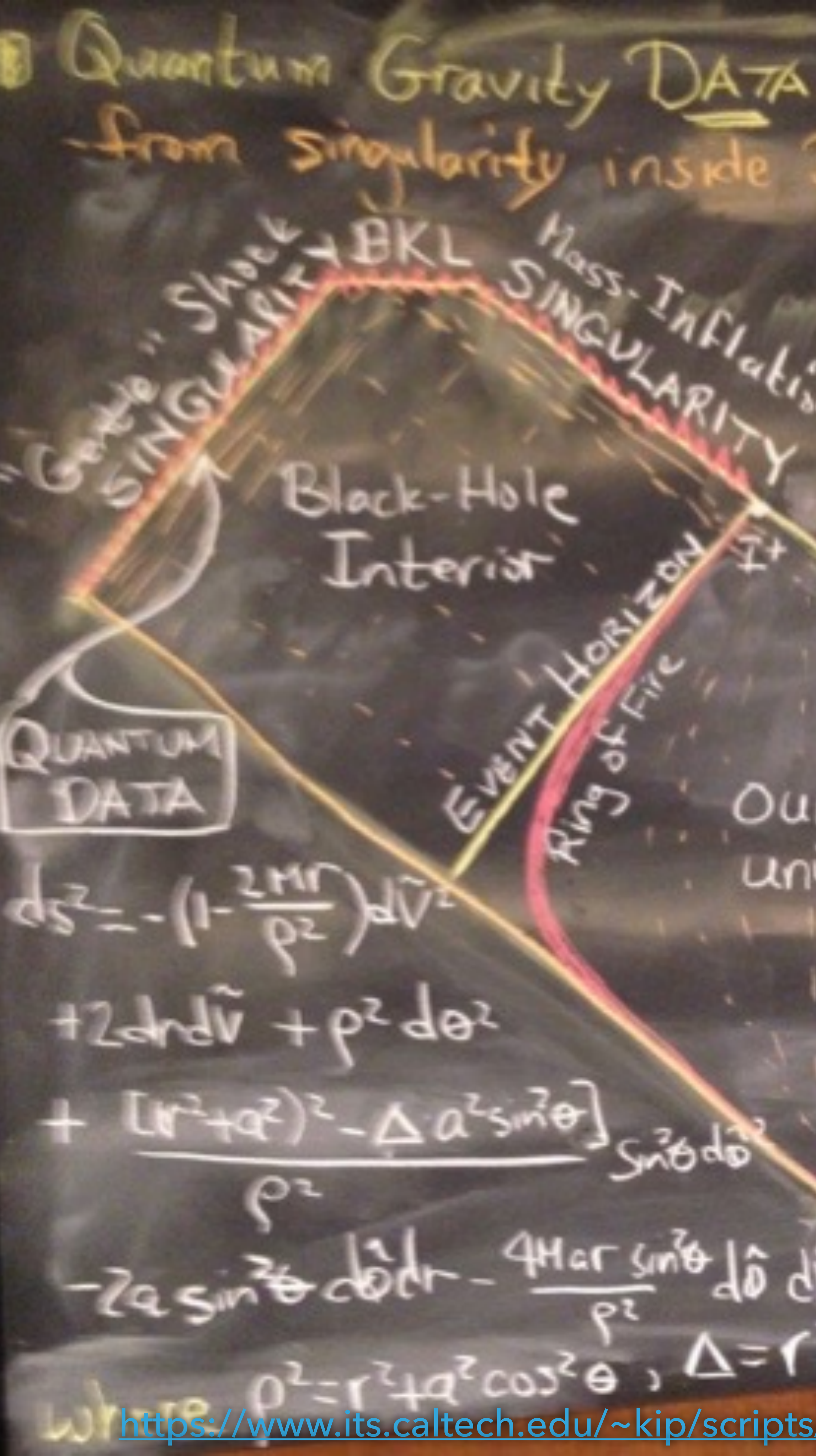
NEUTRON STAR STRUCTURE IN HORAVA-LIFSHITZ GRAVITY

http://www.nasa.gov/sites/default/files/pia18848-wisefacepalm.jpg

OVERVIEW

- Introduction to Horava-Lifshitz gravity
- Kehagias-Sfetsos black hole
- Derivation of TOV equation in HL gravity
- Numerical solutions of TOV in HL and discussion
- White dwarf
- Future work (including overview for parameter estimation in HL)
- Organization of contribution


[Image credit: ESA/Hubble]



(A BRIEF)

INTRODUCTION TO

HL GRAVITY

[Image: Interstellar - Professor Brand's Blackboards]

https://www.its.caltech.edu/~kip/scripts/INTERSTELLAR/BrandBlackBoards/blackboards.html

GENERAL RELATIVITY - SUCCESSFUL BUT INCOMPLETE

- General relativity has emerged as a highly successful model of gravitation and cosmology, which has so far passed many unambiguous observational and experimental tests.
- However, there are strong indications the theory is incomplete. [Maddox 1998, pp. 52–59, 98–122; Penrose 2004, sec. 34.1, ch. 30]
- The problem of quantum gravity and the question of the reality of spacetime singularities remain open.[Quantum Gravity Section]
- Observational data that is taken as evidence for dark energy and dark matter could indicate the need for new physics.[Cosmology Section]

https://en.wikipedia.org/wiki/General_relativity#Current_status

POWER COUNTING RENORMAILIZABILITY

- Considering a scalar field ϕ, its scaling dimension s can be read from the kinetic term $I_{\rm kin} = \frac{1}{2}\int dtd^3x\dot{\phi}^2 \to b^0 I_{\rm kin}$ as $1+3-2-2s=0$, where $t \to bt,\ x^i \to bx^i$ and $\phi \to b^{-s}\phi$.
- Then, the scaling dimension of a potential term $\int dtd^3x\phi^n$ is given by $d=-1-3+ns=n-4$, where the theory is renormalizable only if $d \le 0$ for all the potential terms.
- For an anisotropic scaling $t \to b^z t,\ x^i \to bx^i$, s and d become $s=(3-z)/2,\ d=-z-3+n(3-z)/2$ (≤ 0 for $z \ge 3$) .

TOWARDS UV COMPLETE THEORY OF GR

- Horava proposed a UV complete theory of GR by introducing the anisotropic scaling.
- In the IR limit, the Lorentz symmetry should be recovered.
- To deal with the anisotropic scaling, the ADM decomposition is adopted:

$$ds^2 = -N^2c^2dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt)\,.$$

KINETIC TERMS

- Foliation-Preserving Diffeomorphism:

$$\delta t = f(t), \qquad \delta x^i = \zeta^i(t, \vec{x})$$

- The extrinsic curvature transforms **covariantly** under the foliation-preserving diffeomorphism:

$$K_{ij} = \frac{1}{2N}\left[\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i\right].$$

- General form of **Kinetic terms** of HL gravity:

$$I_{\rm kin} = \frac{2}{\kappa^2}\int dt d^3x \sqrt{g} N \left[K_{ij}K^{ij} - \lambda K^2\right].$$

POTENTIAL TERMS

- z=3 potential terms:

$$\nabla R \nabla R, \qquad R \nabla \nabla R, \qquad RRR$$

- z=2 potential terms:

$$\nabla \nabla R, \qquad RR$$

- z=1 potential terms:

$$R$$

- z=0 potential terms:

$$1$$

HORAVA-LIFSHITZ GRAVITY

- "detailed balance" condition reduces 9 coefficients to 3:

$$I_{\text{pot}} = \frac{\kappa^2}{8} \int dt d^3x \sqrt{g} N E^{ij} \mathcal{G}_{ijk\ell} E^{k\ell}, \qquad \sqrt{g} E^{ij} = \frac{\delta W[g_{k\ell}]}{\delta g_{ij}},$$

$$W = \mu \int d^3x \sqrt{g} (R - 2\Lambda_W) + \frac{1}{\zeta^2} \int_\Sigma \mathrm{Tr} \left(\Gamma \wedge d\Gamma + \frac{2}{3} \Gamma \wedge \Gamma \wedge \Gamma \right),$$

where W is the three dimensional topologically massive gravity and $\mathcal{G}_{ijk\ell}$ is the inverse of the De Witt metric.

DEFORMED HORAVA-LIFSHITZ GRAVITY

- Deformed HL gravity (*softly* broken detailed balance):

$$I_{\mathrm{HL}} = \int dt d^3x \sqrt{g} N \left[\frac{2}{\kappa^2} \left(K_{ij}K^{ij} - \lambda K^2 \right) - \frac{\kappa^2}{2\zeta^4} \left(C_{ij} - \frac{\mu\zeta^2}{2} R_{ij} \right) \left(C^{ij} - \frac{\mu\zeta^2}{2} R^{ij} \right) \right.$$
$$\left. + \frac{\kappa^2\mu^2(4\lambda-1)}{32(3\lambda-1)} \left(R^2 + \frac{4(\omega - \Lambda_W)}{4\lambda-1} R + \frac{12\Lambda_W^2}{4\lambda-1} \right) \right],$$

where ω is introduced to include asymptotically flat solutions and C_{ij} is the Cotton-York tensor,

$$C^{ij} = \varepsilon^{ik\ell} \nabla_k \left(R^j_\ell - \frac{1}{4} R \delta^j_\ell \right).$$

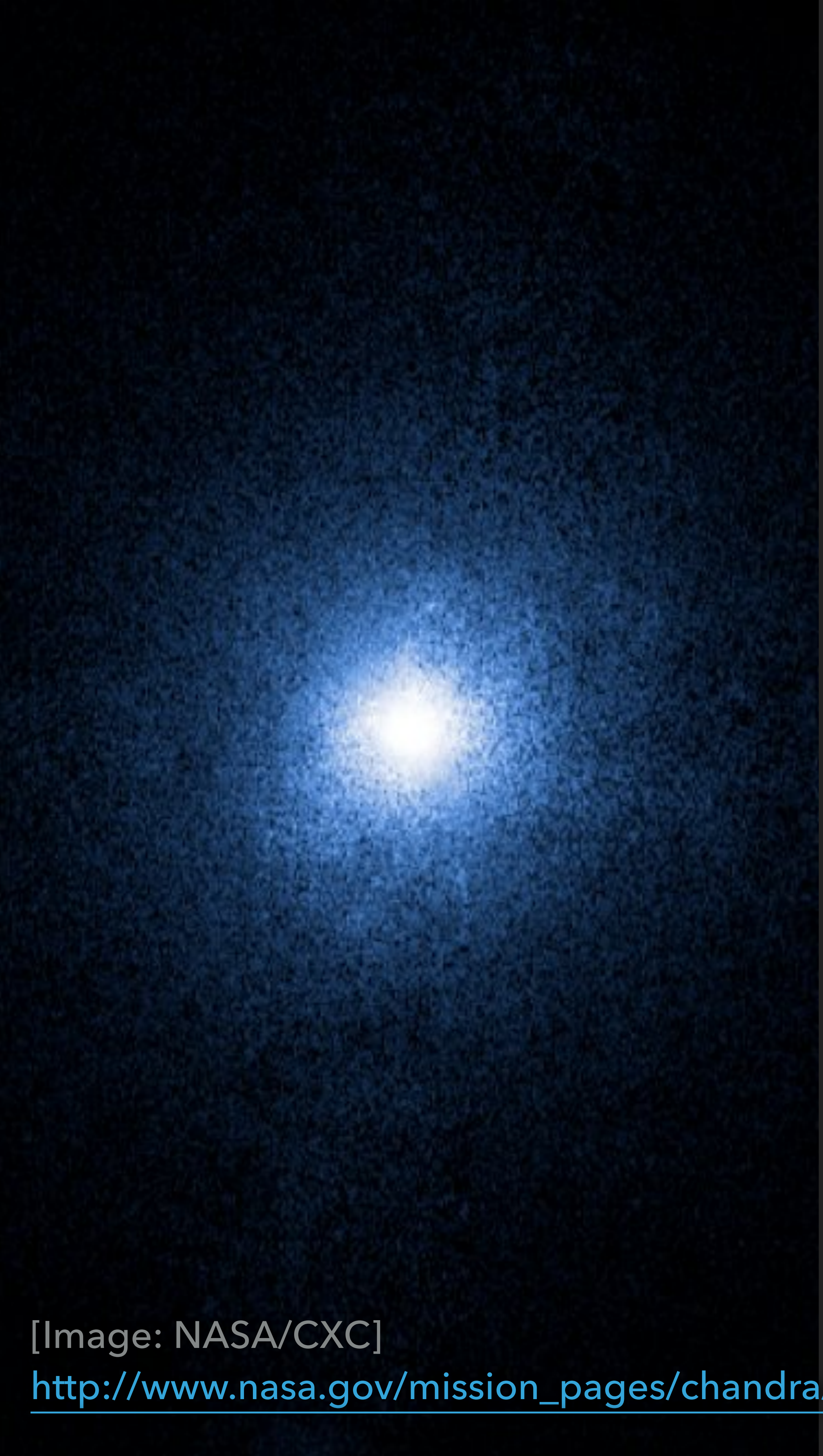
IR LIMIT OF DEFORMED HORAVA-LIFSHITZ GRAVITY

- In the IR limit, GR is recovered with λ=1:

$$I_{\mathrm{EH}} = \frac{1}{16\pi G_N}\int d^4x\sqrt{g}N\left[(K_{ij}K^{ij} - K^2) + R - 2\Lambda\right],$$

where the fundamental coefficients are identified as

$$c = \frac{\kappa^2}{4}\sqrt{\frac{\mu^2(\omega - \Lambda_W)}{3\lambda - 1}},\ G_N = \frac{\kappa^2 c^2}{32\pi},\ \Lambda = -\frac{3\Lambda_W^2}{2(\omega - \Lambda_W)}.$$



SPHERICALLY SYMMETRIC SOLUTION

KEHAGIAS-SFETSOS BH

[Image: NASA/CXC]
http://www.nasa.gov/mission_pages/chandra/multimedia/photo09-065.html

KEHAGIAS-SFETSOS BLACK HOLE

- Spherically symmetric metric ansatz:

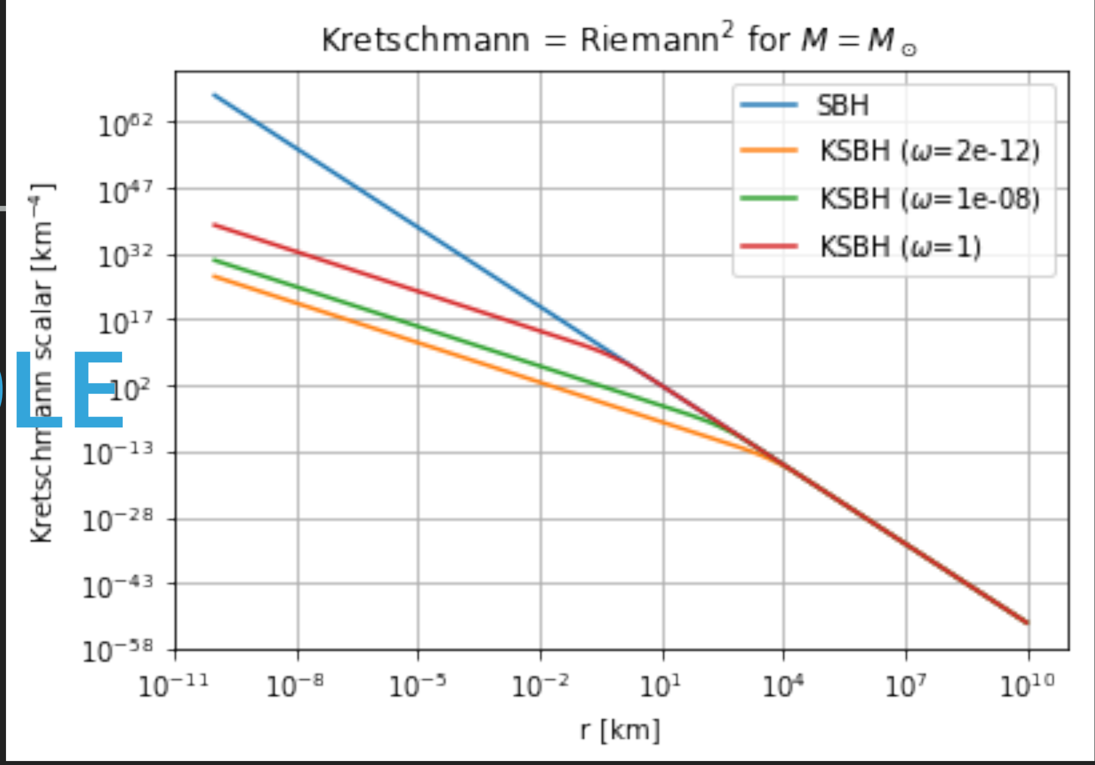
$$ds^2 = -N^2(r)dt^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\,.$$

- Asymptotically flat (Λ_W=0) vacuum solution with λ=1:

$$N^2 = f = 1 + r^2\omega - \sqrt{r\omega(r^3\omega + 4M)}$$

$$\approx 1 - \frac{2M}{r} + O\left(\frac{M^2}{r^4\omega}\right) \text{ for } r^2\omega \gg \frac{M}{r}\,.$$

KEHAGIAS-SFETSOS BLACK HOLE



- Kretschmann curvature scalar:

$$K = R_{\alpha\beta\gamma\sigma}R^{\alpha\beta\gamma\sigma} \xrightarrow{r \ll r_c} \frac{81M\omega}{4r^3}\left[1 + O(r/r_c)^{3/2}\right],$$

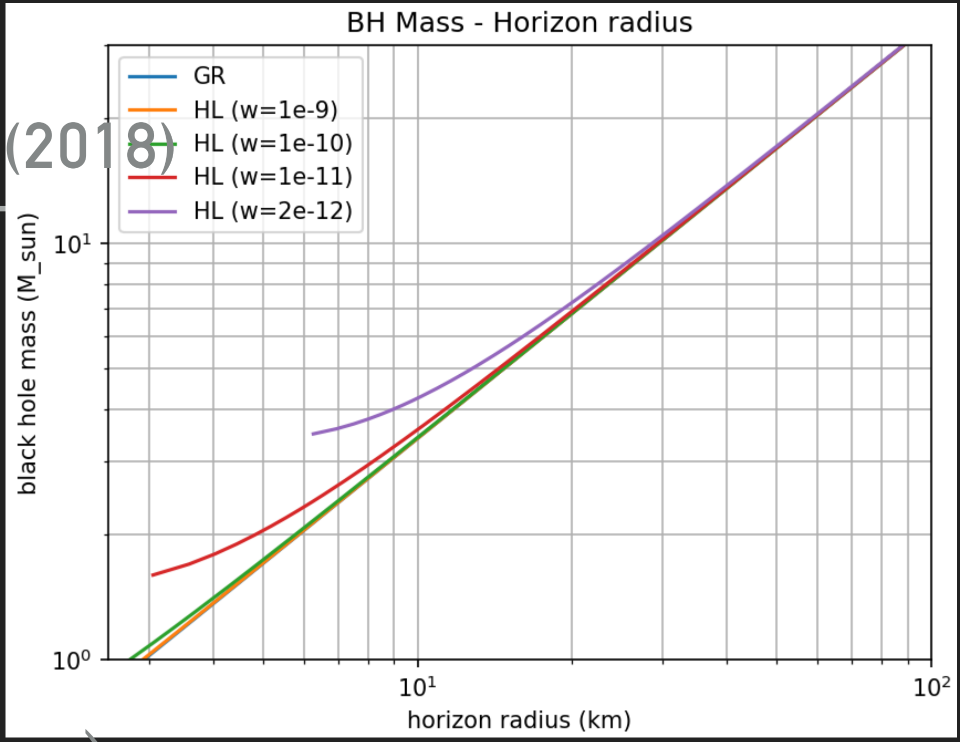
$$\xrightarrow{r \gg r_c} \frac{48M^2}{r^6}\left[1 + O(r_c/r)^3\right],$$

where $r_c = (4M\omega^{-1})^{1/3}$.

- Note that the singularity at the origin is milder than that of Schwarzschild BH:

$$K_{\rm Sch} = \frac{48M^2}{r^6}.$$

KEHAGIAS-SFETSOS BLACK HOLE



- Horizons:

$$r_{\pm} = M\left(1 \pm \sqrt{1-(2\omega M^2)^{-1}}\right).$$

- It looks like there exists a naked singularity for $M < \sqrt{2\omega}$.
- However, it is shown that the naked singularity is non-singular to the quantum fields.

CONSTRAINTS ON KS IN DHL

$$\tilde{\omega} = \omega M^2$$

- The light deflections by the Sun, the Jupiter and the Earth systems constrains [M. Liu et al., GRG (2011)]

$$\tilde{\omega}_{\odot} > 8.27650 \times 10^{-15},\ \tilde{\omega}_{\zeta} > 8.27649 \times 10^{-17},\ \tilde{\omega}_{\oplus} > 1.1725 \times 10^{-16}.$$

- The range-residual of the Mercury in the solar system and the orbital motion of the S2 star around the supermassive black hole of mass $4 \times 10^6 M_{\odot}$ at the center of the Milky Way constrain ω by [L. Iorio et al., IJMPD (2011)]

$$\tilde{\omega}_{\odot} > 7.2 \times 10^{-10}, \qquad \tilde{\omega}_{\S} > 8 \times 10^{-10}$$

- The weak lensing by galaxies of mass $\sim 10^{10} M_{\odot}$ reads [Z. Horváth et al., PRD (2011)]

$$\omega \gtrsim 10^{-48}\ \text{cm}^{-2}\ (\tilde{\omega} \gtrsim 10^{-16})$$

- These low bounds are converted to $\omega_{\text{min}} \sim 10^{-48} - 10^{-16}\ \text{cm}^{-2}$.

[X-ray: NASA/CXC/Univ. of Wisconsin-Madison/S. Heinz, et al.; Optical: DSS]
http://chandra.si.edu/photo/2015/cirx1/

DERIVING AND SOLVING

TOV EQUATION

EQUATIONS OF MOTION IN DHL

- Starting with $I_{tot}=I_{HL}+I_{mat}$, where I_{mat} is the matter action of a perfect fluid and will be specified by choosing a EoS.
- A static, spherically symmetric metric ansatz:

$$ds^2 = -e^{2\Phi(r)}c^2dt^2 + \frac{dr^2}{1-f(r)} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right).$$

- Equations of motion:

$$\rho = \frac{c^2}{16\pi G_N r^2\omega}\left(2r\omega f + \frac{f^2}{r}\right)',$$

$$p = \frac{c^4}{16\pi G_N r^4\omega}\left[f\left(f-2r^2\omega\right) + 4r\left(1-f\right)\left(f+r^2\omega\right)\Phi'\right],$$

$$p' = -\left(\rho c^2 + p\right)\Phi'.$$

TOLMAN-OPPENHEIMER-VOLKOFF EQUATION IN DHL

- The function f is solved as

$$f = -r^2\omega + \sqrt{r\omega(r^3\omega + 4G_N c^{-2} m)}, \qquad m(r) = \int_0^r dr' 4\pi r'^2 \rho(r') \,.$$

- TOV equation in DHL:

$$m' = 4\pi r^2 \rho,$$

$$p' = \frac{(\rho c^2 + p) r\omega \left[\left(1 + G_N c^{-2}\omega^{-1} m r^{-3}\right) - \sqrt{1 + 4G_N c^{-2}\omega^{-1} m r^{-3}} - 4\pi G_N c^{-4}\omega^{-1} p \right]}{\sqrt{1 + 4G_N c^{-2}\omega^{-1} m r^{-3}} \left[1 + r^2\omega \left(1 - \sqrt{1 + 4G_N c^{-2}\omega^{-1} m r^{-3}} \right) \right]},$$

$$p' = -\left(\rho c^2 + p\right) \Phi' .$$

We can solve TOV equation by considering a specific equation-of-state.

SELECTED EQUATION-OF-STATES

- We select four EoS models which cover the observed maximum mass, ~ $2M_\odot$ [P. B. Demorest et al., Nature (2010); M. Linares et al., ApJ (2018)]
- APR4: derived by variational method but with a specific nucleon potential model [A. Akmal et al., PRC (1998)]
- MPA1: derived by relativistic Brueckner-Hartree-Fock theory [H. Müther et al., PLB (1987)]
- MS1: derived by relativistic mean field theory [H. Müller et al., NPA (1996)]
- WFF1: derived by variational method but with a specific nucleon potential model [R. B. Wiringa et al., PRC (1988)]
- For the crust structure, we impose Skyrme-Lyon model.[F. Douchin et al., A&A (2001)]

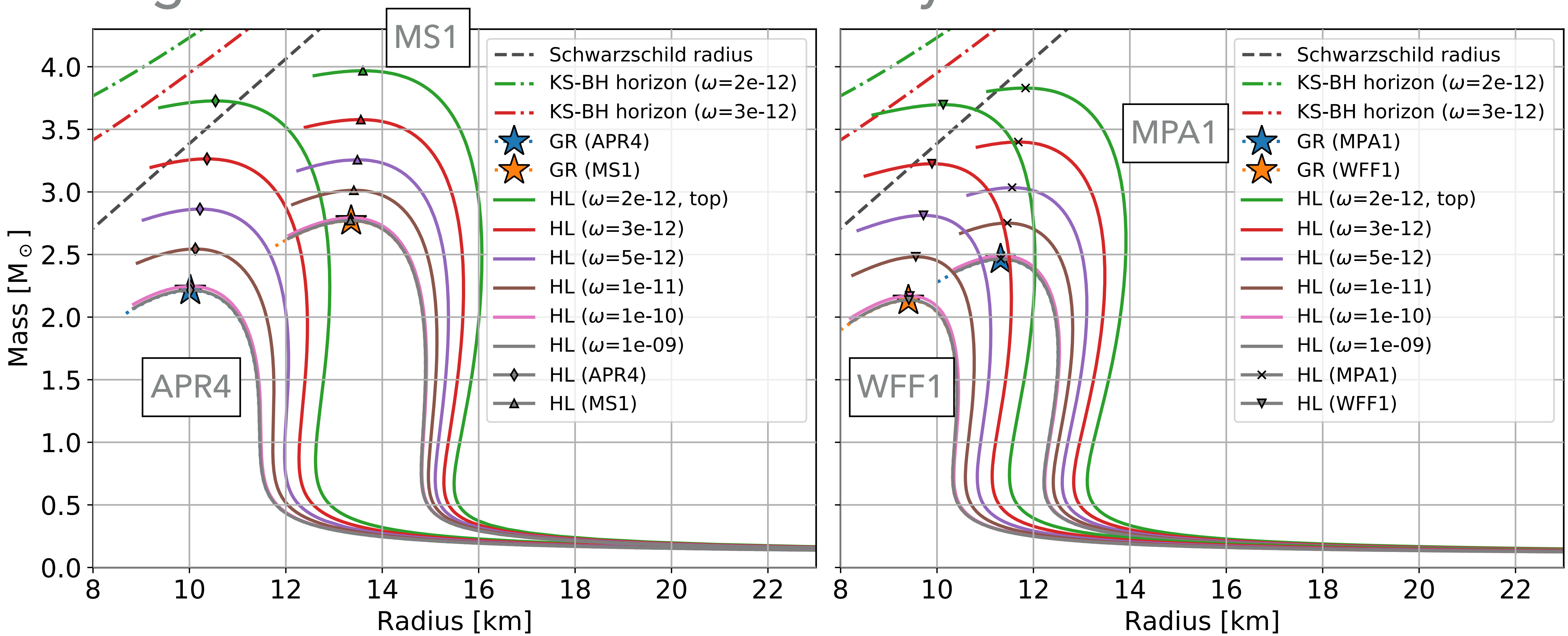
SOLVING TOV NUMERICALLY

- In order to solve the TOV equation numerically, we adopt 5th order Runge-Kutta method with 4th order error control. [J. Dormand and P. Prince, J. of Comp. and Applied Math. 6, 19 (1980); L. F. Shampine, Math. of Comp. 46, 135 (1986)]
- Also, cross-checked with the result solved by separately built 4th order Runge-Kutta method.
- The radius R of a neutron star is determined by p(R)=0.

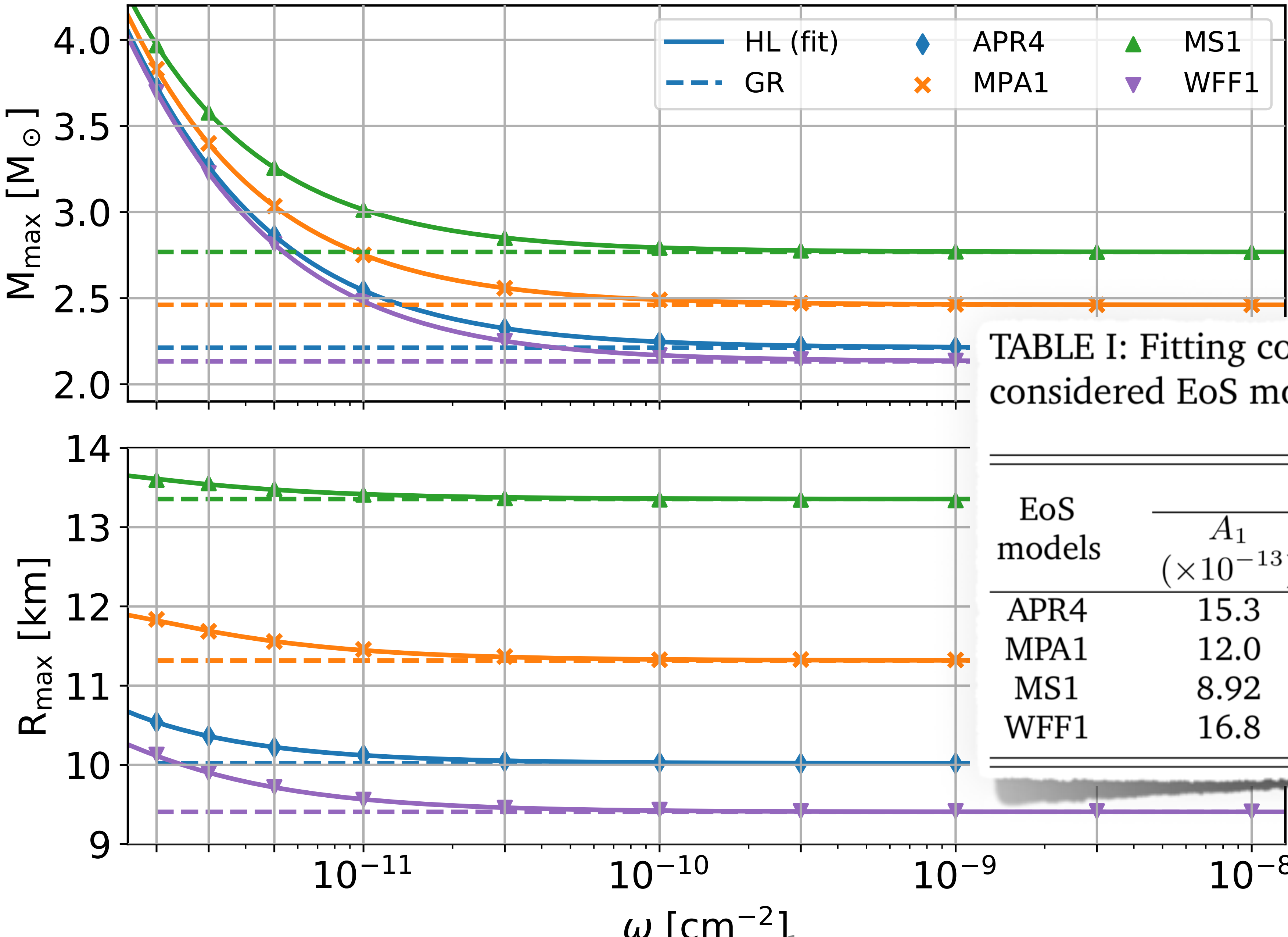
RESULTS

MASS-RADIUS RELATION WITH SELECTED EOS

- Both masses and radii becomes larger in HL than in GR.
- Larger deformation is observed by smaller ω.



MASSES AND RADII OF HEAVIEST NEUTRON STARS



$$M_{\max}^{\rm HL} \approx M_{\max}^{\rm GR}\left[1+\left(A_1\omega^{-1}+A_2\omega^{-2}\right)\right]$$

$$R_{\max}^{\rm HL} \approx R_{\max}^{\rm GR}\left[1+\left(B_1\omega^{-1}+B_2\omega^{-2}\right)\right]$$

TABLE I: Fitting coefficients for $M_{\max}^{\rm HL}$ and $R_{\max}^{\rm HL}$ with all considered EoS models.

EoS models	Fitting Coefficients			
	A_1 ($\times 10^{-13}$)	A_2 ($\times 10^{-26}$)	B_1 ($\times 10^{-14}$)	B_2 ($\times 10^{-27}$)
APR4	15.3	-33.0	10.2	4.17
MPA1	12.0	-16.7	12.0	-60.7
MS1	8.92	-4.78	4.87	-21.2
WFF1	16.8	-42.3	17.4	-46.1

$$Q = M_{\max}, R_{\max}$$

If we require $\dfrac{|\Delta Q|}{Q_{\rm GR}} \lesssim \begin{cases} 10^{-3}, \\ 10^{-2}, \end{cases}$ then $\omega \gtrsim \begin{cases} 1.68\times 10^{-9}\ \text{cm}^{-2}, \\ 1.68\times 10^{-10}\ \text{cm}^{-2}. \end{cases}$

REASONING FOR THE LARGER MASSES AND RADII

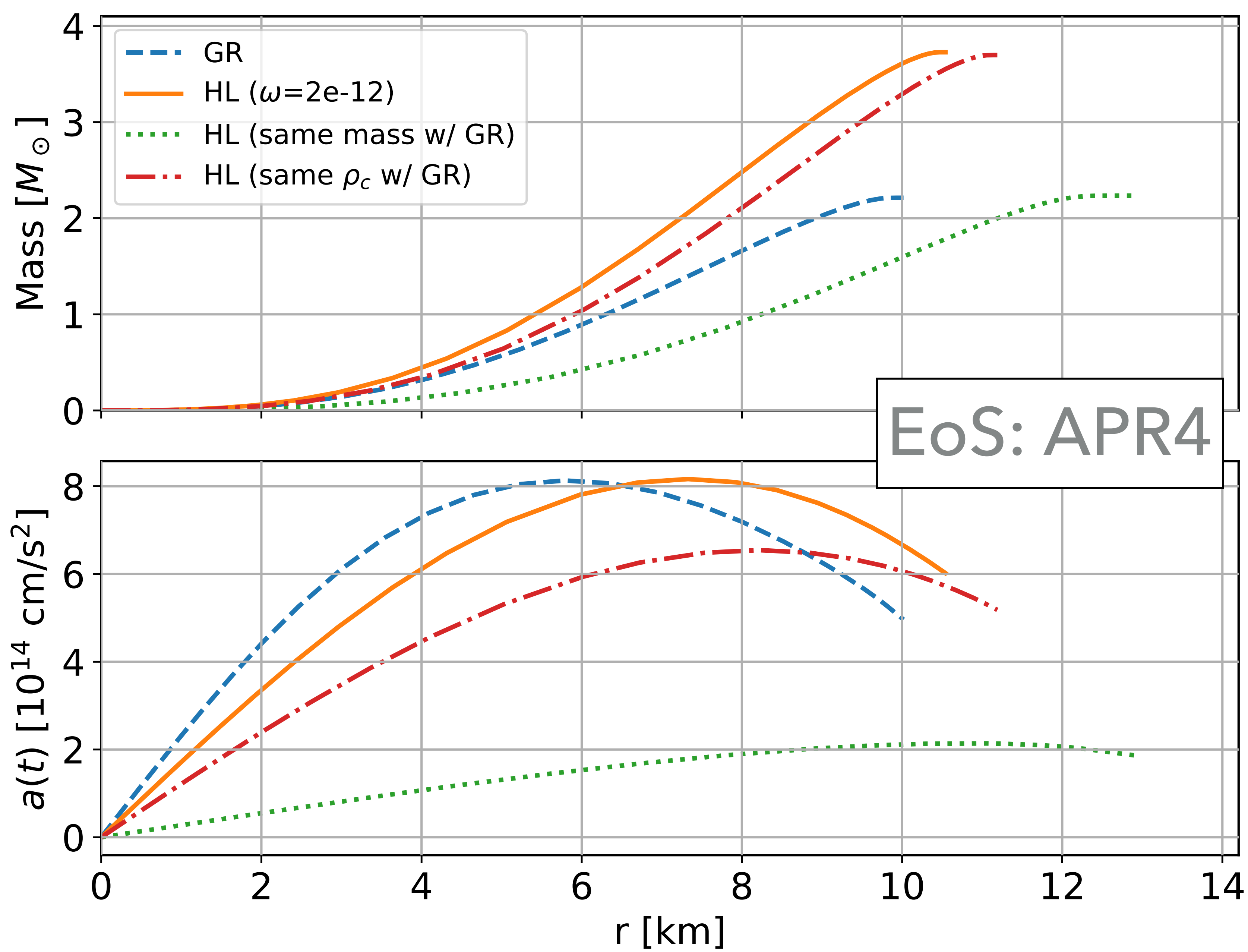
- Comparing the same mass NSs in HL and GR, the radius of NS in HL is larger than that in GR.
  ☞ Larger radius indicates weaker attraction.
- Comparing the heaviest NSs in HL and GR, the mass of NS in HL is larger than that in GR.
  ☞ To balance with the fermion degeneracy pressure, HL needs to confine more masses due to the weaker attraction.
- To show explicitly the weaker attraction of HL, we find the acceleration felt by an observer at a fixed location inside a NS:

$$a(r) = \sqrt{1 - f(r)}\Phi'(r)\,.$$

ACCELERATION INSIDE NEUTRON STARS

- We observe that the acceleration near the center of NS is smaller in HL than in GR.
- Even for the heavier NSs in HL, the acceleration near the center is smaller.

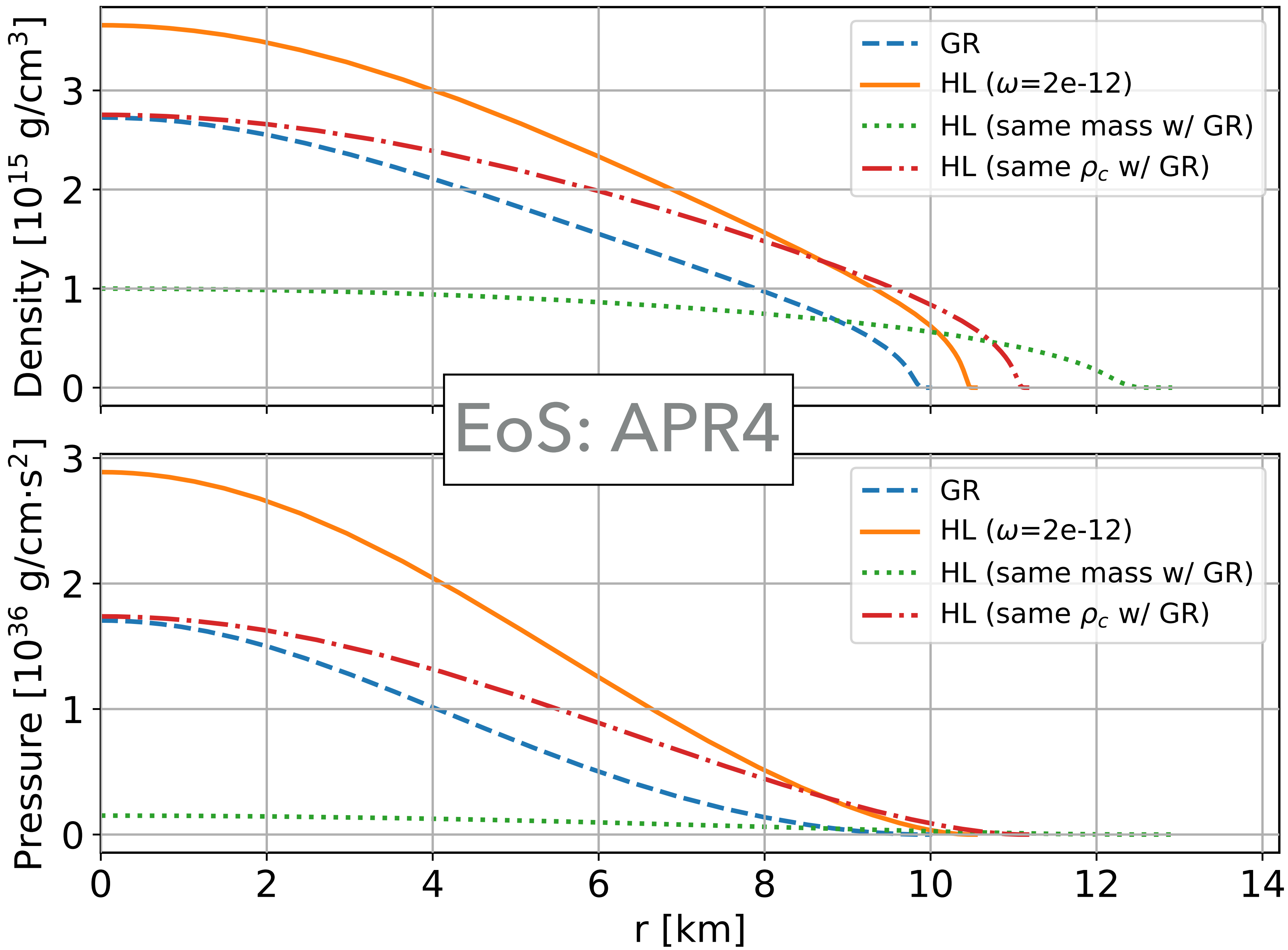
$\omega = 2 \times 10^{-12}$ cm^{-2}



DENSITY AND PRESSURE INSIDE NEUTRON STARS

$$\omega = 2 \times 10^{-12}\ \mathrm{cm}^{-2}$$

- The heaviest NS in HL has larger central density and pressure than the heaviest NS in GR.
- Comparing NSs with the same central density, the density and pressure decreases more slowly in HL than in GR.



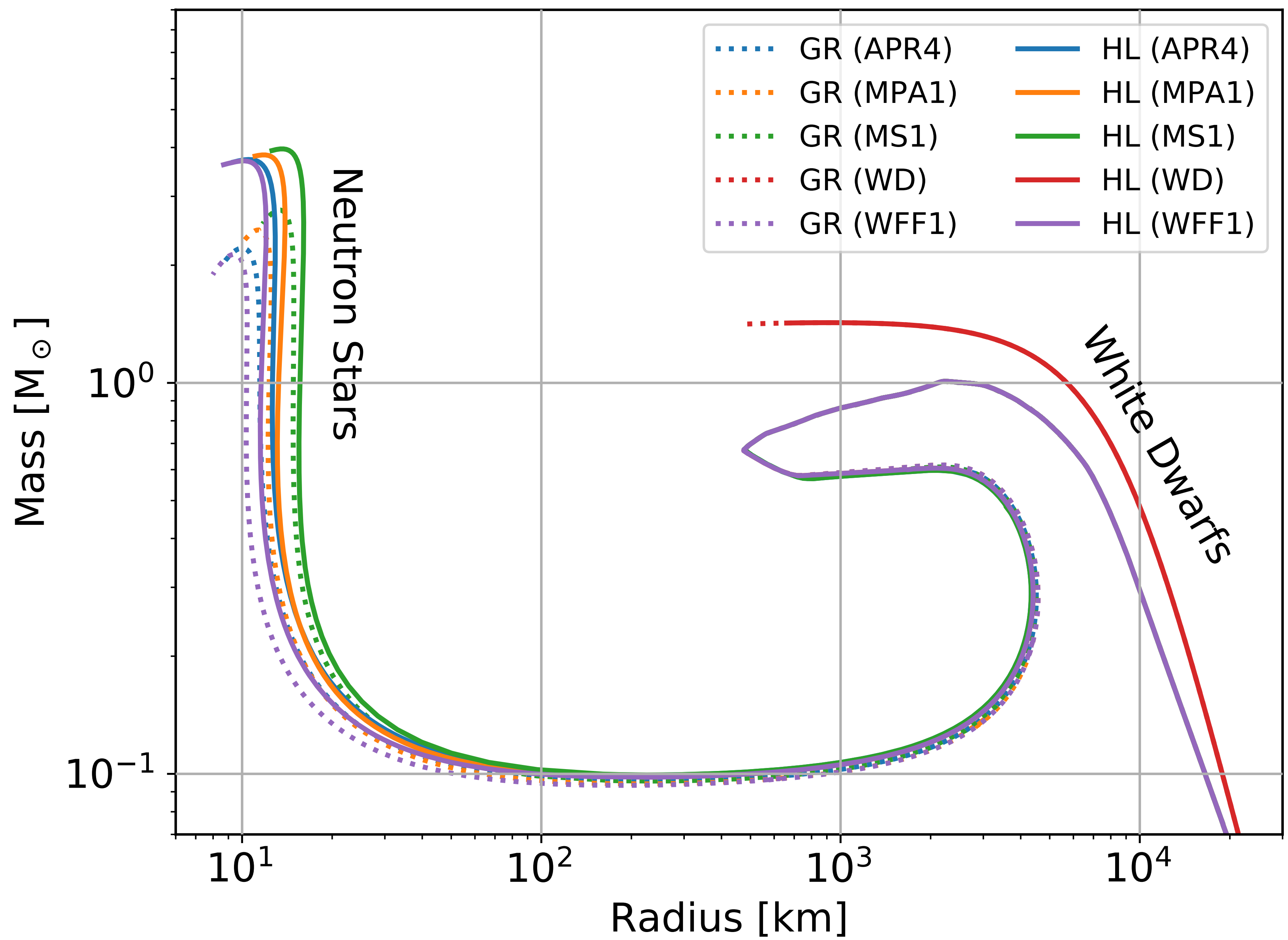
DISCUSSION

SOME REMARKS

- We observe that the gravity in a short scale becomes weaker in HL than in GR.
- Relatively weaker gravity in HL naturally makes NS larger: To balance with the fermion degeneracy pressure, HL gravity must confine more mass than GR.
- In the literature, ω is constrained by $\omega_{min} \sim 10^{-16}$ cm^{-2}, which is far below from our consideration.
- To constrain ω by NS structure, we need accurate observational data of masses and radii of NSs.

WHITE DWARF

- The deviation of mass and radius of a white dwarf is too small to observe.
- The relativistic EoS for white dwarfs is obtained by assuming that the ratio between the atomic mass number A and the number of electrons Z is given by A/Z = 2 [I. Sagert et al., EJP (2006)]



FUTURE WORKS

- A study on the tidal deformation in HL is ongoing.
- The estimation of mass and radius of NS from GW170817 was based on the framework of GR.[B. P. Abbott et al., PRL (2018)]
- We need to estimate them in the framework of HL to compare them with our results.
- We will start with GW waveform in HL gravity.