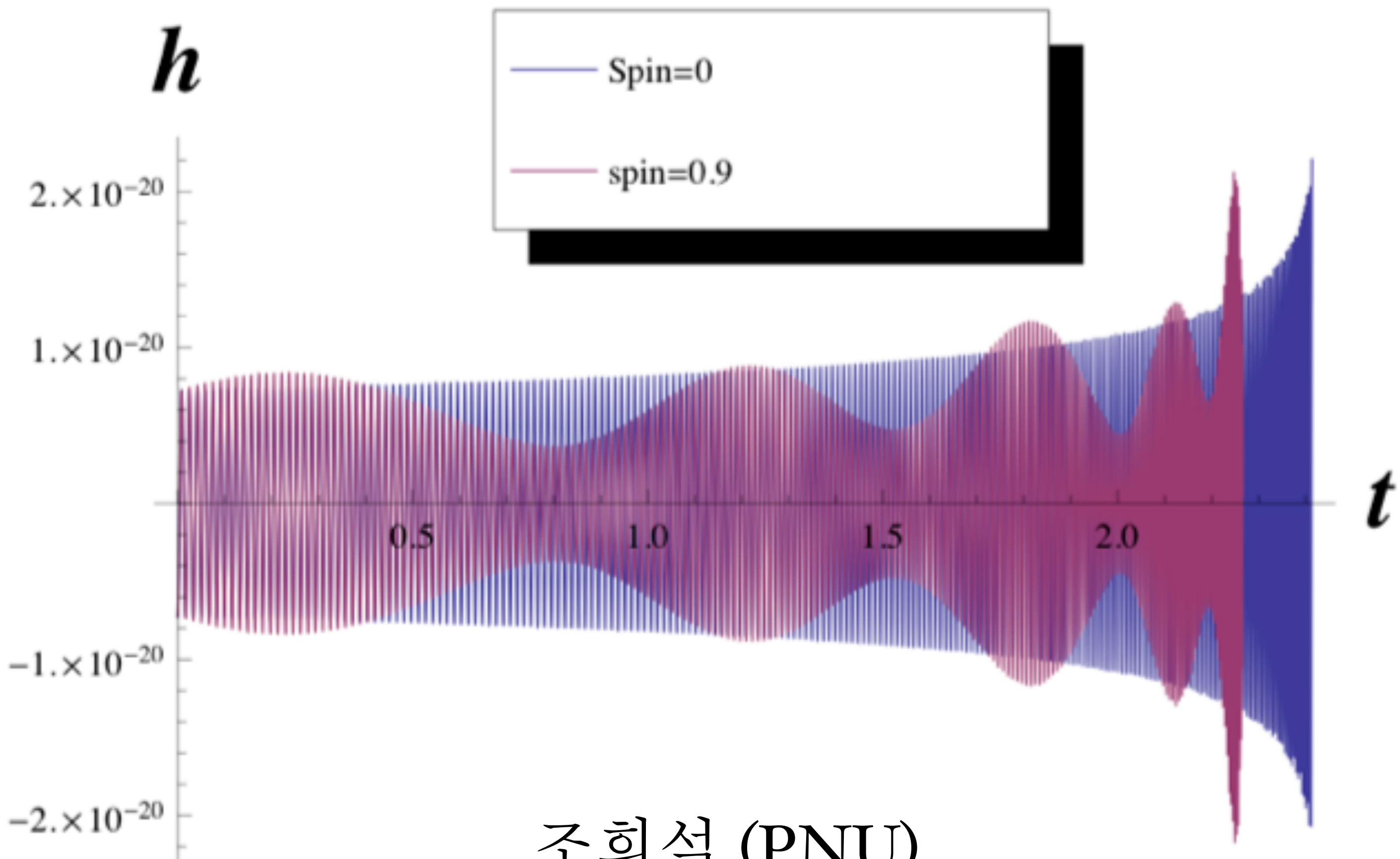
Gravitational waveform modulation from binary black hole spin precession



조희석 (PNU)

3rd GW&NR seminar (20180912 SNU)

Outline

Nonspinning waveforms

Precessing waveforms : waveform modulation

GW data analysis

GW polarizations: nonspinning

$$h_+ = -\frac{2G\mu}{c^2 R} x(1+\cos^2\iota)\cos 2\psi$$

$$h_\times = -\frac{2G\mu}{c^2 R} x(2\cos\iota)\sin 2\psi$$

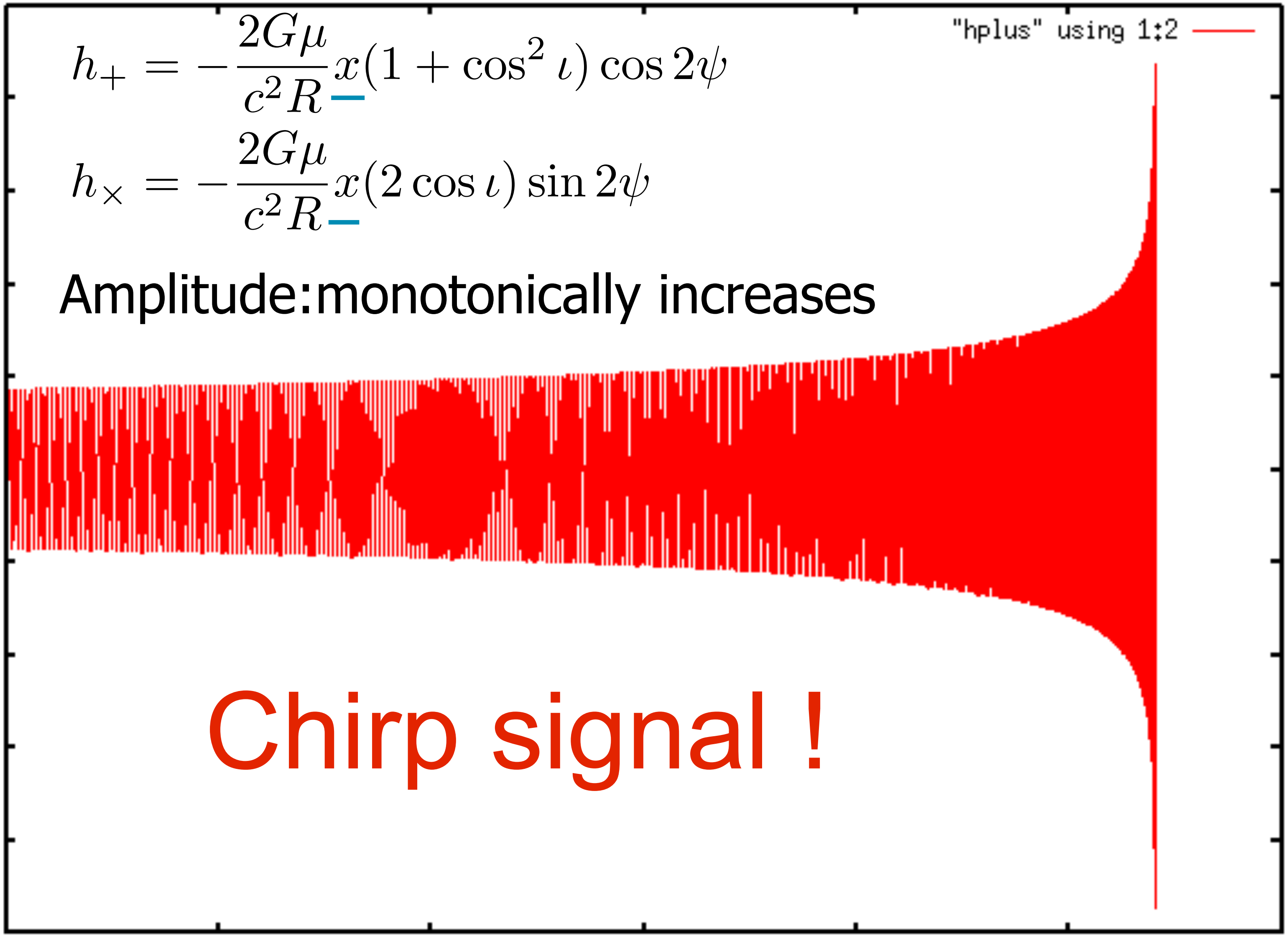
Orbital phase

Inclination

frequency function

$$x \equiv \left(\frac{G\,m\,\omega}{c^3}\right)^{2/3}$$

Waveform: nonspinning



$$h_+ = -\frac{2G\mu}{c^2 R}x(1+\cos^2\iota)\cos 2\psi$$

$$h_\times = -\frac{2G\mu}{c^2 R}x(2\cos\iota)\sin 2\psi$$

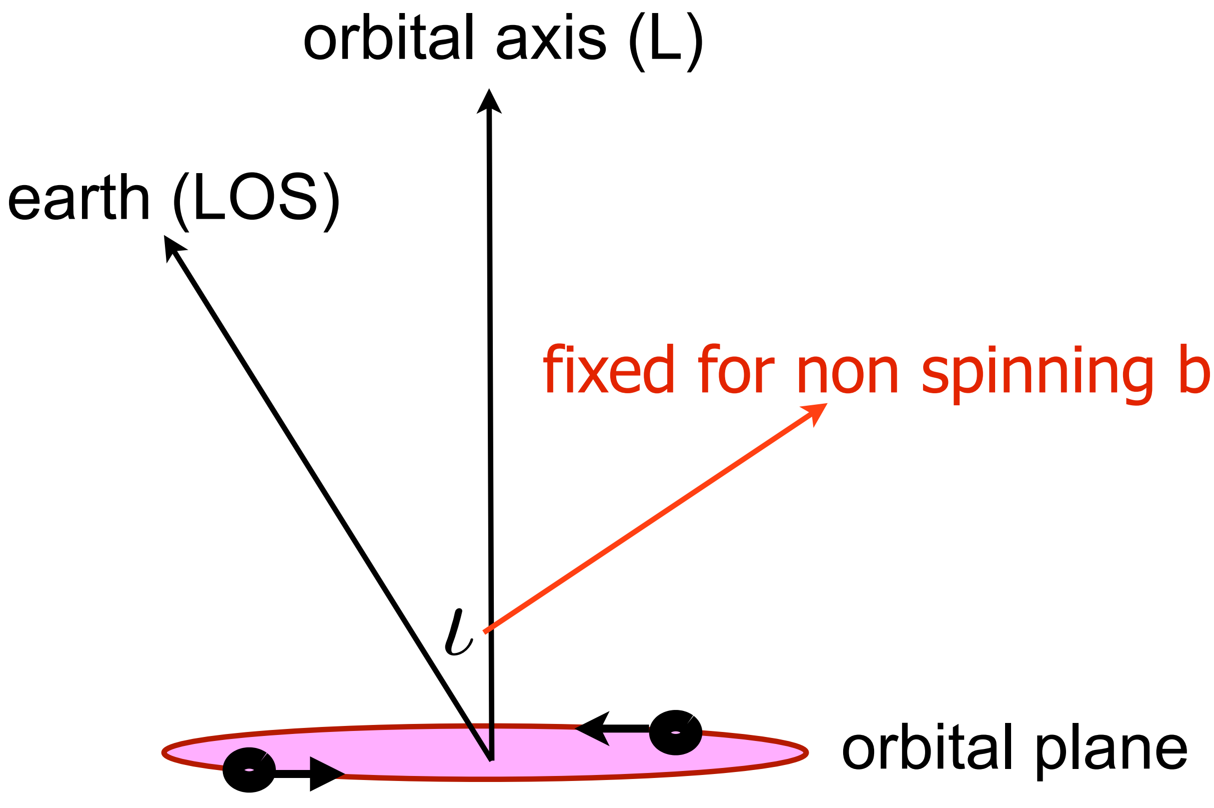
Amplitude:monotonically increases

Chirp signal !

Inclination

$$h_+ = -\frac{2G\mu}{c^2 R} x(1 + \cos^2 \underline{\iota}) \cos 2\psi$$

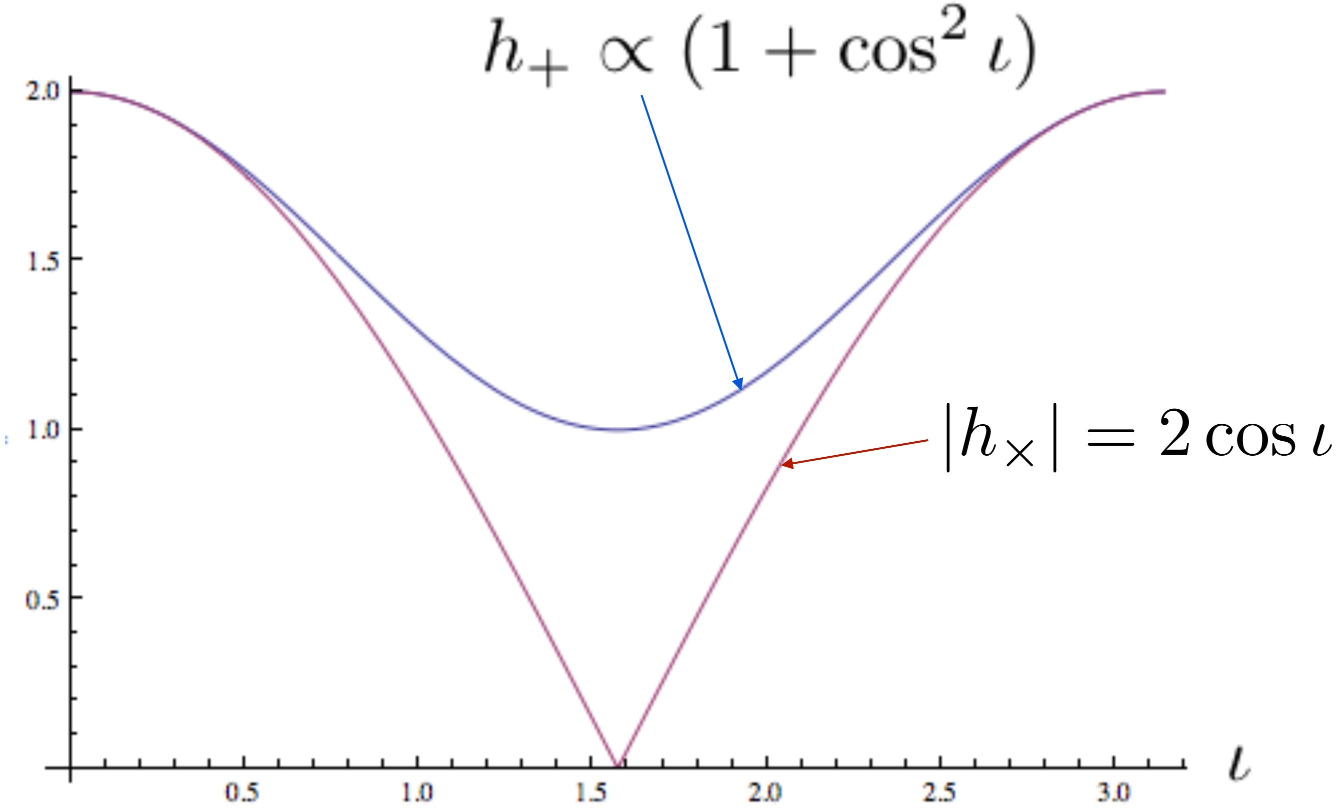
$$h_\times = -\frac{2G\mu}{c^2 R} x(2 \cos \underline{\iota}) \sin 2\psi$$



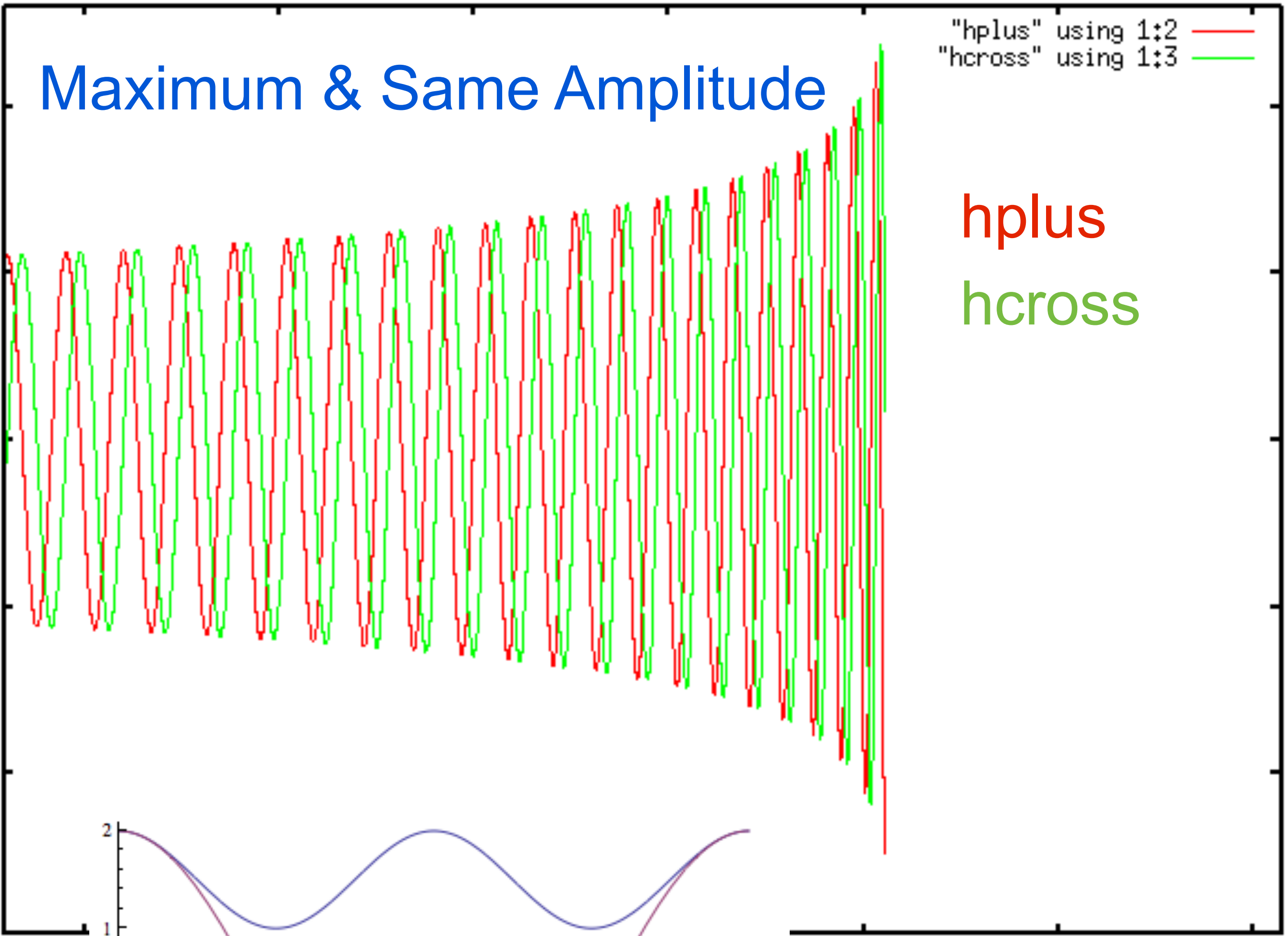
Dependence on inclination

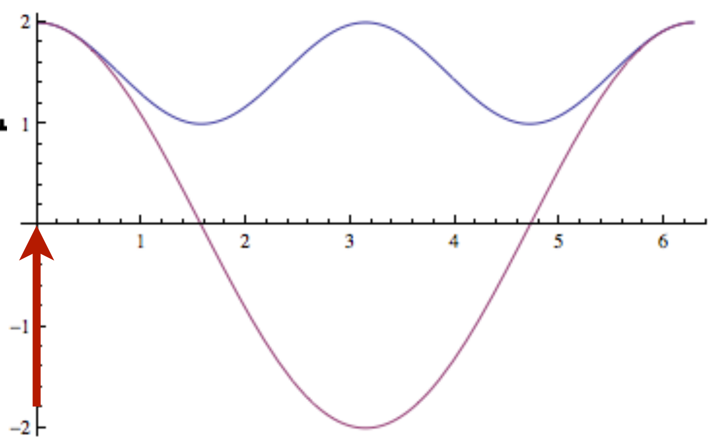
$$h_+ = -\frac{2G\mu}{c^2 R} x(1 + \cos^2 \iota) \cos 2\psi$$

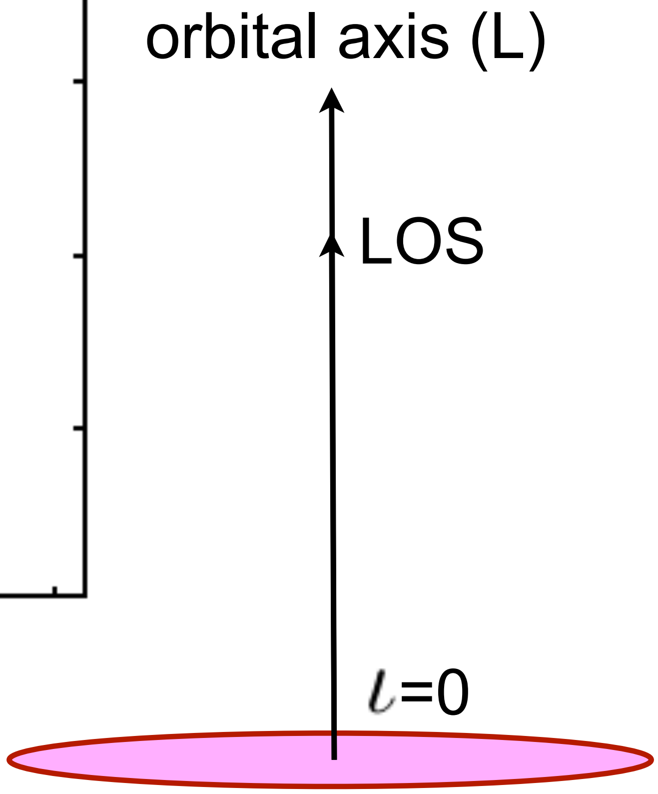
$$h_\times = -\frac{2G\mu}{c^2 R} x(2 \cos \iota) \sin 2\psi$$



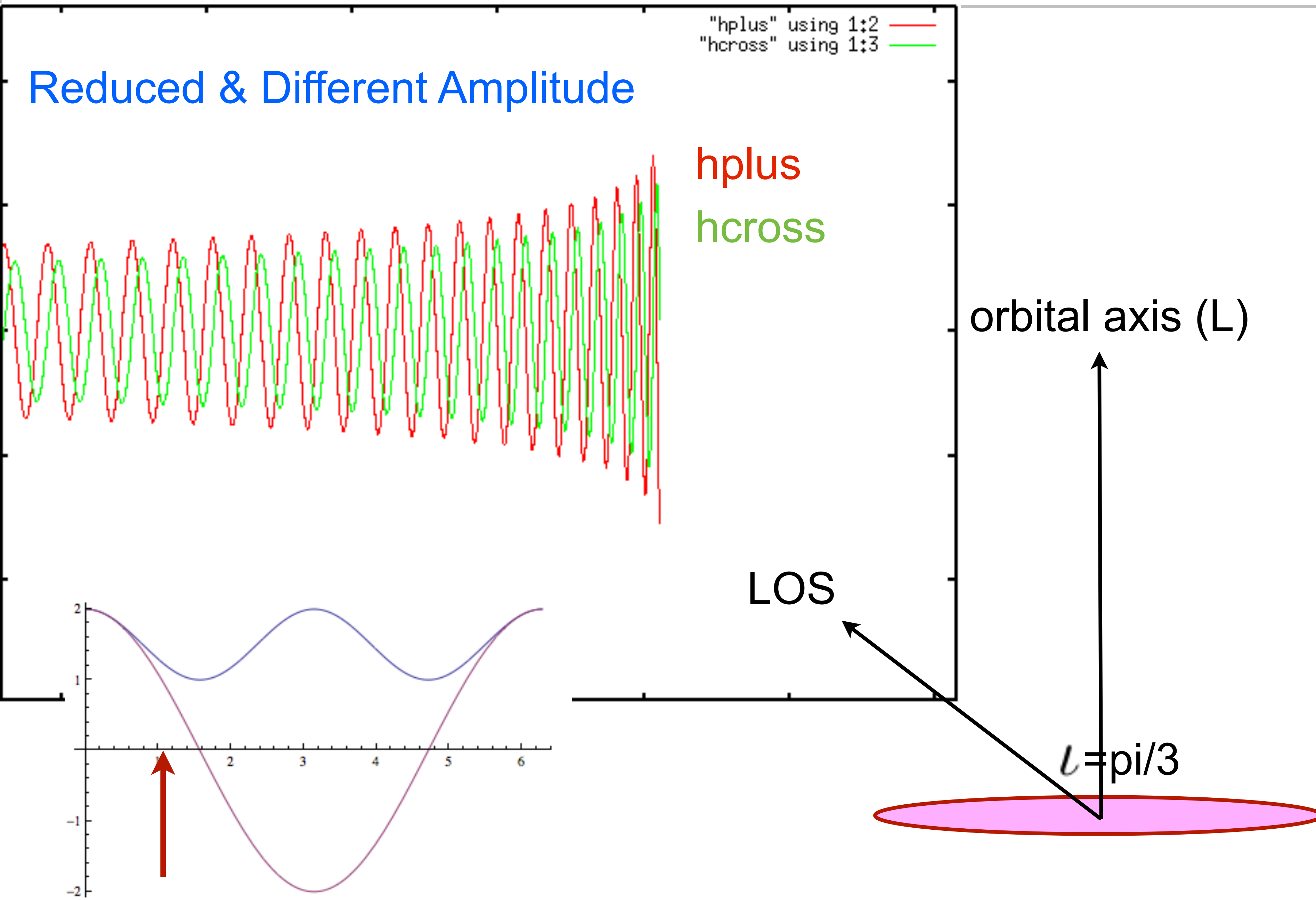
Nonspinning GW polarizations (iota=0)







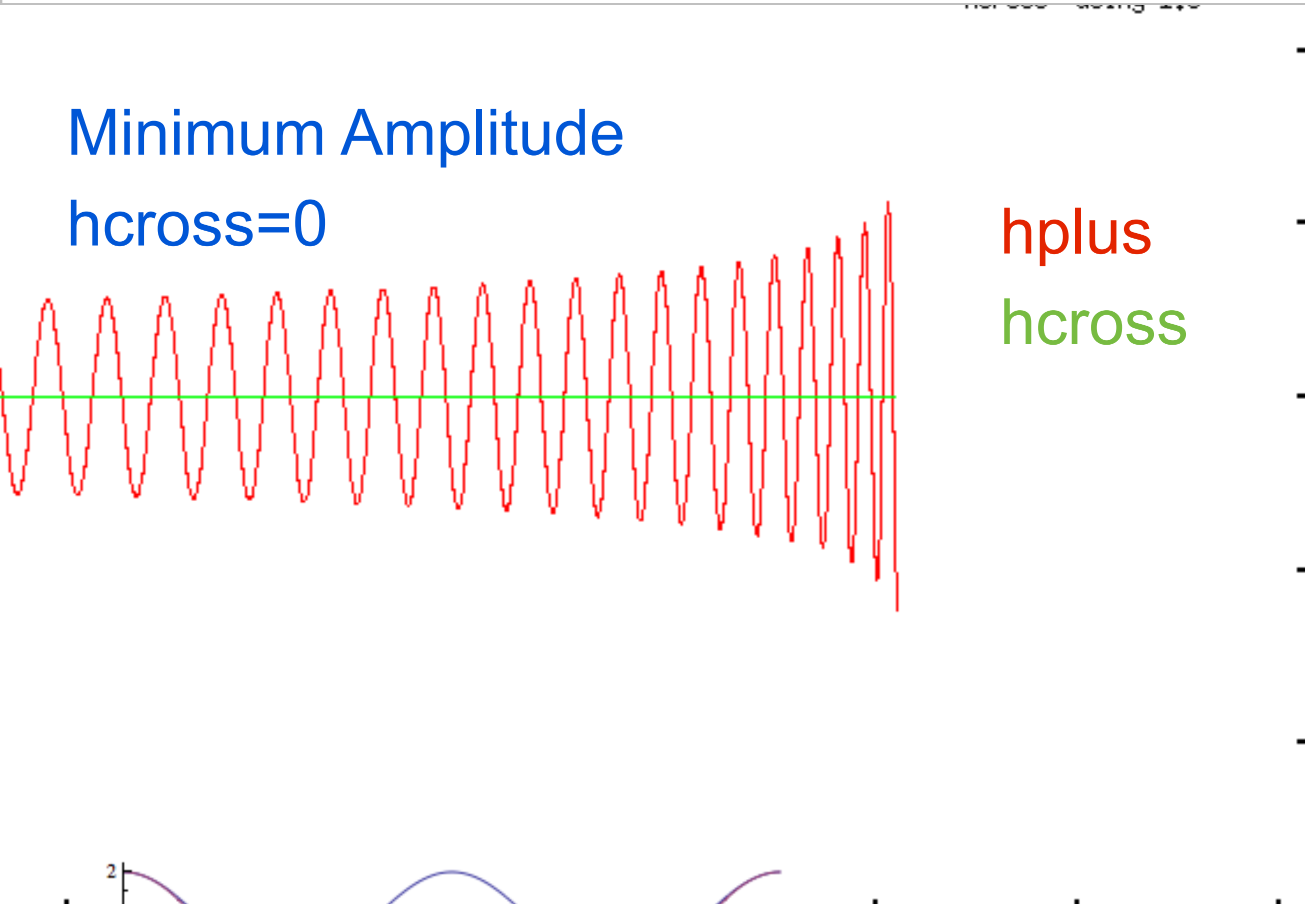
Non spinning polarizations (iota=pi/3)



Non spinning polarizations (iota=pi/2)

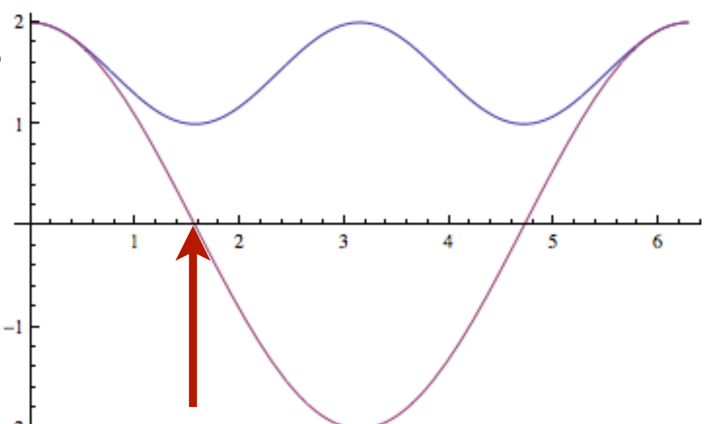
Minimum Amplitude

hcross=0

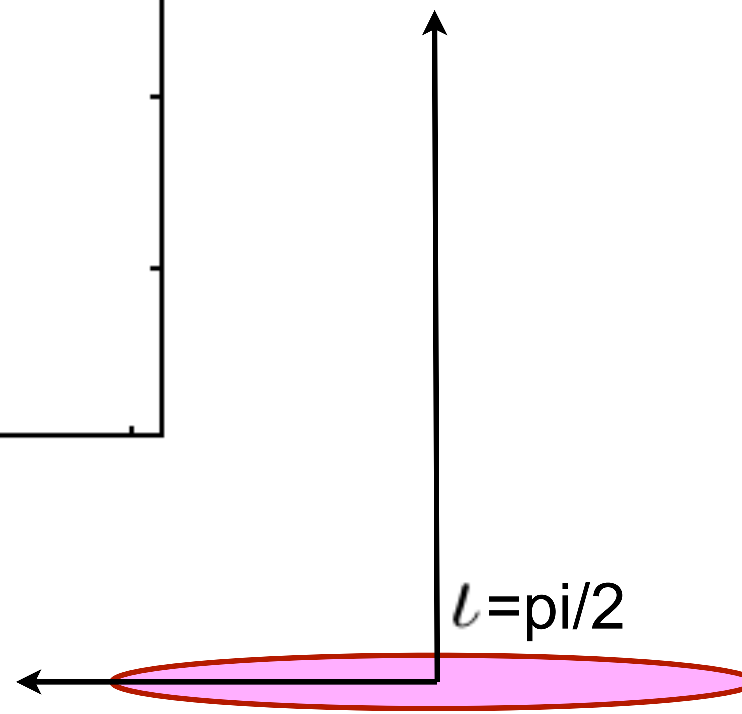


hplus

hcross



orbital axis (L)



GW polarization: nonspinning

$$h_+ = -\frac{2G\mu}{c^2 R} x (1 + \cos^2 \iota) \cos 2\underline{\psi}$$

$$h_\times = -\frac{2G\mu}{c^2 R} x (2 \cos \iota) \sin 2\underline{\psi}$$

Phase evolution is most important !

Post Newtonian Energy & Flux

$$E = -\underline{\frac{\mu c^2 x}{2}}\left\{1 + \left(-\frac{3}{4} - \frac{1}{12}\nu\right)x + \left(-\frac{27}{8} + \frac{19}{8}\nu - \frac{1}{24}\nu^2\right)x^2 + \left(-\frac{675}{64} + \left[\frac{34445}{576} - \frac{205}{96}\pi^2\right]\nu - \frac{155}{96}\nu^2 - \frac{35}{5184}\nu^3\right)x^3\right\} + \mathcal{O}\left(\frac{1}{c^8}\right).$$

Newtonian binding energy of a binary

$$\mathcal{L} = \frac{32c^5}{5G}\nu^2 x^5\left\{1 + \left(-\frac{1247}{336} - \frac{35}{12}\nu\right)x + 4\pi x^{3/2} + \left(-\frac{44711}{9072} + \frac{9271}{504}\nu + \frac{65}{18}\nu^2\right)x^2 + \left(-\frac{8191}{672} - \frac{583}{24}\nu\right)\pi x^{5/2} + \left[\frac{6643739519}{69854400} + \frac{16}{3}\pi^2 - \frac{1712}{105}C - \frac{856}{105}\ln(16x) + \left(-\frac{134543}{7776} + \frac{41}{48}\pi^2\right)\nu - \frac{94403}{3024}\nu^2 - \frac{775}{324}\nu^3\right]x^3 + \left(-\frac{16285}{504} + \frac{214745}{1728}\nu + \frac{193385}{3024}\nu^2\right)\pi x^{7/2} + \mathcal{O}\left(\frac{1}{c^8}\right)\right\}.$$

$$x \equiv \left(\frac{G\,m\,\omega}{c^3}\right)^{2/3} \qquad m = m_1 + m_2 \qquad \mu = m_1 m_2/m \qquad \nu \equiv \frac{\mu}{m} \equiv \frac{m_1 m_2}{(m_1+m_2)^2}.$$

Phase evolution

Energy balance equation : orbital binding energy loss = GW emission energy

$$\boxed{\frac{dE}{dt} = -L} \qquad \frac{dE}{dt} = \frac{dE}{dx}\frac{dx}{dt} \qquad \frac{dx}{dt} = -\frac{L}{(dE/dx)}$$

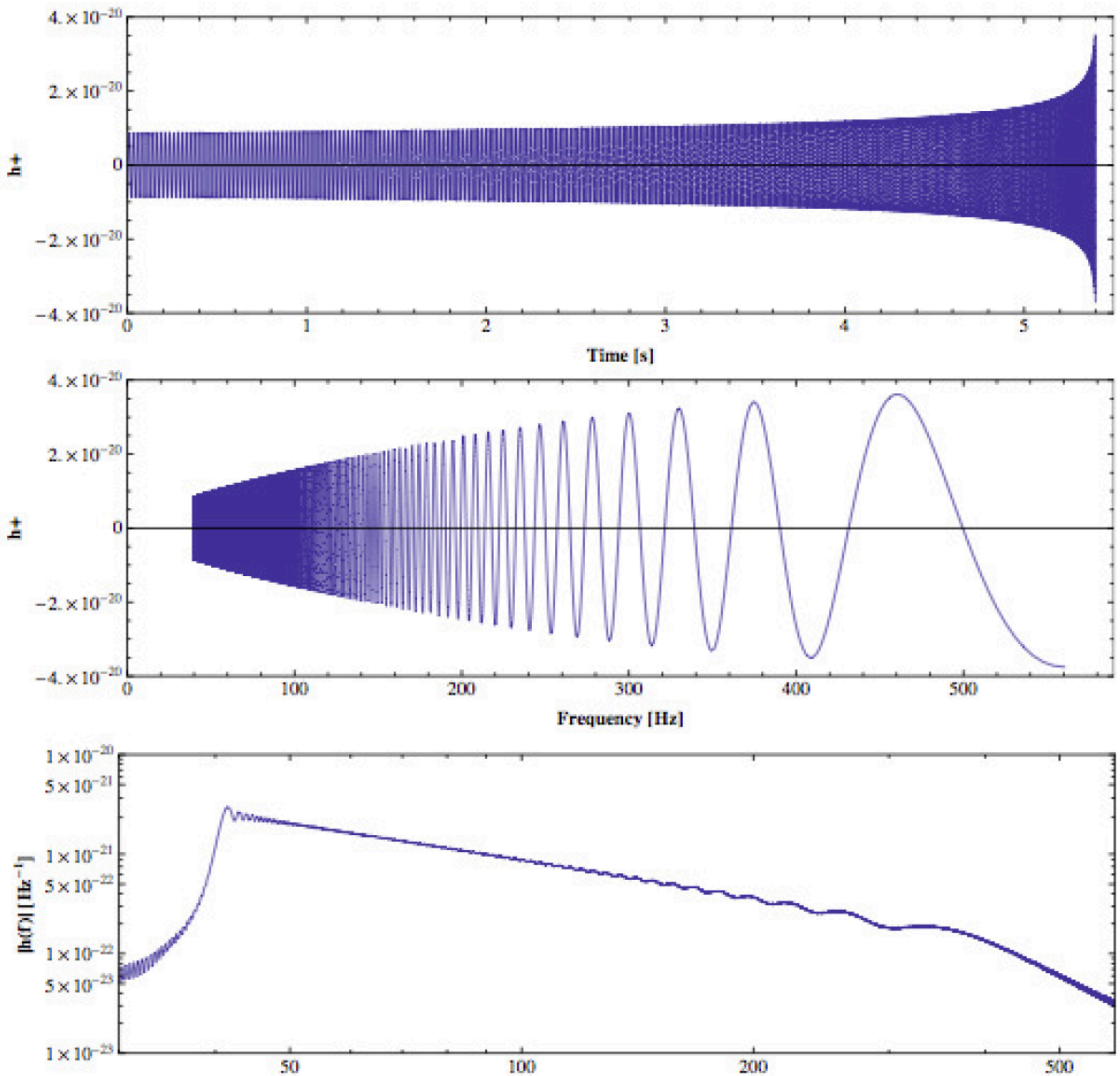
expand with x , $x \equiv \left(\frac{G\,m\,\omega}{c^3}\right)^{2/3}$

$$\begin{aligned}\frac{\dot{\omega}}{\omega^2} = \frac{96}{5}\eta(M\omega)^{5/3}\Bigg(&1 - \frac{743+924\eta}{336}(M\omega)^{2/3} + \left(\frac{34\,103}{18\,144} + \frac{13\,661}{2016}\eta + \frac{59}{18}\eta^2\right)(M\omega)^{4/3} - \frac{1}{672}(4159+14\,532\eta)\pi(M\omega)^{5/3}\eta \\ &+\left[\left(\frac{16\,447\,322\,263}{139\,708\,800} - \frac{1712}{105}\gamma_E + \frac{16}{3}\pi^2\right) + \left(-\frac{273\,811\,877}{1\,088\,640} + \frac{451}{48}\pi^2 - \frac{88}{3}\hat{\theta}\right)\eta + \frac{541}{896}\eta^2 - \frac{5605}{2592}\eta^3 \right. \\ &\left. - \frac{856}{105}\log[16(M\omega)^{2/3}]\right](M\omega)^2 + \left(-\frac{4\,415}{4\,032} + \frac{661\,775}{12\,096}\eta + \frac{149\,789}{3\,024}\eta^2\right)\pi(M\omega)^{7/3}\Bigg),\end{aligned}$$

$$w(t) = \int_0^t \dot{w}(w)dt, \qquad \psi(t) = \int_0^t w(t)dt$$ ---> Numerical integration

Accurate orbital phase evolution !

Nonspinning waveform (10+1.4, fstart=40 Hz)



Evolution of a spinning binary

Orbital angular momentum

$$\dot{S}_1 = \frac{(M\omega)^2}{2M}\left\{\left(4+3\frac{m_2}{m_1}\right)L_N + \frac{1}{M^2}\left[S_2 - 3(S_2\cdot\hat{L}_N)\hat{L}_N\right]\right\}\times S_1,$$
$$\dot{S}_2 = \frac{(M\omega)^2}{2M}\left\{\left(4+3\frac{m_1}{m_2}\right)L_N + \frac{1}{M^2}\left[S_1 - 3(S_1\cdot\hat{L}_N)\hat{L}_N\right]\right\}\times S_2,$$

$$\dot{S}_i = \Omega_i \times S_i;$$

constant magnitude & precession

w/o gravitational radiation

$$J = L_N + S, \qquad \dot{L}_N = -(\dot{S}_1 + \dot{S}_2),$$

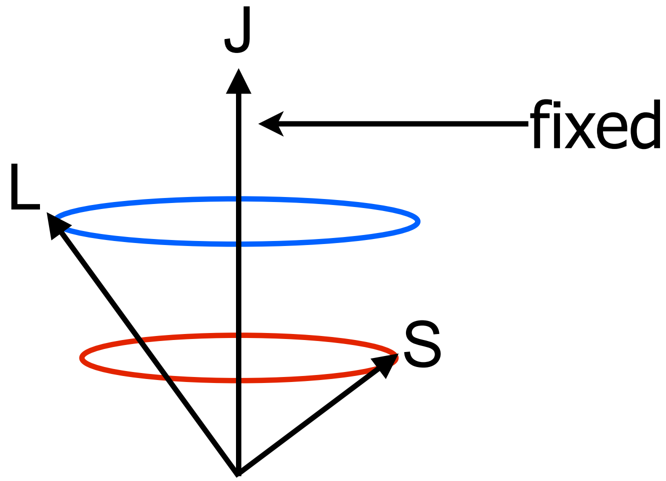
$$\dot{L}_N = \frac{\omega^2}{2M}\left\{\left[\left(4+3\frac{m_1}{m_2}\right)S_1 + \left(4+3\frac{m_2}{m_1}\right)S_2\right] \times L_N - 3[S_1(S_2\cdot\hat{L}_N) + S_2(S_1\cdot\hat{L}_N)] \times \hat{L}_N\right\}.$$

One spin case $\quad S = S_1 + S_2 = S_1$

$$J \times S = (L_N + S) \times S = L_N \times S,$$

$$\dot{L}_N = \frac{(M\omega)^2}{2M}\left[\frac{1}{2}\left(1+3\frac{M}{m_s}\right)\left(J \times L_N\right)\right].$$

$$\dot{S} = \frac{(M\omega)^2}{2M}\left[\frac{1}{2}\left(1+3\frac{M}{m_s}\right)\left(J \times S\right)\right],$$



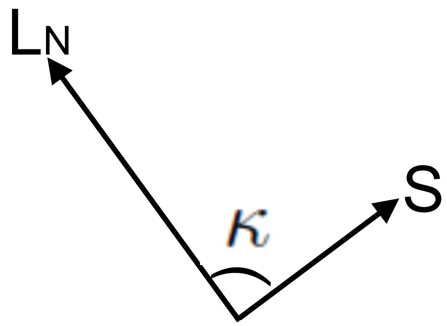
constant magnitude & precession about J

w/ gravitational radiation

$$\dot{L}_N = \frac{(M\omega)^2}{2M}\left[\frac{1}{2}\left(1+3\frac{M}{m_s}\right)\left(J \times L_N\right)\right] - \dot{L}_{GW},$$

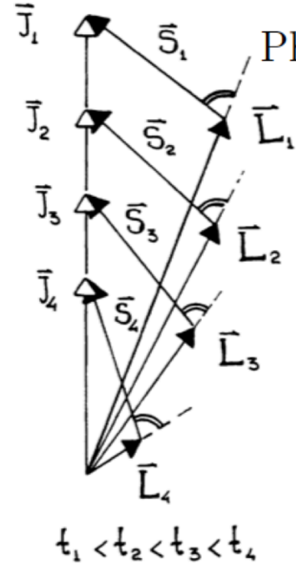
$$\Omega = \frac{(M\omega)^2}{2M}\left[\frac{1}{2}\left(1+3\frac{M}{m_s}\right)\right]|J|.$$

$$\begin{aligned}
\dot{S} &= \Omega\hat{J} \times S, \\
\dot{L}_N &= \Omega\hat{J} \times L_N - \dot{L}_{GW}, \\
\dot{J} &= -\dot{L}_{GW},
\end{aligned}$$



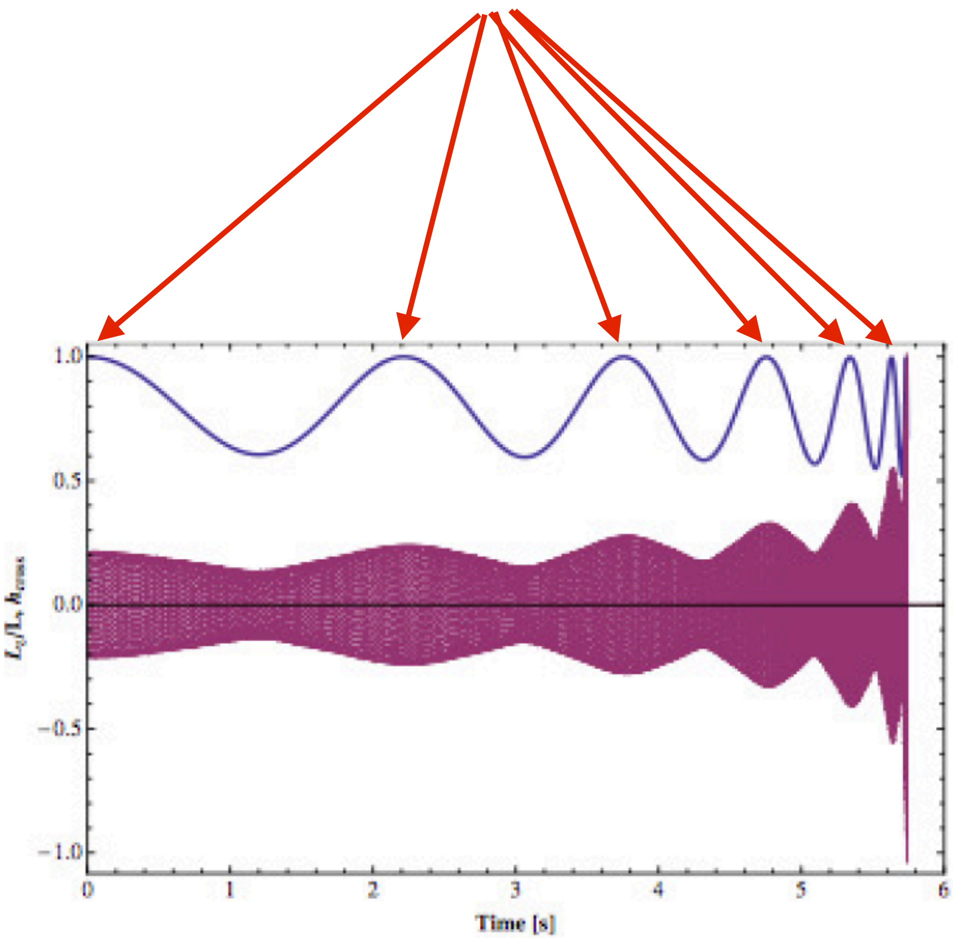
$$\kappa = \cos^{-1}(\hat{L}_N \cdot \hat{S}) \longrightarrow \dot{\kappa} = 0.$$

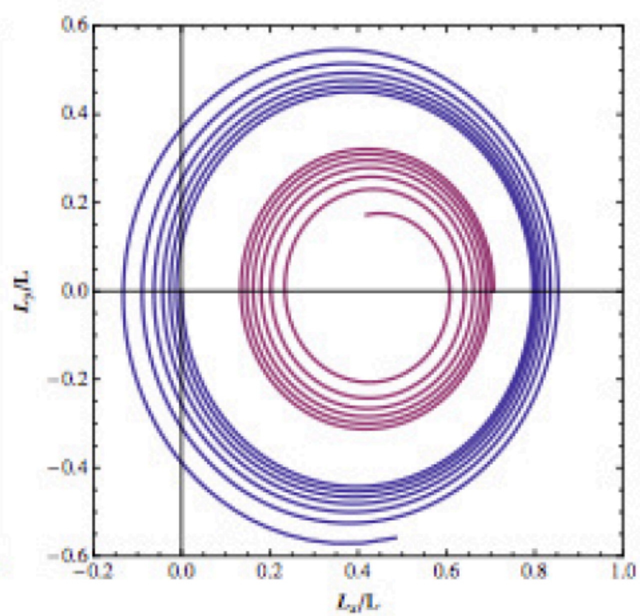
Precession of a spinning binary



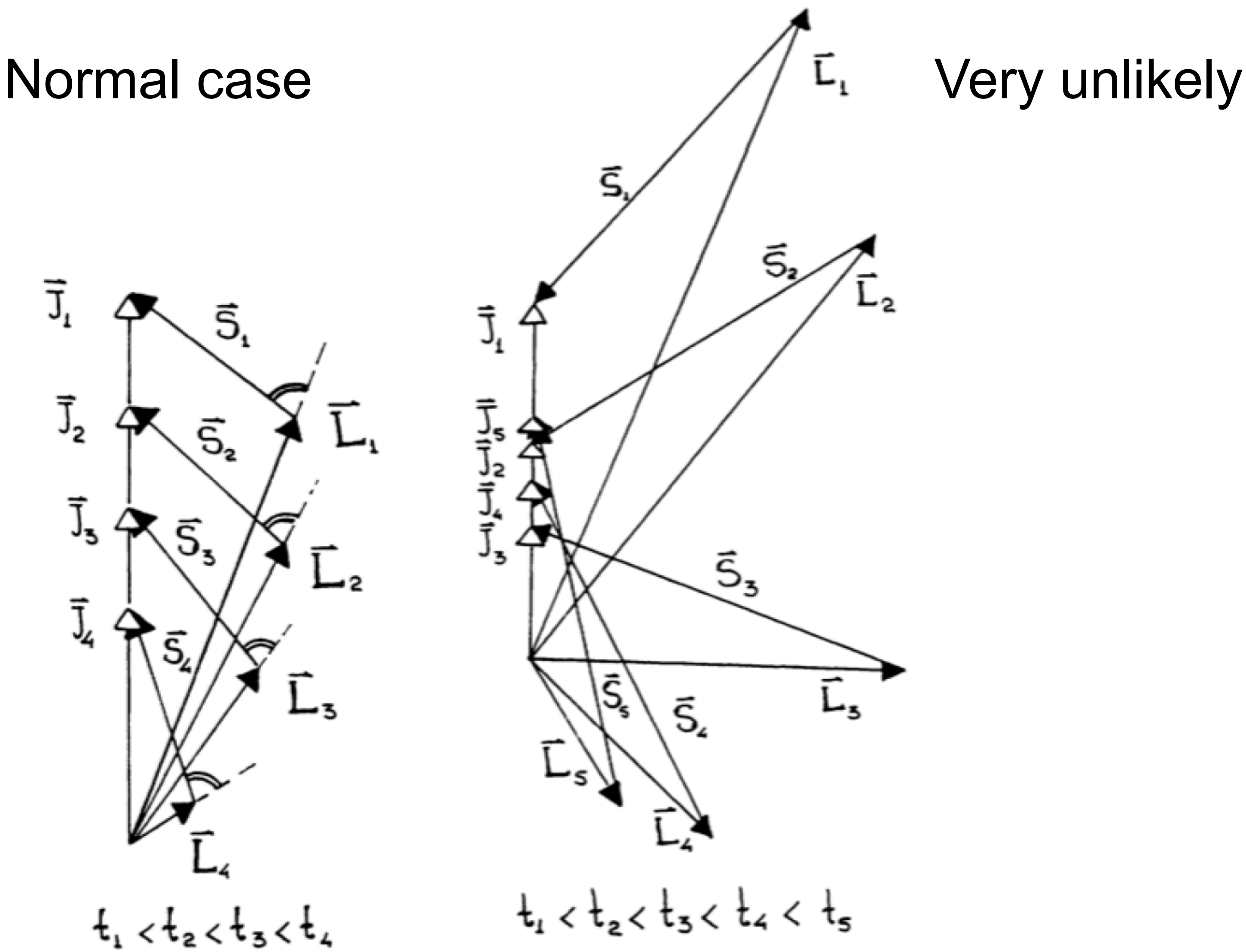
Phys. Rev. D **49**, 6274.

Small Inclination





Transitional Precession (BH with high spin)



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Orbital axis precession

Variation of L_N depends on S, S dot L_N, S cross L_N

---> Precessional motion depends on

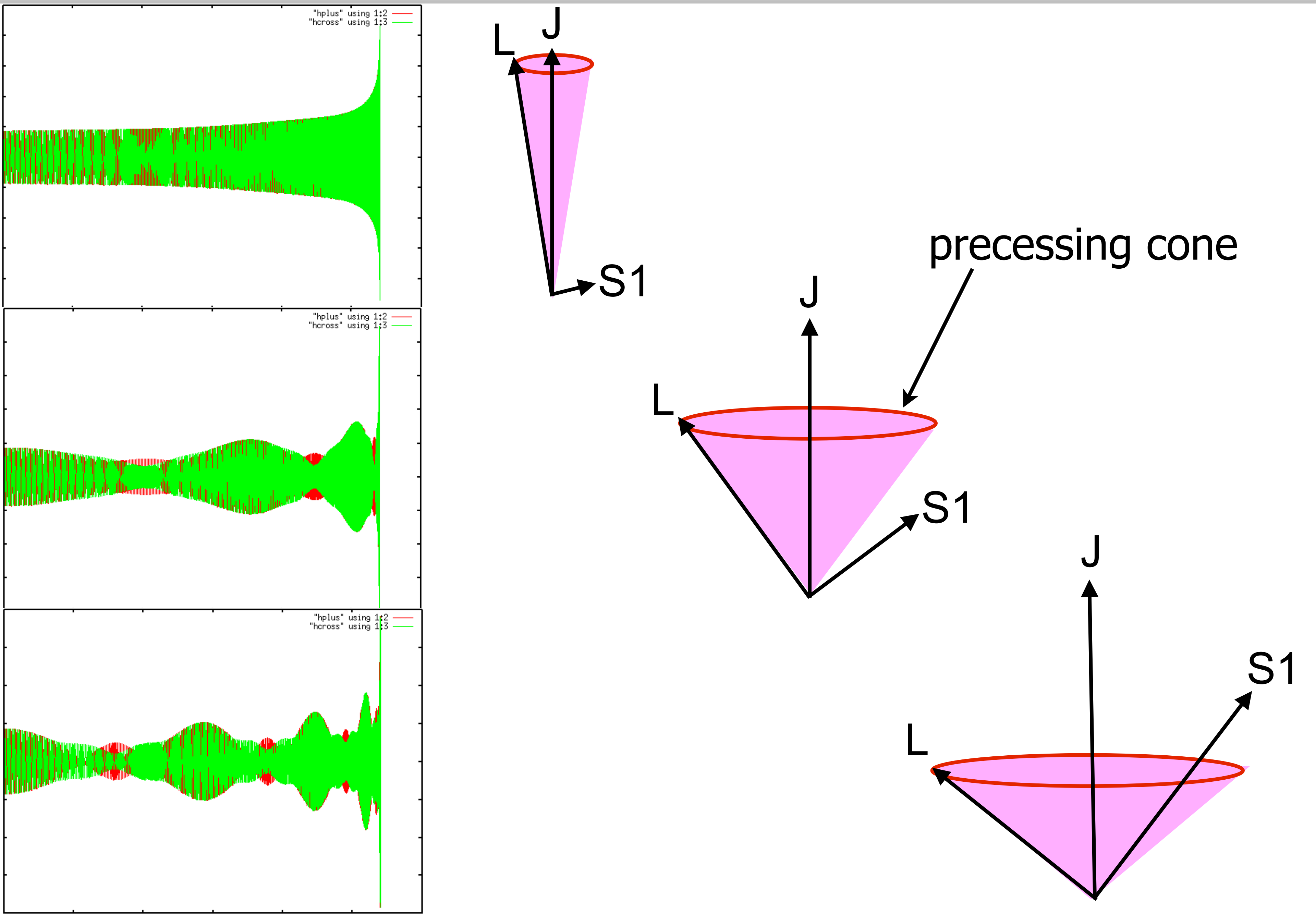
1) Spin magnitude

2) Angle between L_N and S

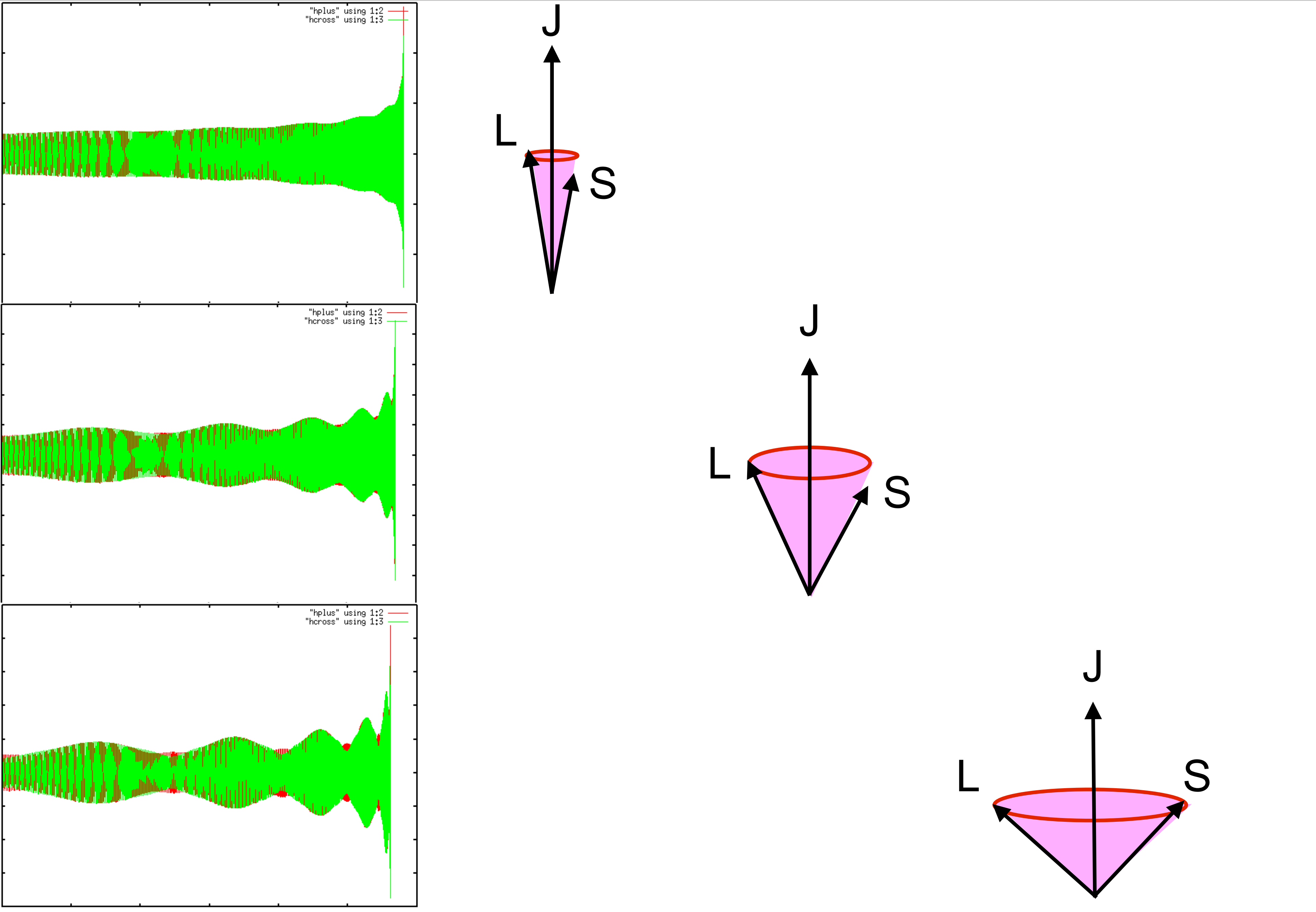
$$\dot{\hat{\mathbf{L}}}_N = -\frac{(M\omega)^{1/3}}{\eta M^2}\,\dot{\mathbf{S}} = \frac{\omega^2}{2M}\Bigg\{\left[\left(4+3\frac{m_2}{m_1}\right)\underline{\mathbf{S}_1} + \left(4+3\frac{m_1}{m_2}\right)\underline{\mathbf{S}_2}\right]\underline{\times\hat{\mathbf{L}}_N}$$

$$-\frac{3\,\omega^{1/3}}{\eta\, M^{5/3}}\left[\underline{(\mathbf{S}_2\cdot\hat{\mathbf{L}}_N)}\,\underline{\mathbf{S}_1} + \underline{(\mathbf{S}_1\cdot\hat{\mathbf{L}}_N)}\,\underline{\mathbf{S}_2}\right]\underline{\times\hat{\mathbf{L}}_N}\Bigg\}$$

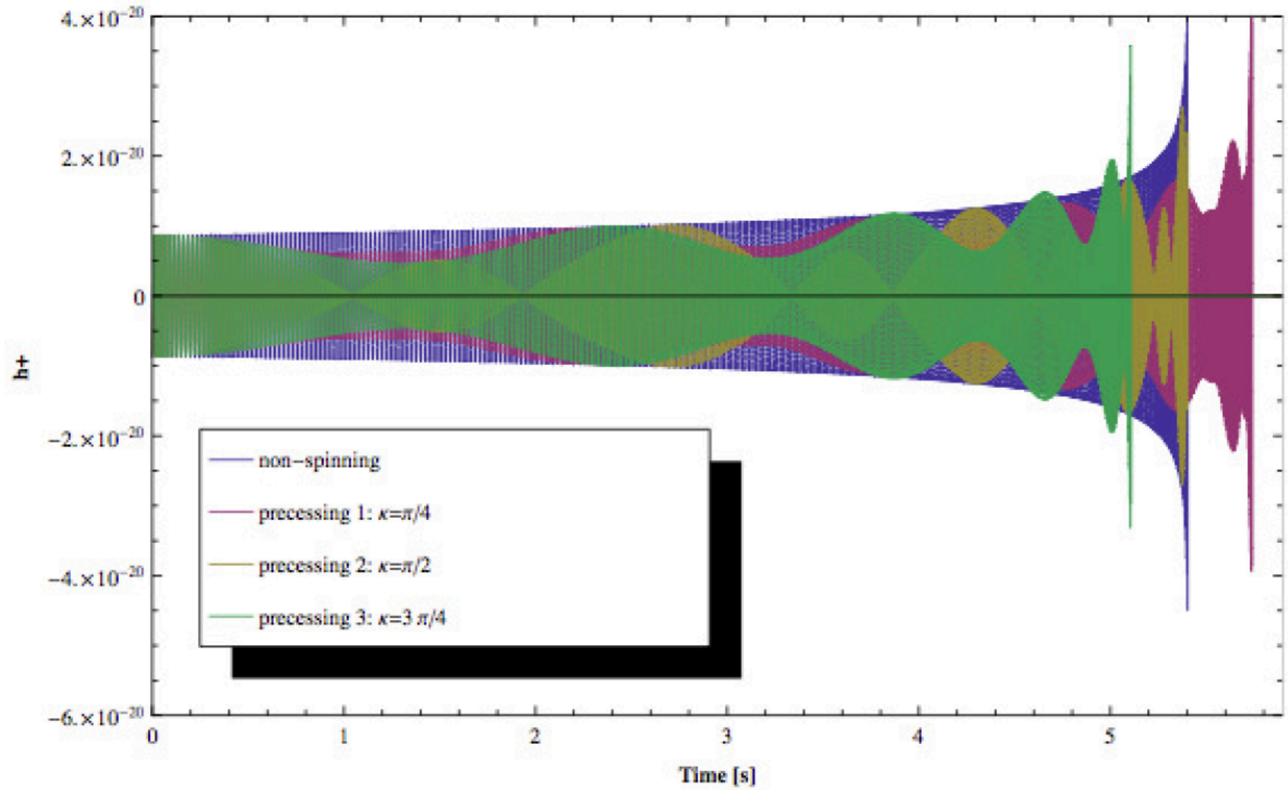
Amplitude modulation by Spin (angle=pi/2)

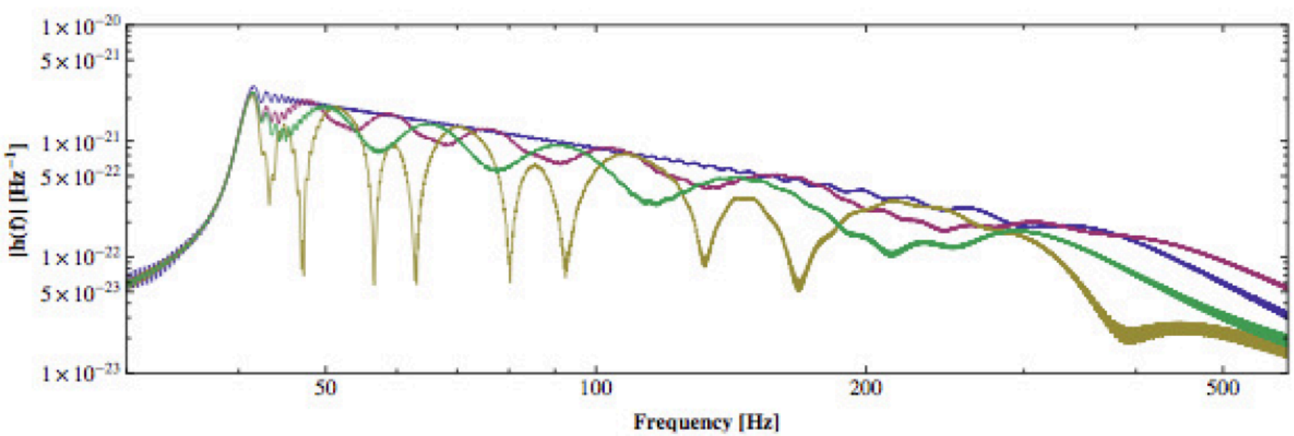


Amplitude modulation by Angle (spin=0.99)



10+1.4 BH-NS, fstart=40 Hz, BH spin=1, NS spin=0





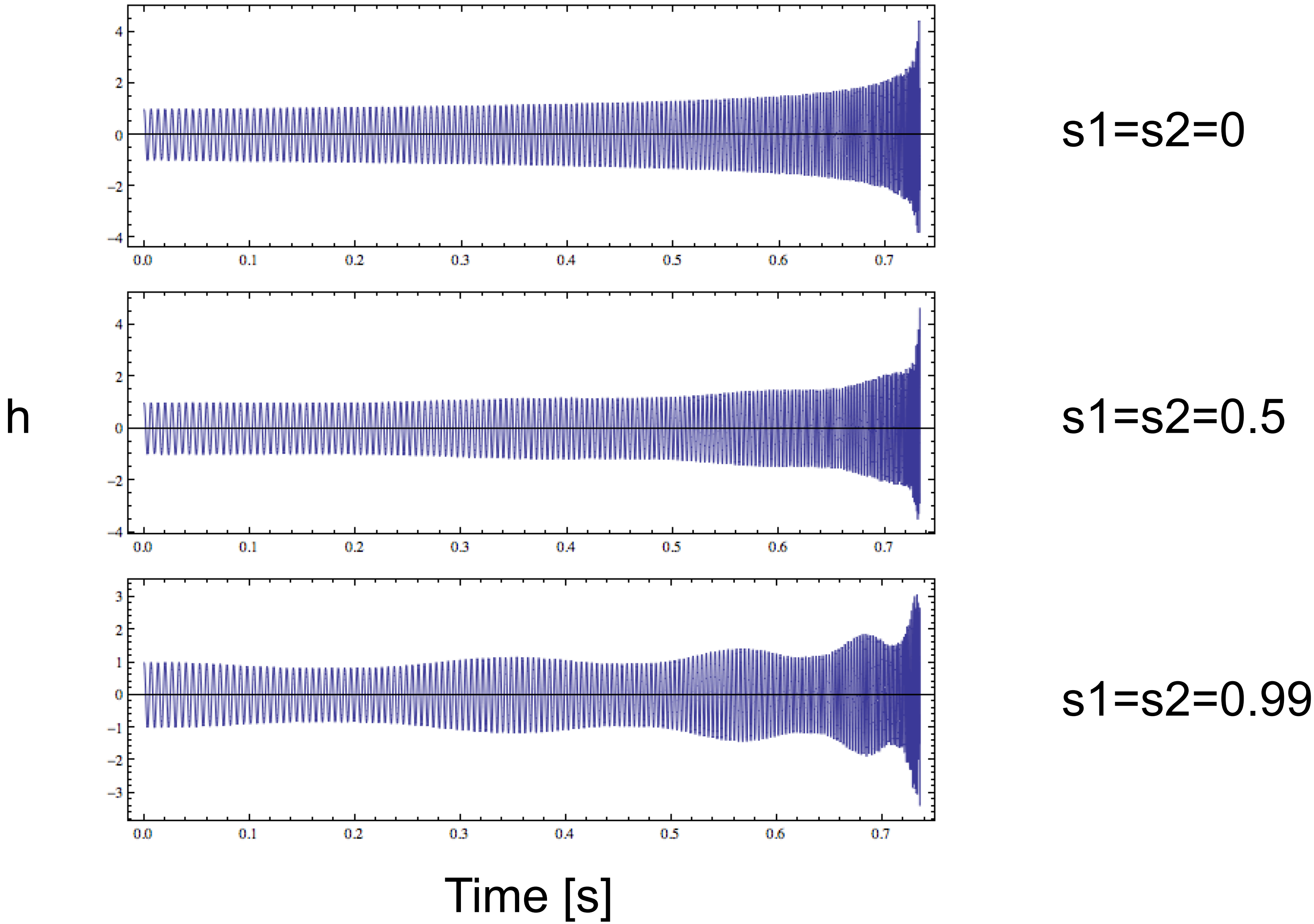
Post Newtonian Energy & Flux for precessing binaries

SO & SS corrections to the nonspinning eqns.

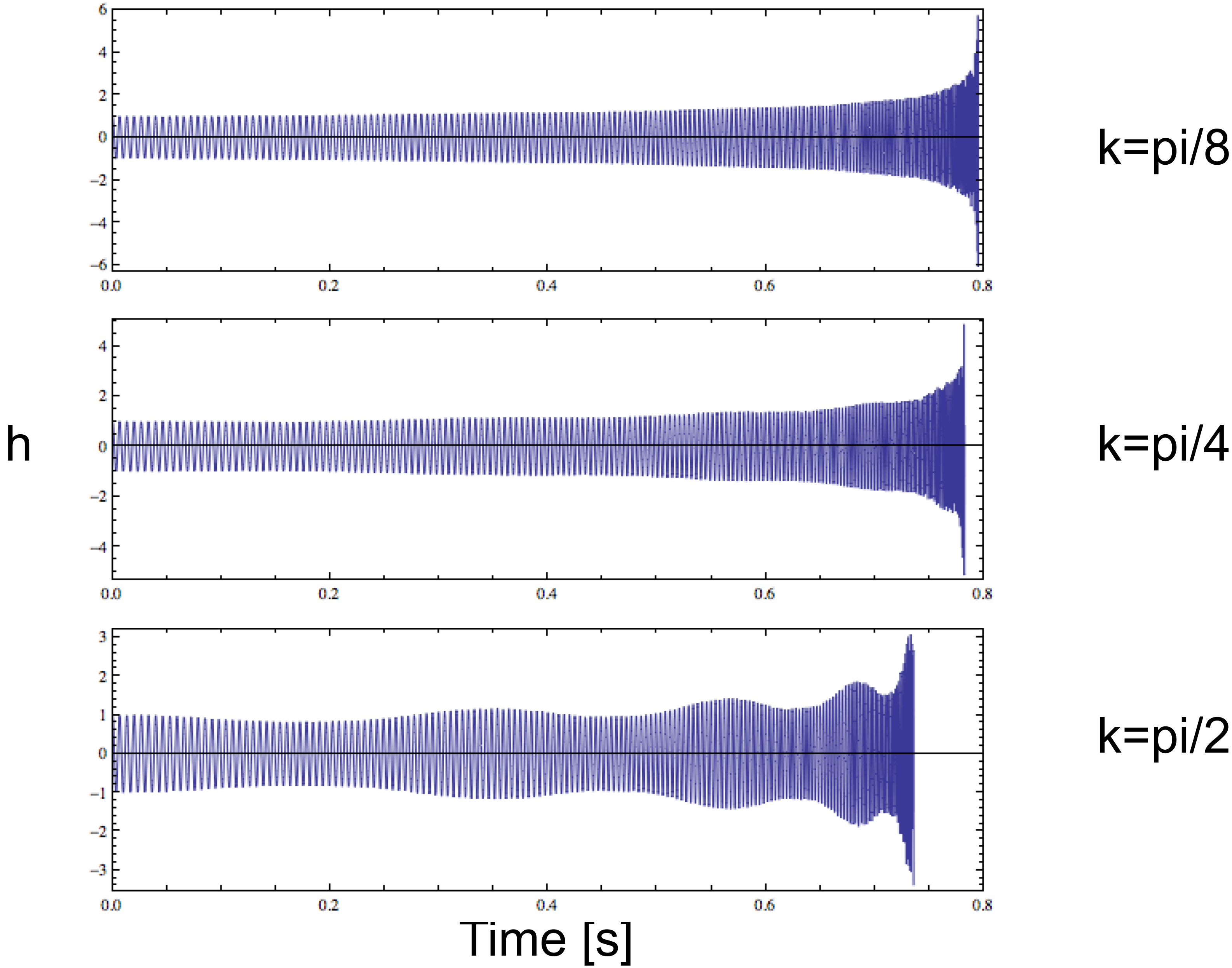
$$
\begin{aligned}
E &= -\frac{\mu x}{2}\left\{1 + A_{1PN}x + \frac{8}{3}\hat{L}_N \cdot \left[\left(1+\frac{3m_2}{4m_1}\right)S_1 + \left(1+\frac{3m_1}{4m_2}\right)S_2\right]x^{3/2}\right. \\
&+ \left.\left(A_{2PN} + \frac{1}{\nu}[S_1\cdot S_2 - 3(\hat{L}_N\cdot S_1)(\hat{L}_N\cdot S_2)]\right)x^2 + A_{3PN}x^3\right\}, \qquad (3.44) \\
\mathcal{L} &= -\frac{32\nu^2x^5}{5}\left\{1 + B_{1PN}(M\omega)^{2/3}\right. \\
&+ \left(B_{1.5PN} - \frac{1}{12}\sum_{i=1,2}\left[\chi_i(\hat{L}_N\cdot\hat{S}_i)\left(73\frac{m_i^2}{M^2} + 75\nu\right)\right]\right)(M\omega) \\
&+ \left(B_{2PN} - \frac{1}{48}\nu\chi_1\chi_2[223(\hat{S}_1\cdot\hat{S}_2) - 649(\hat{L}_N\cdot\hat{(S)}_1)(\hat{L}_N\cdot\hat{(S)}_2)]\right)(M\omega)^{4/3} \\
&+ \left. B_{2.5PN}(M\omega)^{5/3} + B_{3PN}(M\omega)^2 + B_{3.5PN}(M\omega)^{7/3}\right\}, \qquad (3.45)
\end{aligned}
$$

Precessing phase = Nonspinning phase + corrections

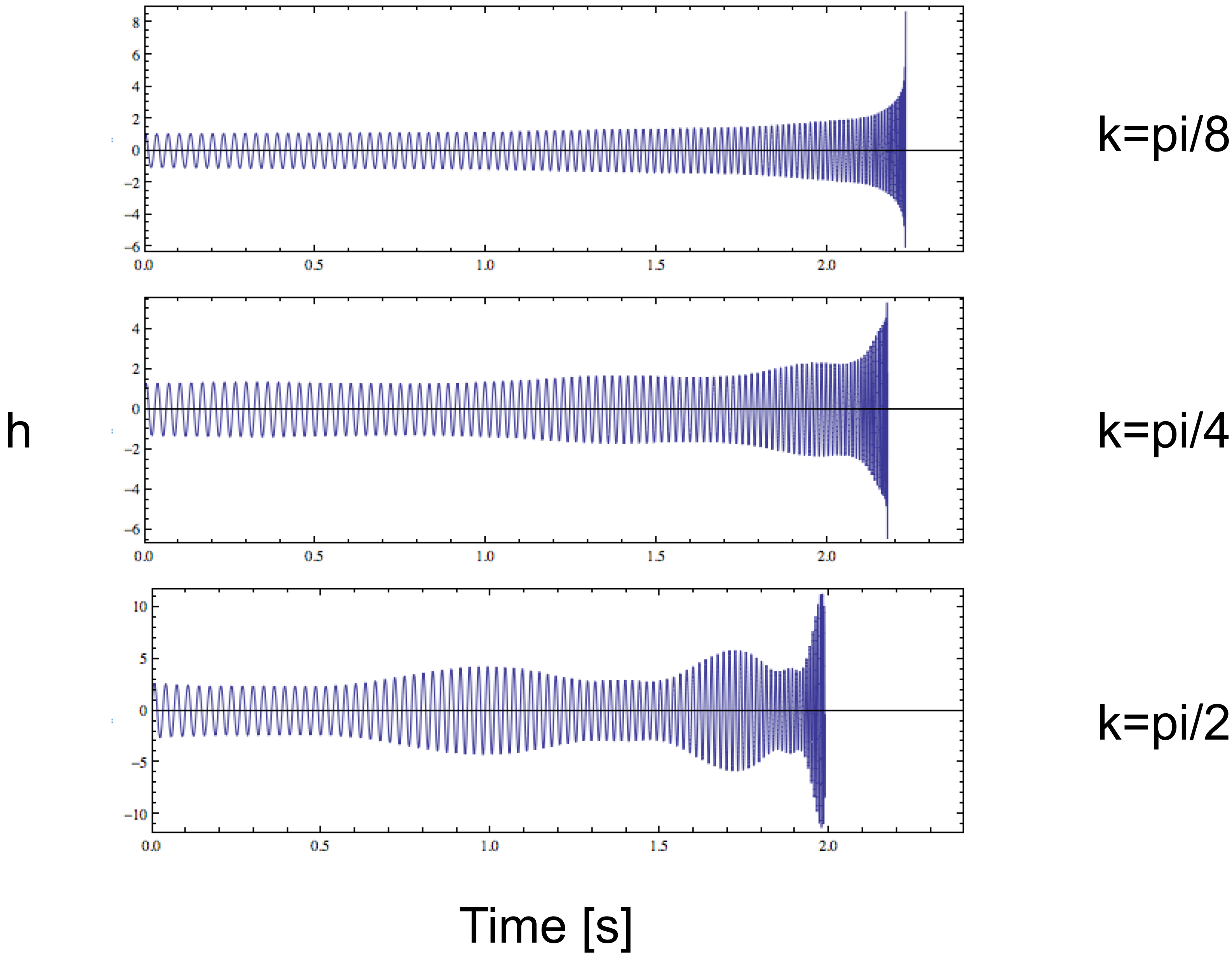
1.4+1.4 NS binary, k=pi/2, fstart=150 Hz



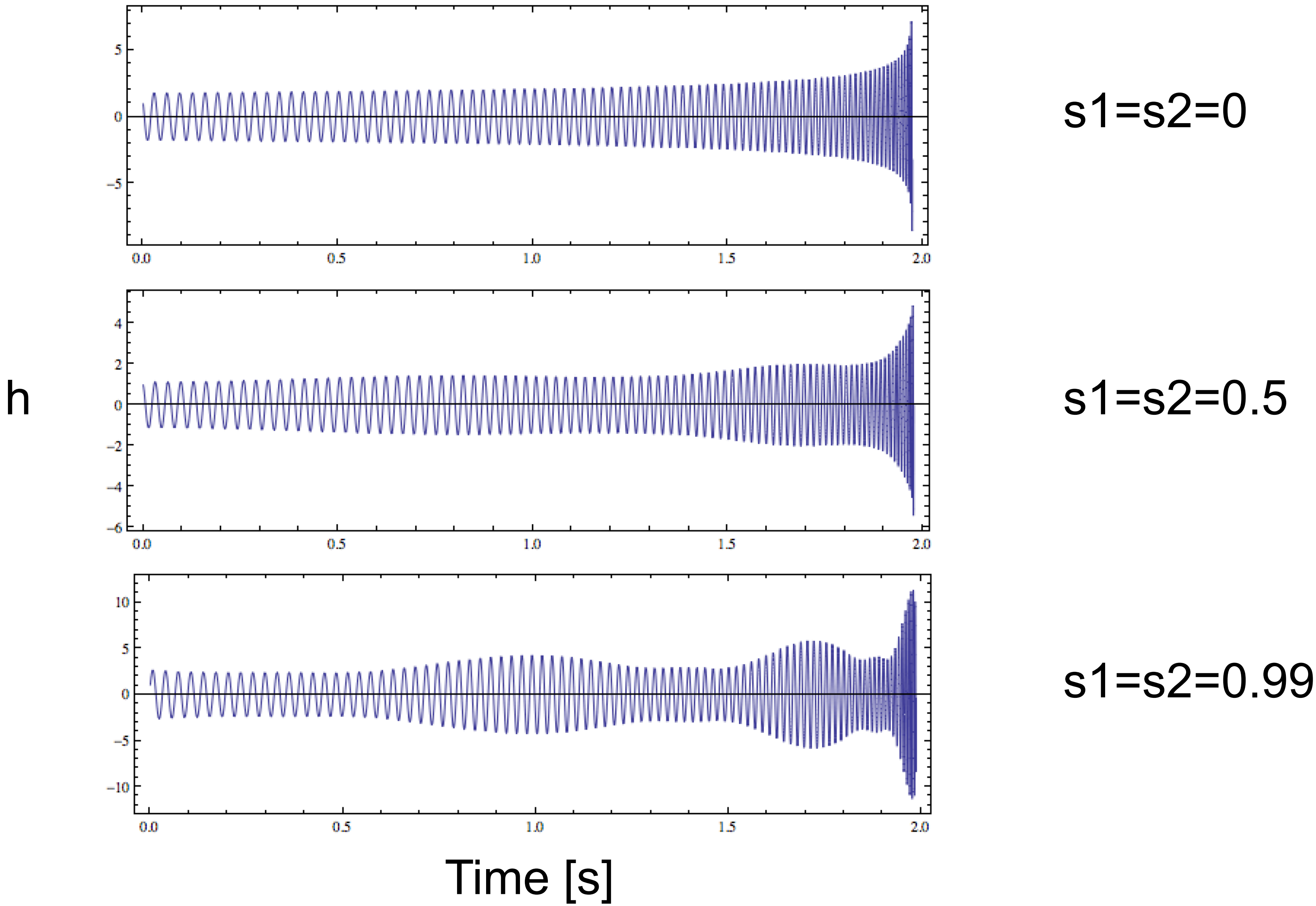
1.4+1.4 NS binary, s1=s2=0.99, fstart=150 Hz



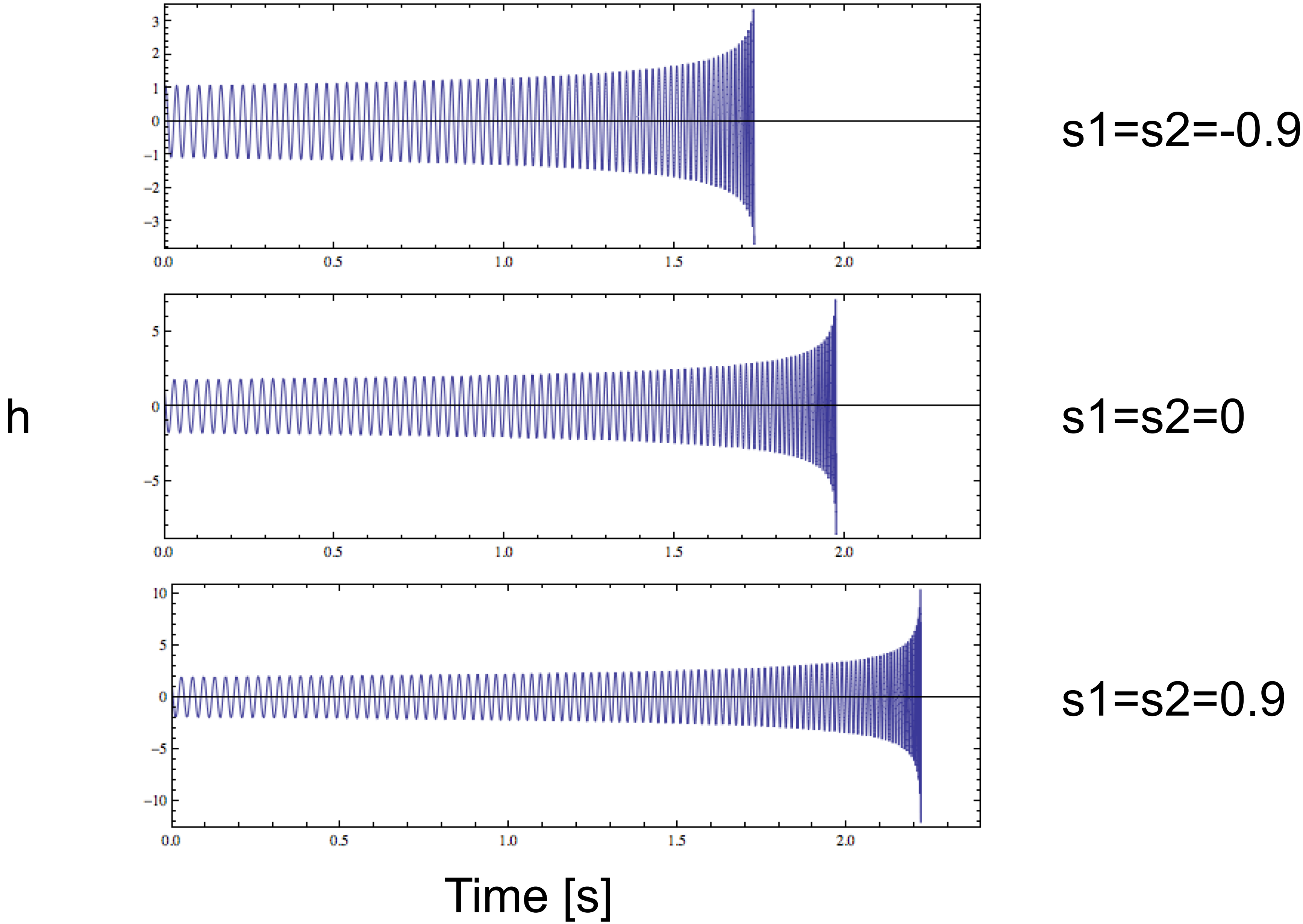
10+10 BH binary, s1=s2=0.99, fstart=30 Hz



10+10 BH binary, k=pi/2, fstart=30 Hz



Alined case, 10+10 BH binary, k=0, fstart=30 Hz

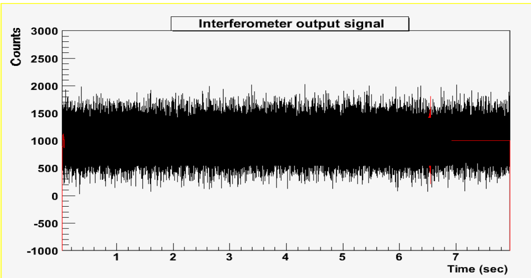


Matched filtering : detection

m1, m2, spin

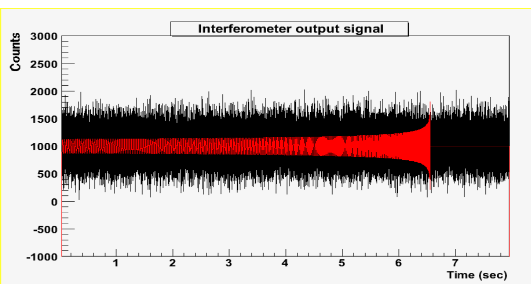
Detector output (data)

$$S(t) = 4 \int_{f_{low}}^{f_{final}} \frac{\tilde{d}(f)\ \tilde{T}^*(f)}{S_n(f)}\ e^{2\pi i f t}\ df$$

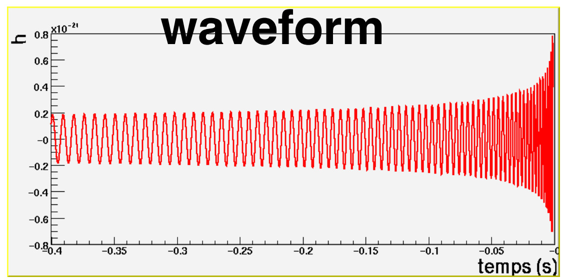


or

$\otimes$



Theoretical waveform



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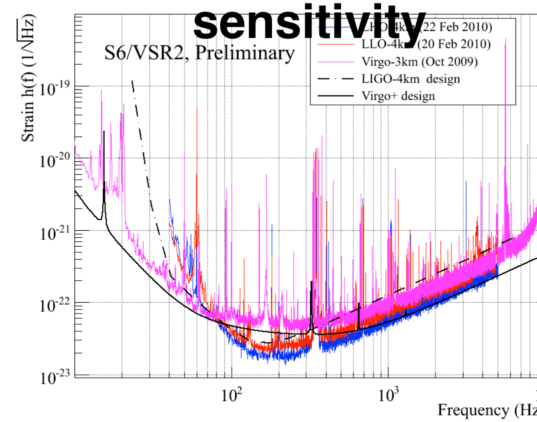
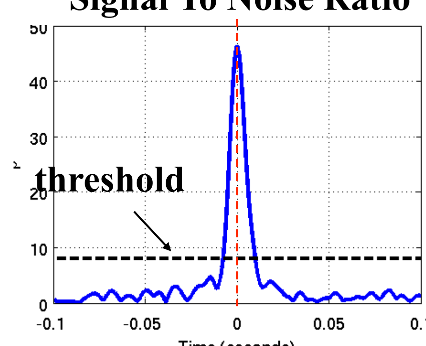
Detector noise sensitivity



$$\tilde{d}(f) = \tilde{n}(f) + \tilde{s}(f)$$

==>

Signal To Noise Ratio



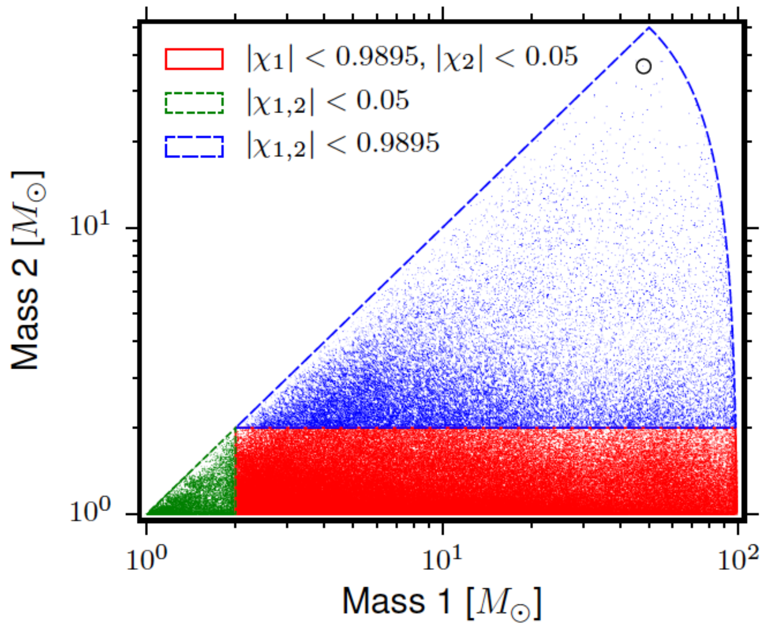
Template bank for GW150914

$$S(t) = 4\int_{f_{low}}^{f_{final}} \frac{\tilde{d}(f)\,\tilde{T}^*(f)}{S_n(f)} e^{2\pi i f t}\, df$$

•Request Minimum Match = 0.97

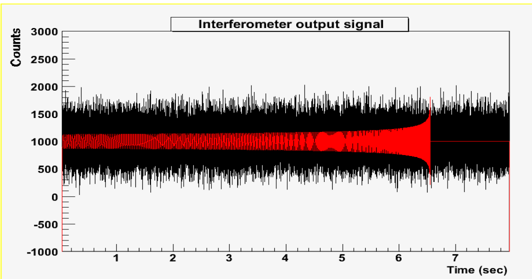
⇒ Low Mass region is more densely populated than high mass region

GW150914: 250000 templates (mass1, mass2, spin1, spin2)



Overlap: parameter estimation

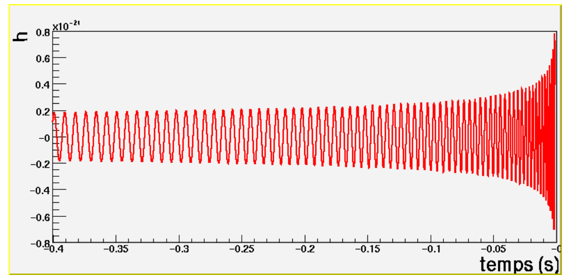
Detector output (data)



$$\tilde{d}(f) = \tilde{n}(f) + \tilde{s}(f)$$

$\otimes$

Theoretical waveform



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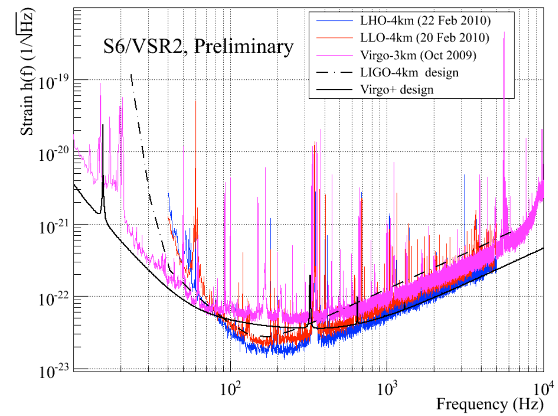
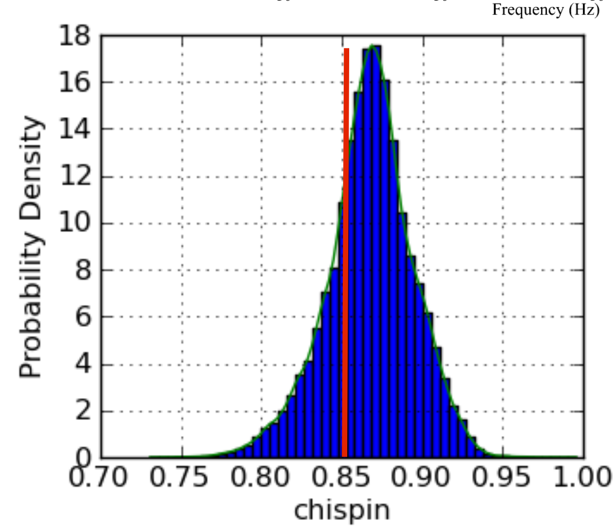
Detector noise sensitivity



9~15 parameters: mass, spin, ecc, tidal, ...

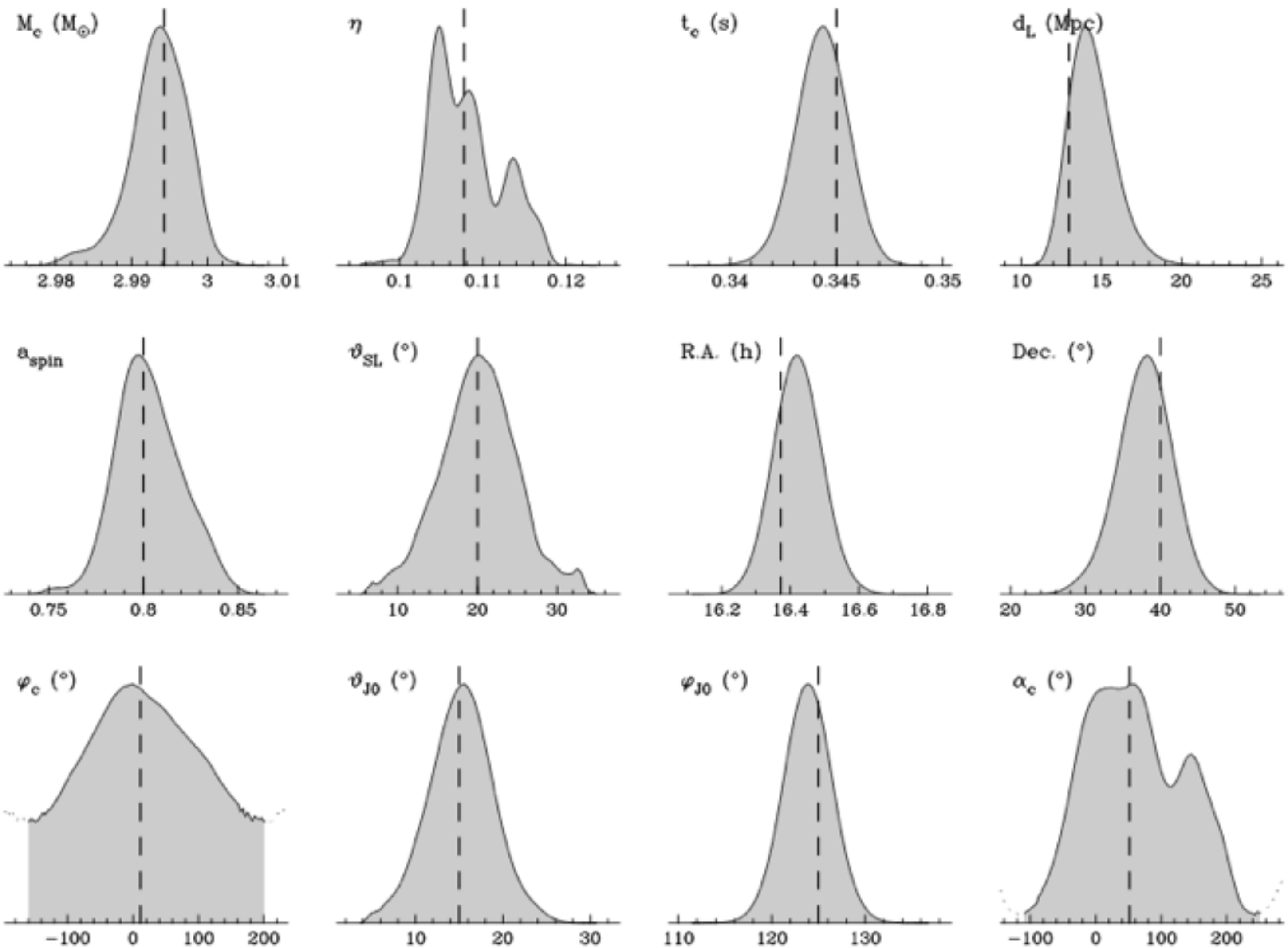
$$\langle d|h\rangle = 4\int_0^\infty \frac{df}{S_n(f)} \mathrm{Re}[\tilde{d}(f)\tilde{h}(f)^*]$$

==>



parameters

Posterior PDF Examples

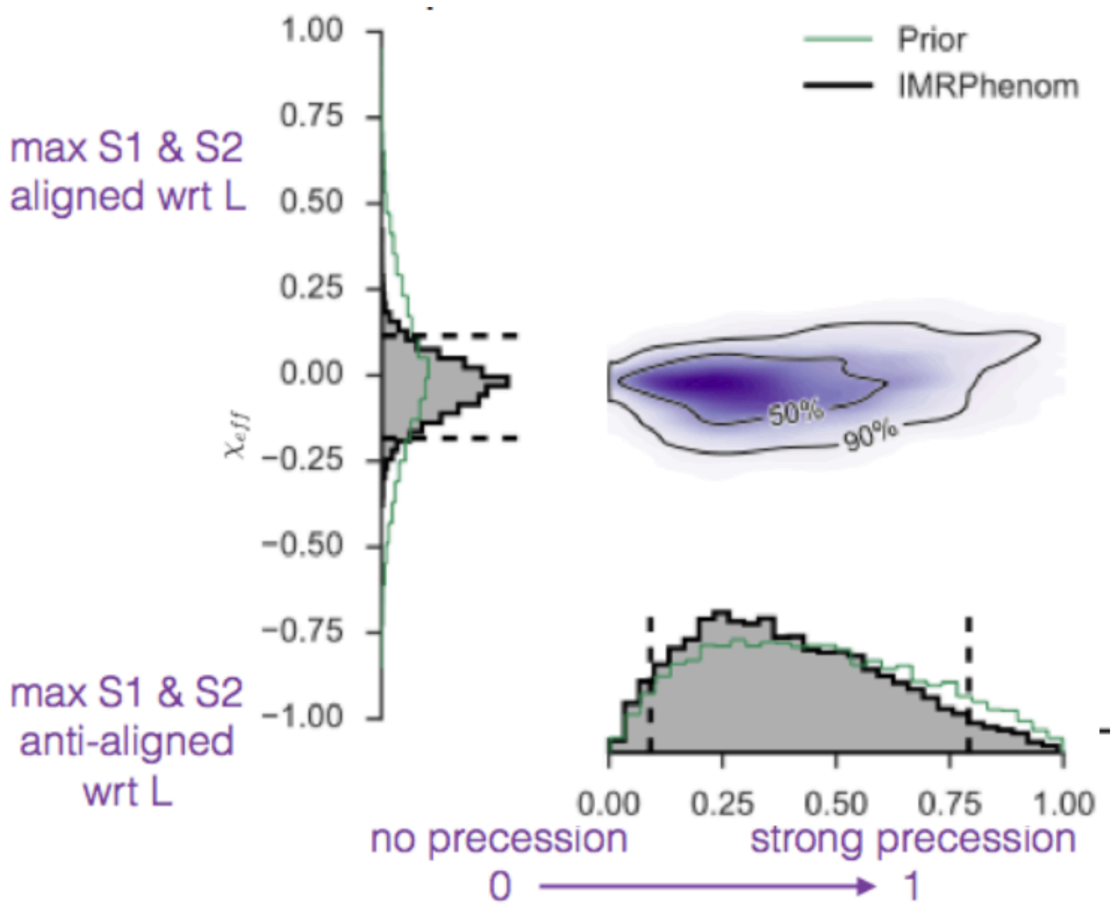


How to measure the lensing effect from GWs?

Possible, if the lensing effect is parameterized
in the wave function, such as mass, spin, eccentricity, ….

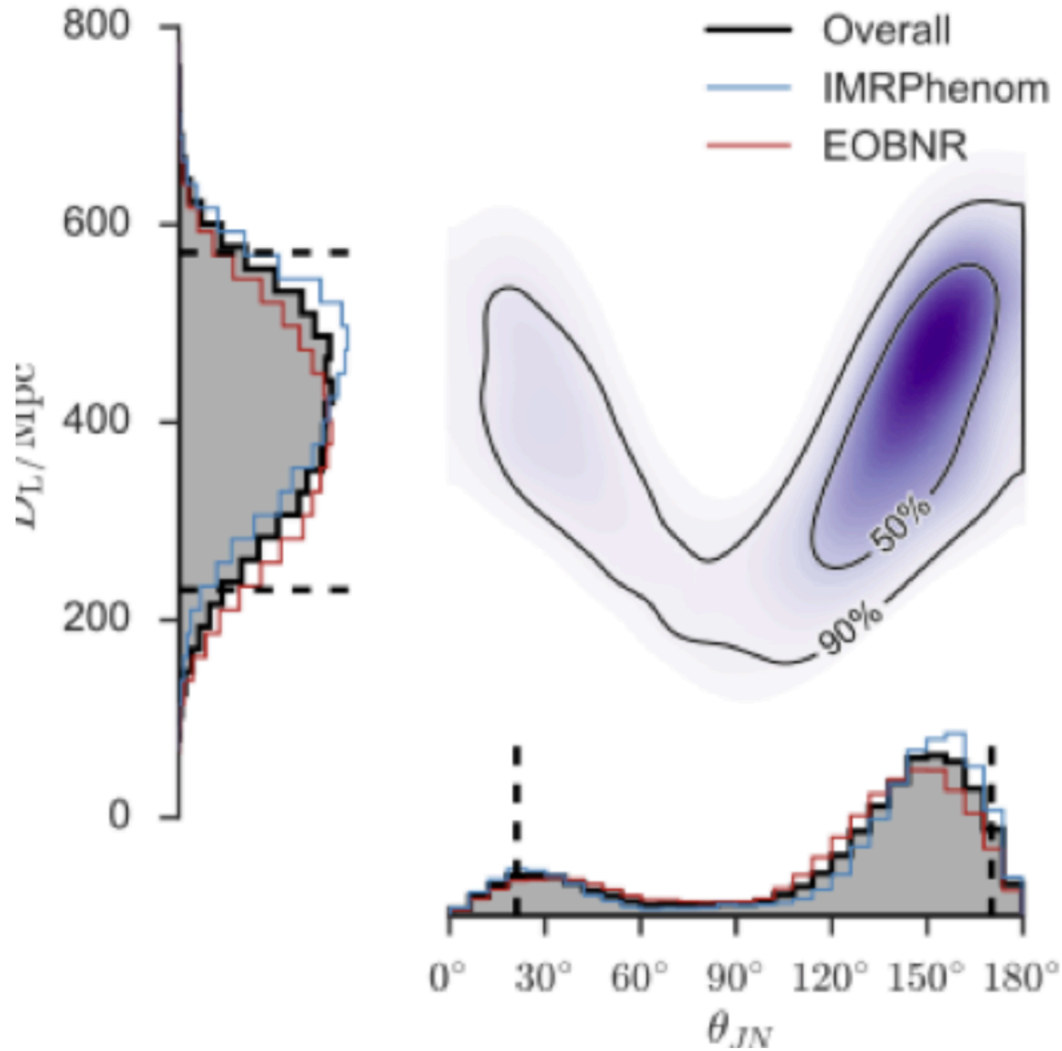
but, how well can it be measured ?

Results for GW150014



Precession effect is very hard to measure, lensing effect ?

Degeneracy



Degeneracy btw precession & lensing effects ?

GW data analysis study including lensing effect will be very interesting !!

Thanks