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Neutron Star Equation of State

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Contents

Motivation

- **Detections of gravitational waves**
- **Observations of x-ray binaries**
- **Heavy ion experiments**

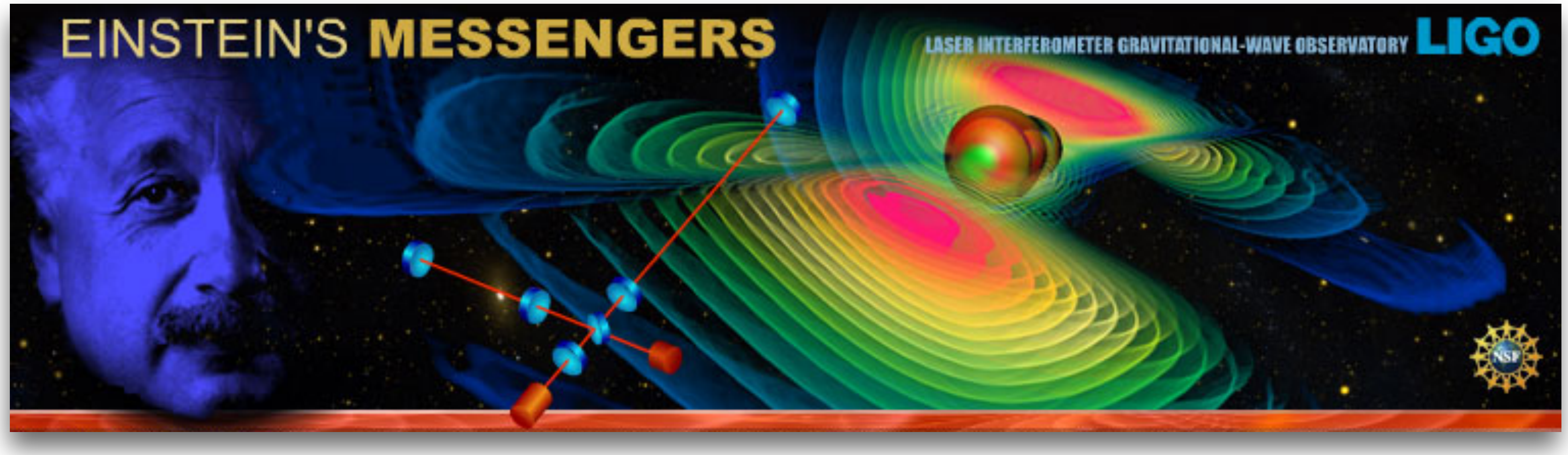
RAON

LIGO Laser Interferometer Gravitational-Wave Observatory

NICER Neutron star Interior Composition ExploreR

LIGO

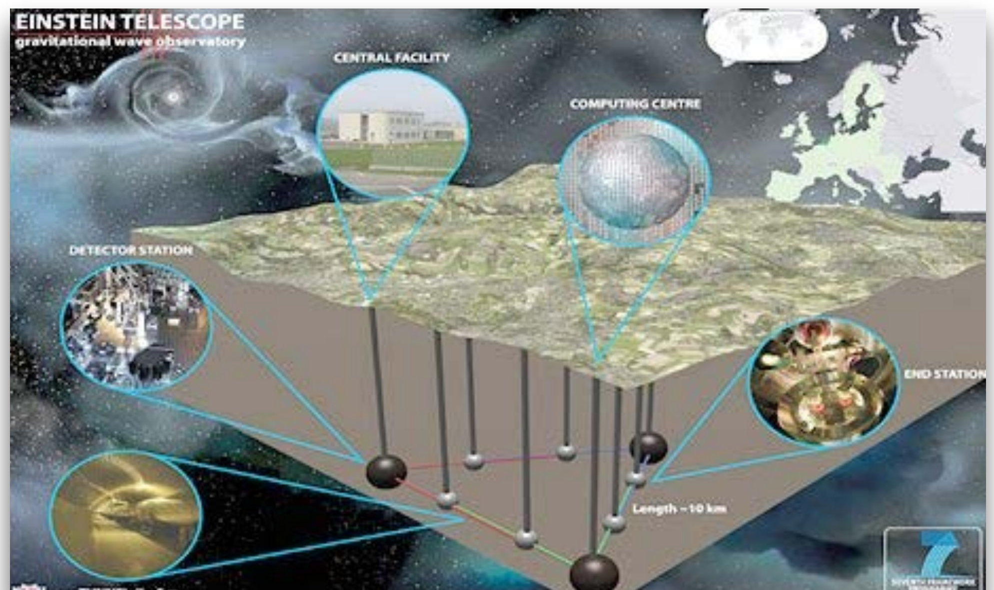
- First detection of gravitational-waves
- First detection of black hole binary
- First observation of heavy black holes
- New possibilities: grativational-wave astronomy
- Neutron stars, black holes, supernovae, gamma-ray bursts, …
- … …



Future Gravitational-Wave Observatories

Einstein Telescope

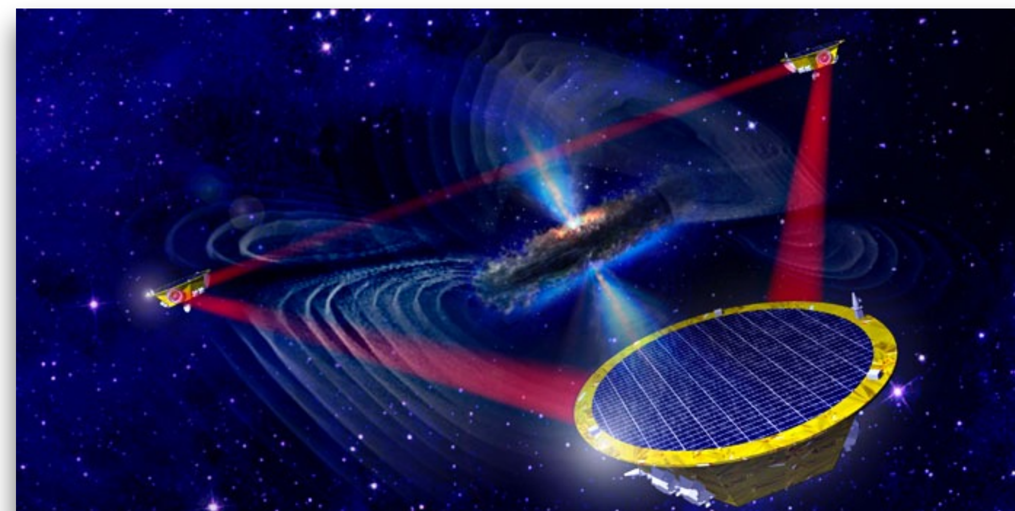
ESA / 2030? (designing stage)



10 km

eLISA

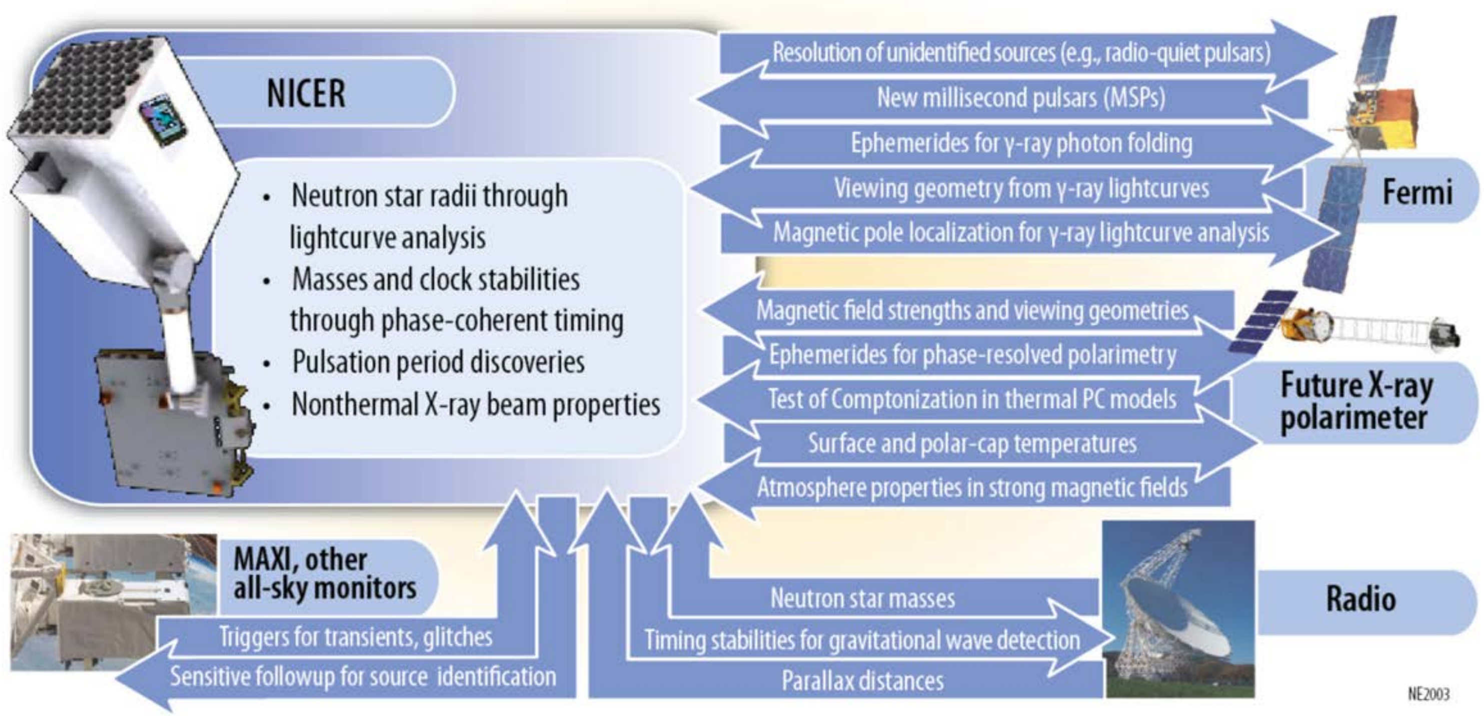
2034



10^6 km

NICER Neutron star Interior Composition ExploreR

- **launch**: early 2017, SpaceX
- **platform**: ISS ELC (ExPRESS Logistics Carrier)
- **instrument**: X-ray (0.2-12 keV)
- **objective**
  - ***structure***: neutron star radii to 5%, cooling timescales
  - ***dynamics***: stability of pulsars as clocks, properties of outbursts, oscillations, and precession
  - ***energetics***: intrinsic radiation patterns, spectra, and luminosities



RAON

2017

06
중이온가속기(RAON)
부지 인수 완료
(부지조성은
한국토지주택공사(LH) 수행)

09
중이온가속기 시설건설사업
실시설계 적격 심의 완료

10
건물 시공 착수

2018

10
RAON 설치 시작

2020

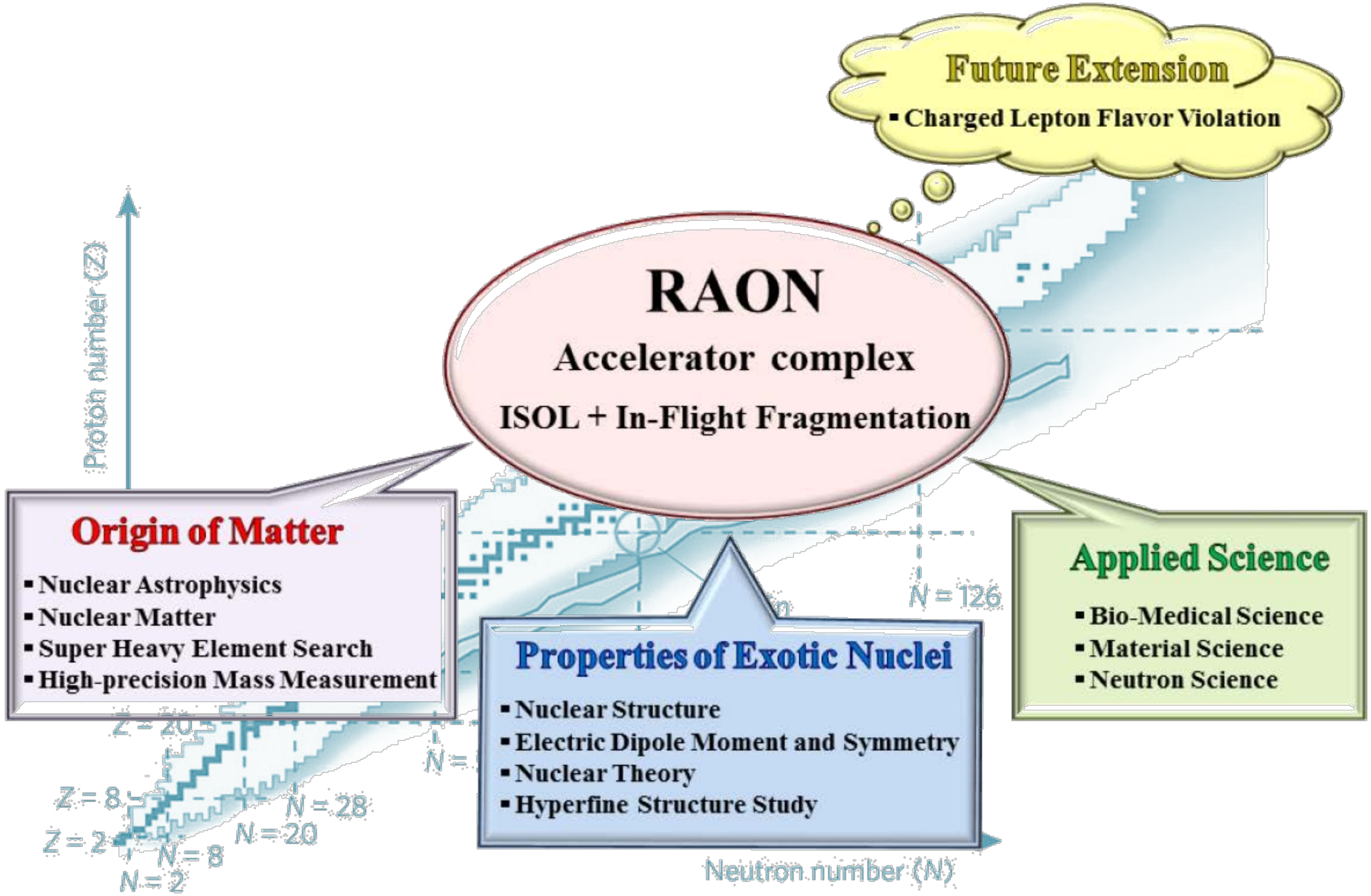
12
건물 시공 완료

2021

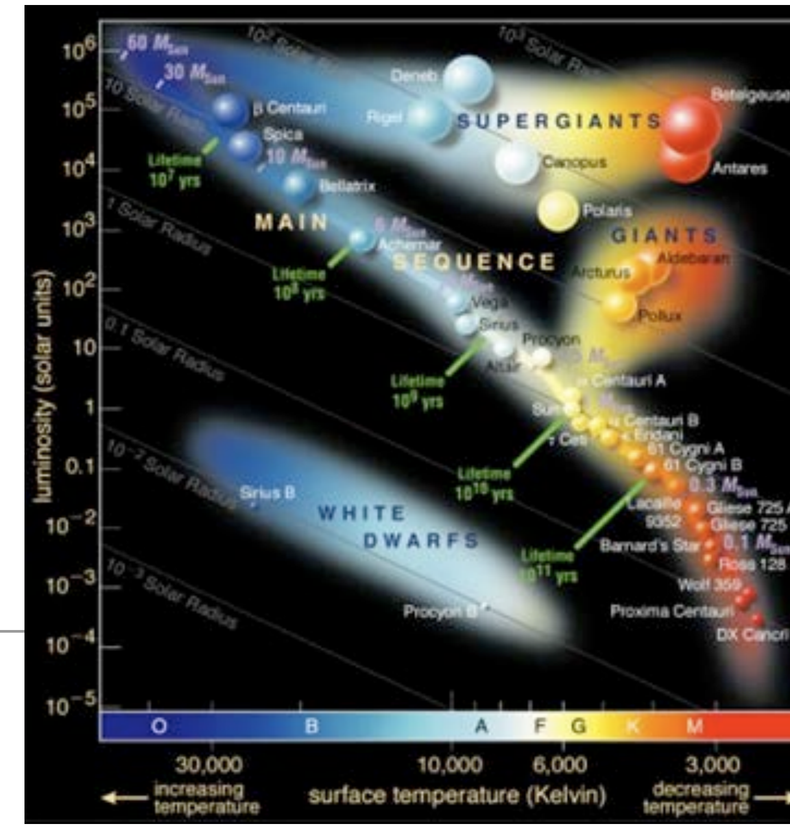
12
RAON
설치 완료

Rare Isotope Physics

slide from Kevin Insik Hahn



Physics of White Dwarfs



S. Chandrasekhar (1967)
Stellar Structure

S.L. Shapiro & S.A. Teukolsky (1983)
Black Holes, White Dwarfs, and Neutron Stars : The Physics of Compact Object

D. Maoz (2006)
Astrophysics in a Nutshell

Thermodynamics

Hydrostatic equilibrium
$$\frac{dP(r)}{dr} = -\frac{GM(r)\rho(r)}{r^2}$$

Mass continuity
$$\frac{dM(r)}{dr} = 4\pi r^2 \rho(r)$$

Radiative energy transport
$$\frac{dT(r)}{dr} = -\frac{3L(r)\kappa(r)\rho(r)}{4\pi r^2 4acT(r)^3}$$

Energy conservation
$$\frac{dL(r)}{dr} = 4\pi r^2 \rho(r)\epsilon(r)$$

$$P = P(\rho, T, \text{composition})$$

$$\kappa = \kappa(\rho, T, \text{composition})$$

$$\epsilon = \epsilon(\rho, T, \text{composition})$$

Ideal Degenerate Electron Gas

$$\Delta x \Delta p_x > h \qquad d^3 p dV > h^3$$

$$\rho_{\rm quantum} \approx \frac{m_p}{(\lambda_e/2)^3} = \frac{8 m_p (3 m_e k T)^{3/2}}{h^3}$$

$$\rho_{\rm quantum}(\rm sun, center) \approx 640\ g/cm^3$$

$$\rho_{\rm quantum}(T = 10^8\ {\rm K}) \approx 11,000\ {\rm g/cm^3}$$

$$\boxed{n_e = \int_0^{p_f} \frac{8\pi}{h^3} p^2 dp = \frac{8\pi}{3h^3} p_f^3 = Z n_+ = Z \frac{\rho}{A m_p}}$$

$$\rho_{\rm WD} \sim 10^6\ {\rm g/cm^3}$$

$$T_{\rm WD} \sim 10^7\ {\rm K}$$

Quantum degeneracy pressure dominates : *thermal pressure negligible*

$$\frac{dP(r)}{dr} = -\frac{G M(r) \rho(r)}{r^2}$$

$$p \propto \rho^\gamma \propto \rho^{(n+1)/n}$$

$$P_{\rm deg} \approx 3 \times 10^{22} {\rm dyne/cm^2}$$

$$P_{\rm th} \approx 2 \times 10^{20} {\rm dyne/cm^2}$$

Polytropic Structure

n : polytropic index

ratio of specific heats $\gamma = \frac{c_p}{c_V}$ $pV^\gamma = \text{constant}$

$$\boxed{\text{ideal gas} \quad p \propto \rho^\gamma \propto \rho^{(n+1)/n}}$$

nonrelativistic

$$\gamma = \frac{5}{3} \qquad n = \frac{3}{2} \qquad P_e = \left(\frac{3}{\pi}\right)^{2/3} \frac{h^2}{20 m_e m_p^{5/3}} \left(\frac{Z}{A}\right)^{5/3} \rho^{5/3}$$

ultrarelativistic

$$\gamma = \frac{4}{3} \qquad n = 3 \qquad P_e = \left(\frac{3}{8\pi}\right)^{1/3} \frac{hc}{4 m_p^{4/3}} \left(\frac{Z}{A}\right)^{4/3} \rho^{4/3}$$

Lane-Emden Equation

$$\frac{dP(r)}{dr} = -\frac{GM(r)\rho(r)}{r^2}$$

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho(r)$$

$$\frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2}{\rho}\frac{dP}{dr}\right) = -4\pi G\rho$$

$$P = K\rho^\gamma$$

$$\rho = \rho_c \theta^n$$

$$r = a\xi$$

$$a = \left[\frac{(n+1)K\rho_c^{1/n-1}}{4\pi G}\right]^{1/2}$$

$$\boxed{\frac{1}{\xi}\frac{d}{d\xi}\xi^2\frac{d\theta}{d\xi} = -\theta^n}$$

White Dwarfs

$\mu_e = A/Z \approx 2 \text{ for He, C, O,..}$

Non-relativistic $\gamma = \frac{5}{3} \quad n = \frac{3}{2}$

$$R = 1.122 \times 10^4 \left(\frac{\rho_c}{10^6 \text{ g cm}^{-3}}\right)^{-1/6} \left(\frac{2}{\mu_e}\right)^{5/6} \text{ km}$$

$$M = 0.7011 \left(\frac{R}{10^4 \text{ km}}\right)^{-3} \left(\frac{2}{\mu_e}\right)^5 M_\odot$$

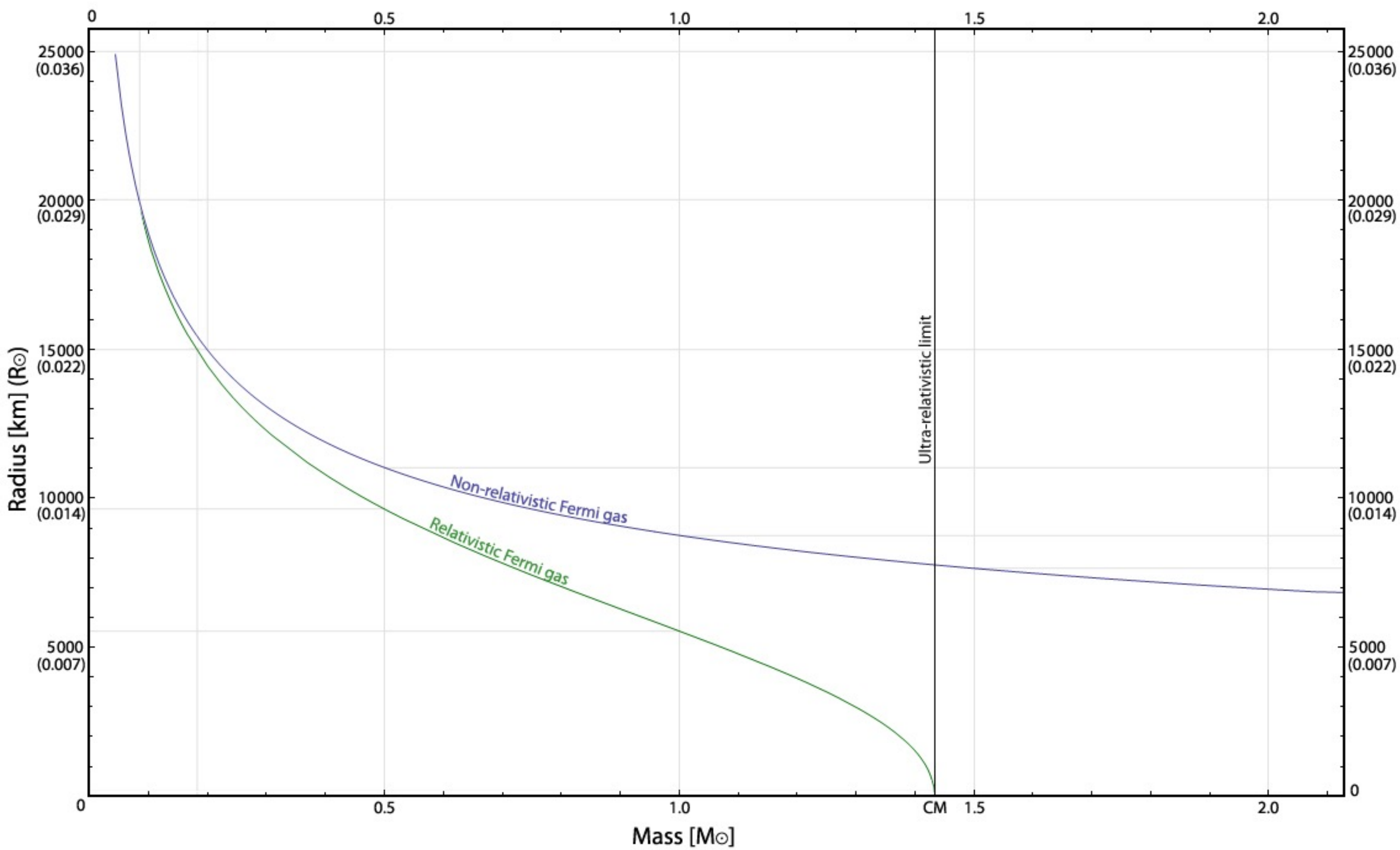
Ultra-relativistic $\gamma = \frac{4}{3} \quad n = 3$

$$R = 3.347 \times 10^4 \left(\frac{\rho_c}{10^6 \text{ g cm}^{-3}}\right)^{-1/3} \left(\frac{2}{\mu_e}\right)^{2/3} \text{ km}$$

$$M = 1.457 \left(\frac{2}{\mu_e}\right)^2 M_\odot$$

independent of central density

White Dwarfs

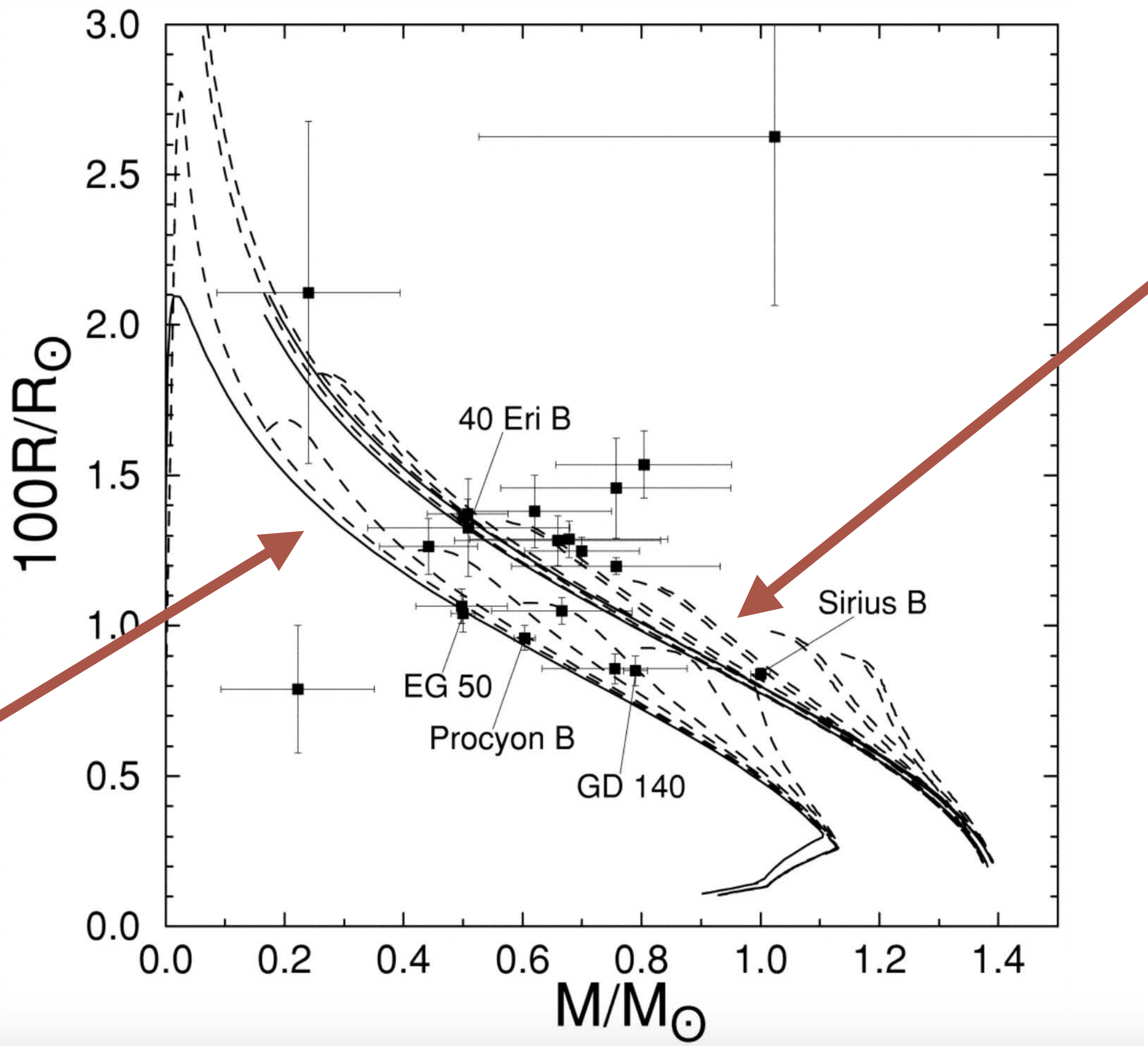


iopscience

White Dwarfs

ApJ 530, 949 (2000)

^{4}He, ^{12}C



^{56}Fe

Physics of Neutron Stars

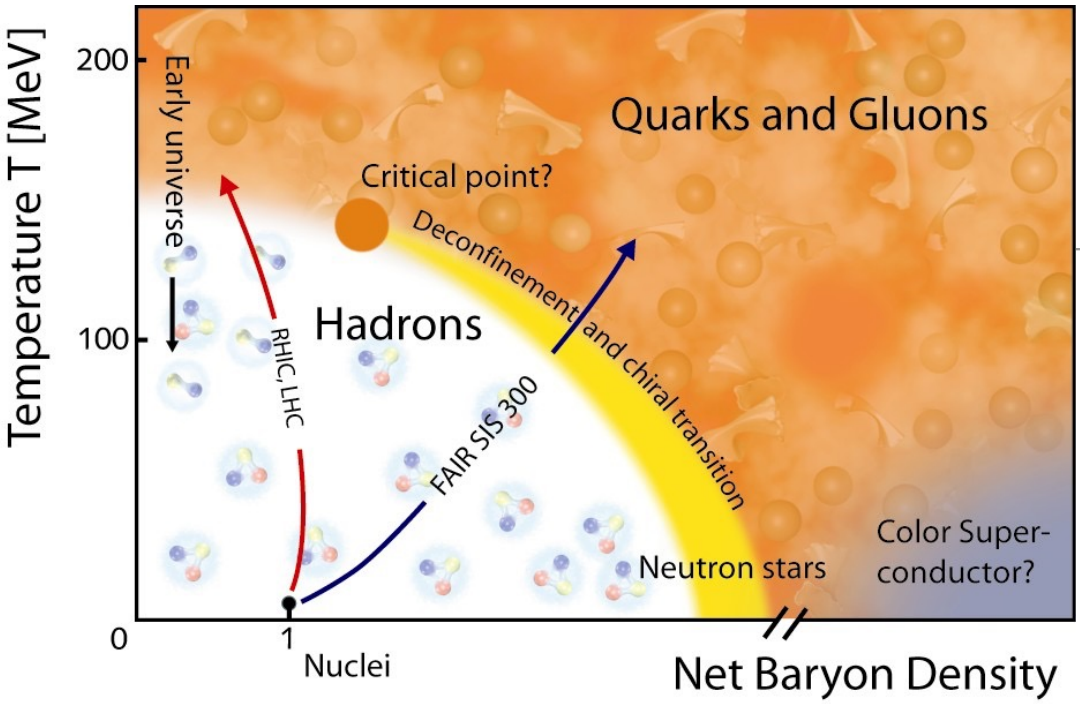
S.L. Shapiro & S.A. Teukolsky (1983)
Black Holes, White Dwarfs, and Neutron Stars: The Physics of Compact Object

J. M. Lattimer (2016)
Neutron Stars are Gold Mines (G.E.Brown Memorial, World Scientific, 2017)

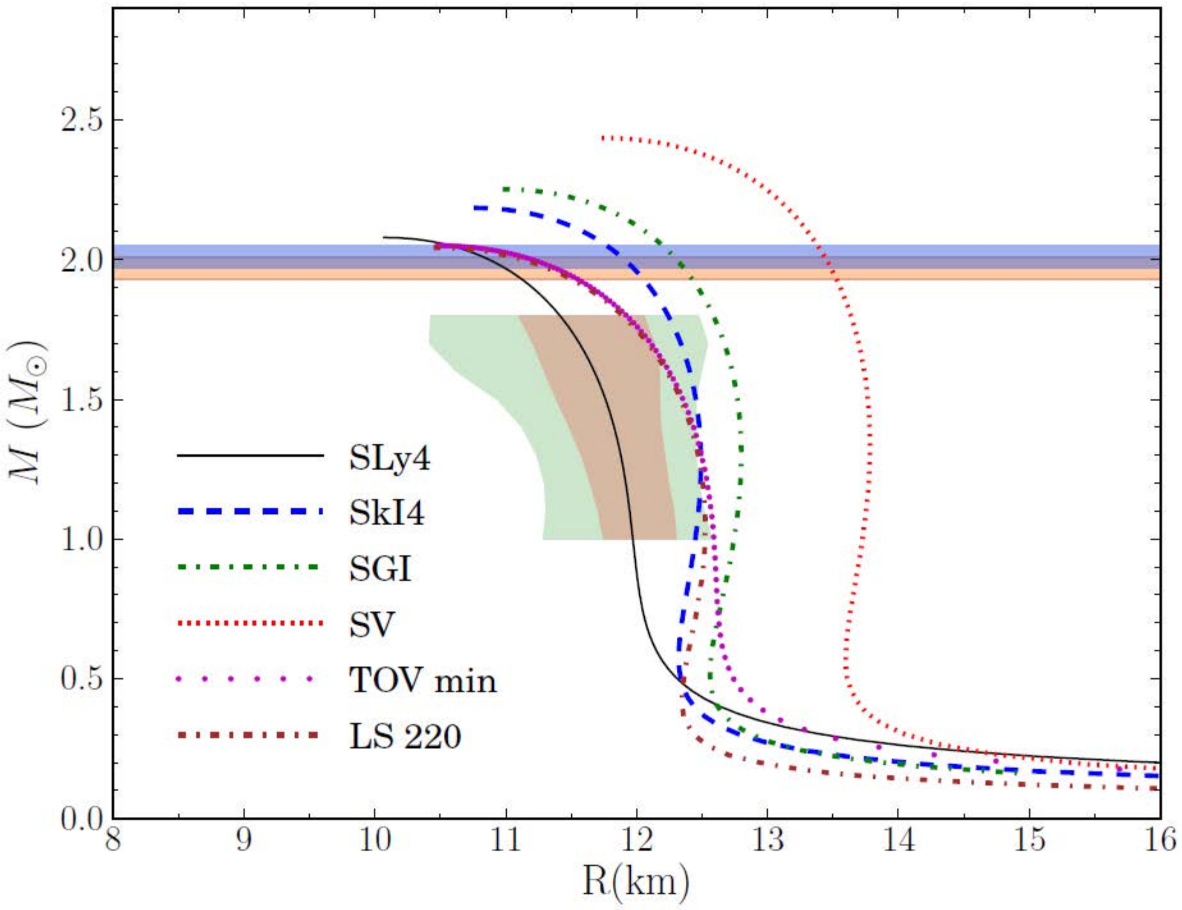
… …

Physics of Dense Matter

- Chiral perturbation theory
- QCD effective models
- Color superconductivity
- Color-flavor locking
- AdS/QCD



Neutron Star
Equation of State

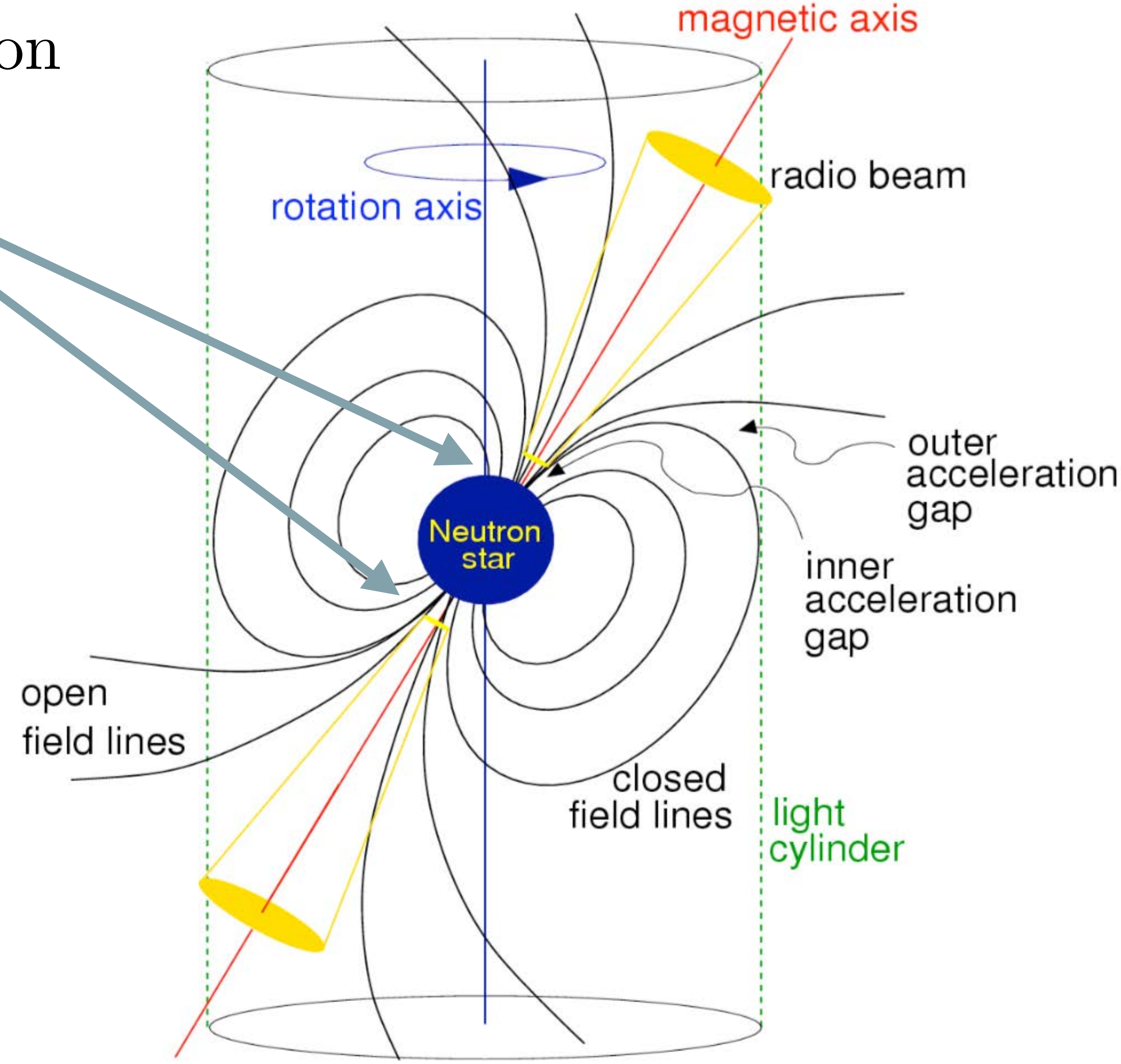


Properties of Neutron Star

e^+e^- pair creation

Pulsar

$$M = 1.5 \sim 2.0 M_\odot$$
$$R = 10 \sim 15 \text{ km}$$
$$A \sim 10^{57} \text{ nucleons}$$



Educated Guesses

white dwarfs

$\rho_{\rm WD} \sim 10^6 \ {\rm g/cm^3}$

$T_{\rm WD} \sim 10^7 \ {\rm K}$

$$R = 3.347 \times 10^4 \left(\frac{\rho_c}{10^6 \ {\rm g \ cm^{-3}}} \right)^{-1/3} \left(\frac{2}{\mu_e} \right)^{2/3} \ {\rm km}$$

$$M = 1.457 \left(\frac{2}{\mu_e} \right)^2 M_\odot$$

Quantum degeneracy pressure of protons/neutrons

neutron stars

$\rho \sim 10^{14} {\rm g/cm^3}$

$$R_{\rm NS} \approx 2.3 \times 10^9 \ {\rm cm} \left(\frac{m_e}{m_n} \right) \left(\frac{1}{\mu_n} \right)^{5/3} \left(\frac{M}{M_\odot} \right)^{-1/3} \approx 14 \ {\rm km} \left(\frac{M}{1.4 M_\odot} \right)^{-1/3}$$

$$M_{\rm NS,Chan} \approx 0.2 \left(\frac{2}{\mu_n} \right)^2 \left(\frac{hc}{G m_p^2} \right)^{3/2} m_p \approx 4 \times 1.4 M_\odot = 5.6 M_\odot$$

$$\mu_e = A/Z \rightarrow \mu_n = 1$$

Neutron Star Formation from Fe Core Collapse

When Fe core reaches Chandrasekhar Limit

Nuclear Photodisintegration

$$\gamma + {}^{56}\mathrm{Fe} \rightarrow 13\ {}^{4}\mathrm{He} + 4\ \mathrm{n}\ (124\ \mathrm{MeV\ is\ consumed})$$

$$\gamma + {}^{4}\mathrm{He} \rightarrow 2\ \mathrm{p} + 2\ \mathrm{n}\ (28.3\ \mathrm{MeV\ is\ consumed})$$

$$E_{\mathrm{consumed}}(10^{57}\mathrm{p}) \sim 1.4 \times 10^{52}\ \mathrm{erg} \sim 10^{11}\mathrm{year\ with}\ L_{\odot}$$

Neutralisation

$$\mathrm{e}^{-} + \mathrm{p} \rightarrow \mathrm{n} + \nu_{\mathrm{e}}$$

$$\mathrm{e}^{-} + {}^{56}\mathrm{Fe} \rightarrow {}^{56}\mathrm{Mn} + \nu_{\mathrm{e}}$$

$$\mathrm{e}^{-} + {}^{\mathrm{Mn}}\mathrm{Fe} \rightarrow {}^{56}\mathrm{Cr} + \nu_{\mathrm{e}}$$

$$\tau_{\mathrm{fall}} \sim \mathrm{a\ few\ seconds}$$

$$R_{\mathrm{Fe}} \sim 1500\mathrm{km}$$

$$R_{\mathrm{NS}} \sim \mathcal{O}(10\mathrm{km})$$

General Relativity

hydrostatic equilibrium

$$\frac{dP}{dr} = -\frac{GM\rho}{r^2}$$

$$\frac{dM}{dr} = 4\pi r^2 \rho$$

$$\frac{dP}{dr} = -\frac{GM\rho}{r^2}\left(1 + \frac{P}{\rho c^2}\right)\left(1 + \frac{4\pi P r^3}{Mc^2}\right)\left(1 - \frac{2GM}{rc^2}\right)^{-1}$$

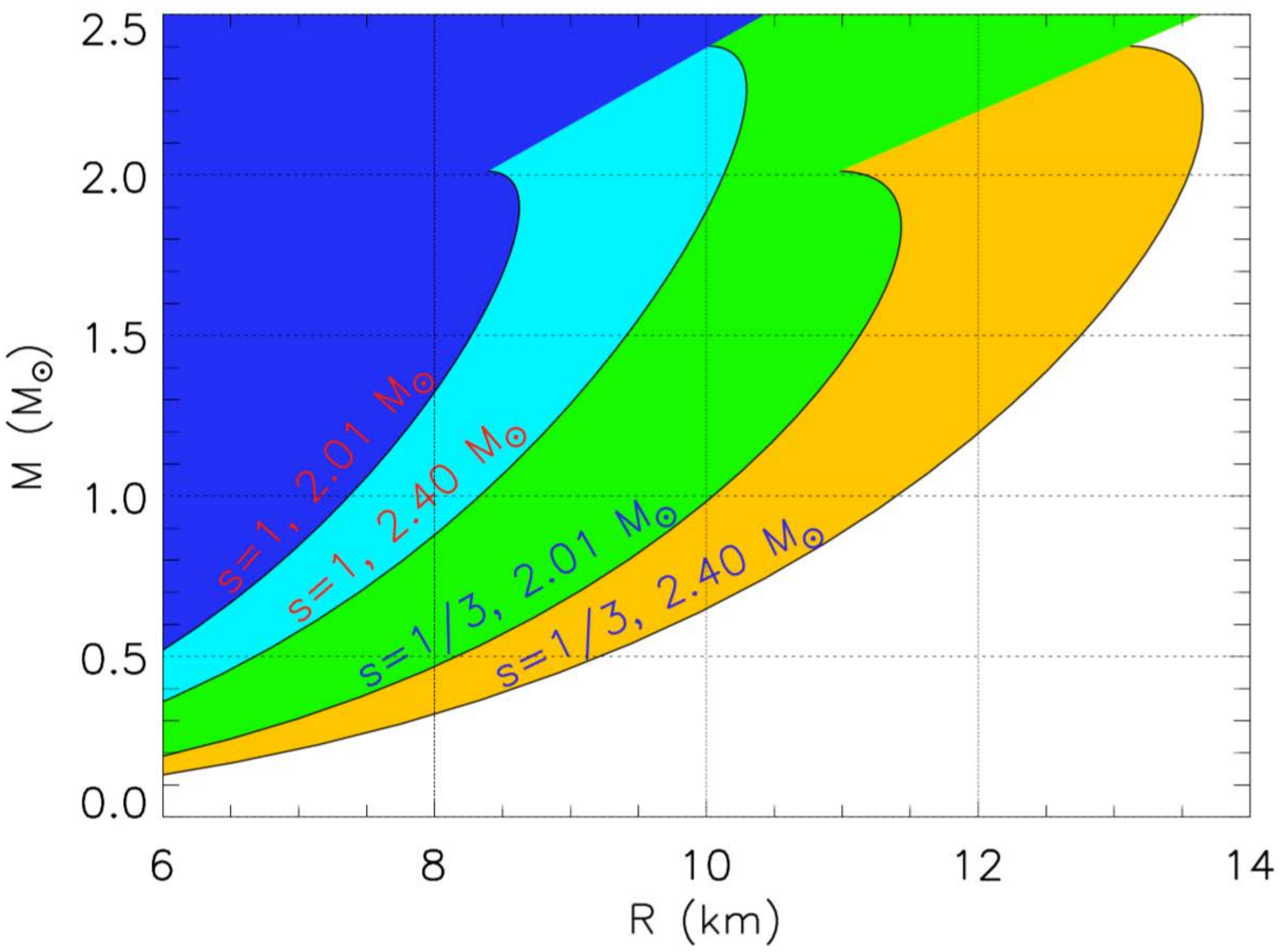
$$\frac{dM}{dr} = 4\pi r^2 \left(\frac{\epsilon}{c^2}\right)$$

includes all energy sources

physics of dense nuclear matter
(strong interaction)

Causality Limit

$$s = c_s^2 = \frac{\partial p}{\partial \epsilon} \; (c_s : \text{speed of sound})$$

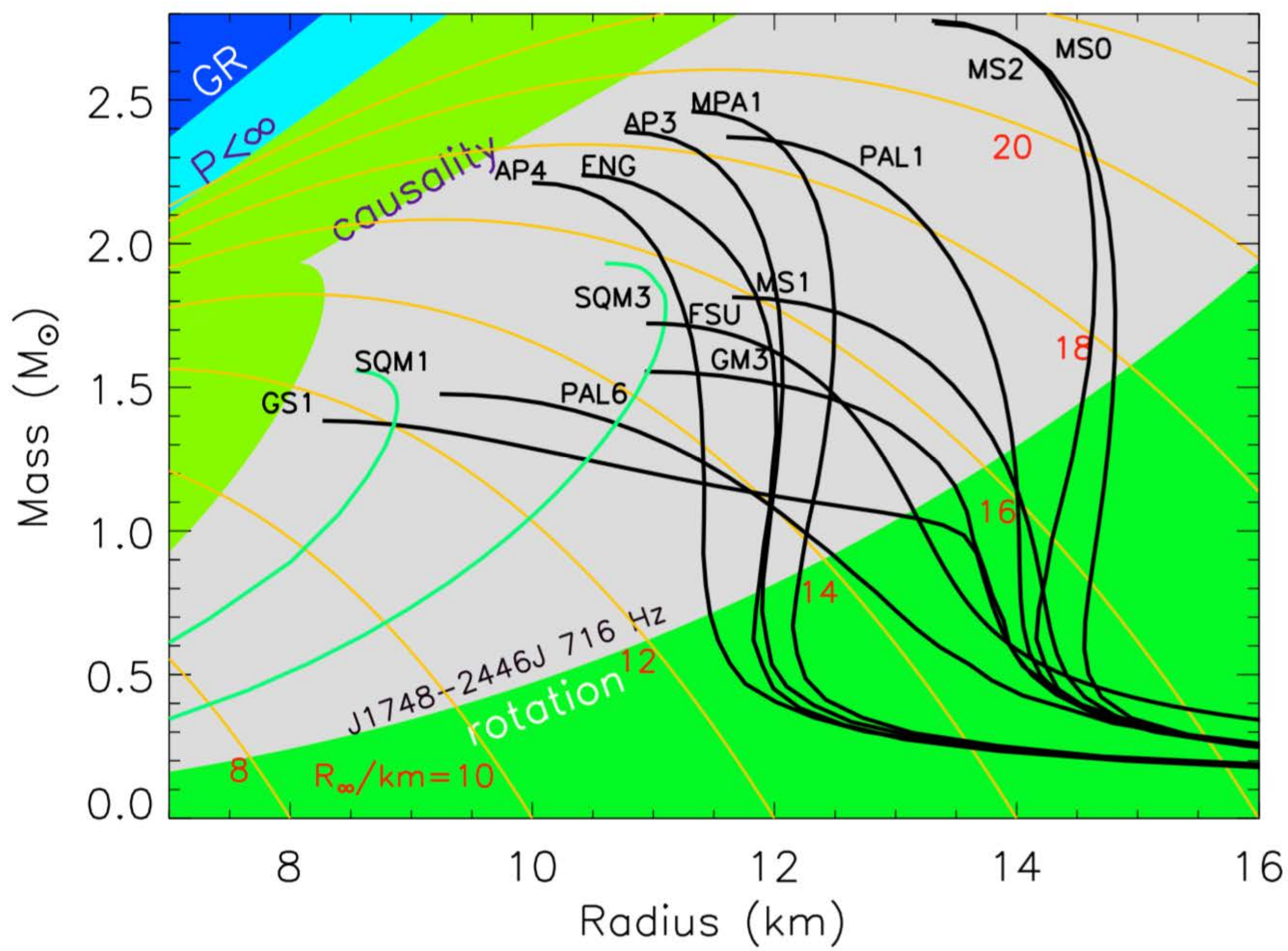


compactness $\beta = \dfrac{2GM}{rc^2}$

$$\beta_{\max} = 0.271$$

$$s = \frac{1}{3} \text{ for ideal gas}$$

Neutron Star Equation of States



Q) How to understand ?

Polytropic Structure *without general relativity*

$$\gamma = (n+1)/n$$

$$P = K\rho^{\gamma}$$

$$M \propto K^{1/(3\gamma-4)} R^{(\gamma-2)/(3\gamma-4)}$$

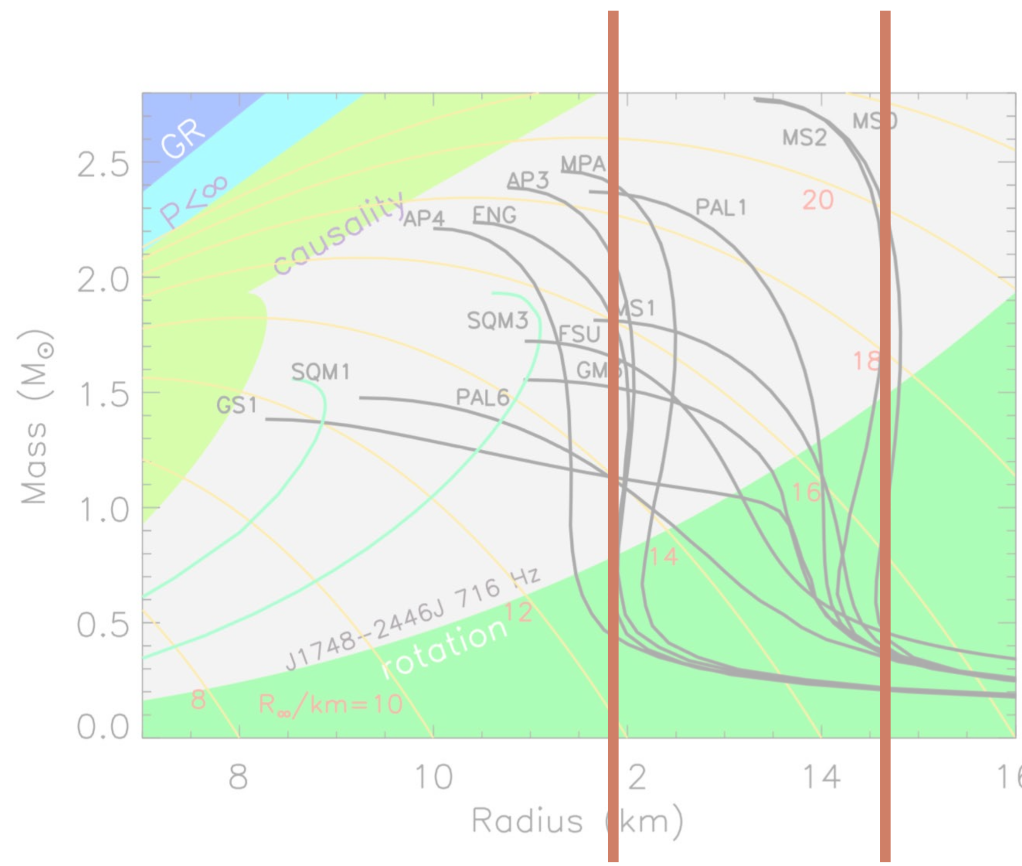
$$R \propto \left(\frac{(n+1)K}{4\pi G}\right)^{1/2} \rho_c^{(1-n)/2n}$$

R **is independent of** ***M***

$$\gamma \approx 2 \; (n \approx 1)$$

non-relativistic $\gamma = \frac{5}{3}$

ultra-relativistic $\gamma = \frac{4}{3}$



Polytropic Structure

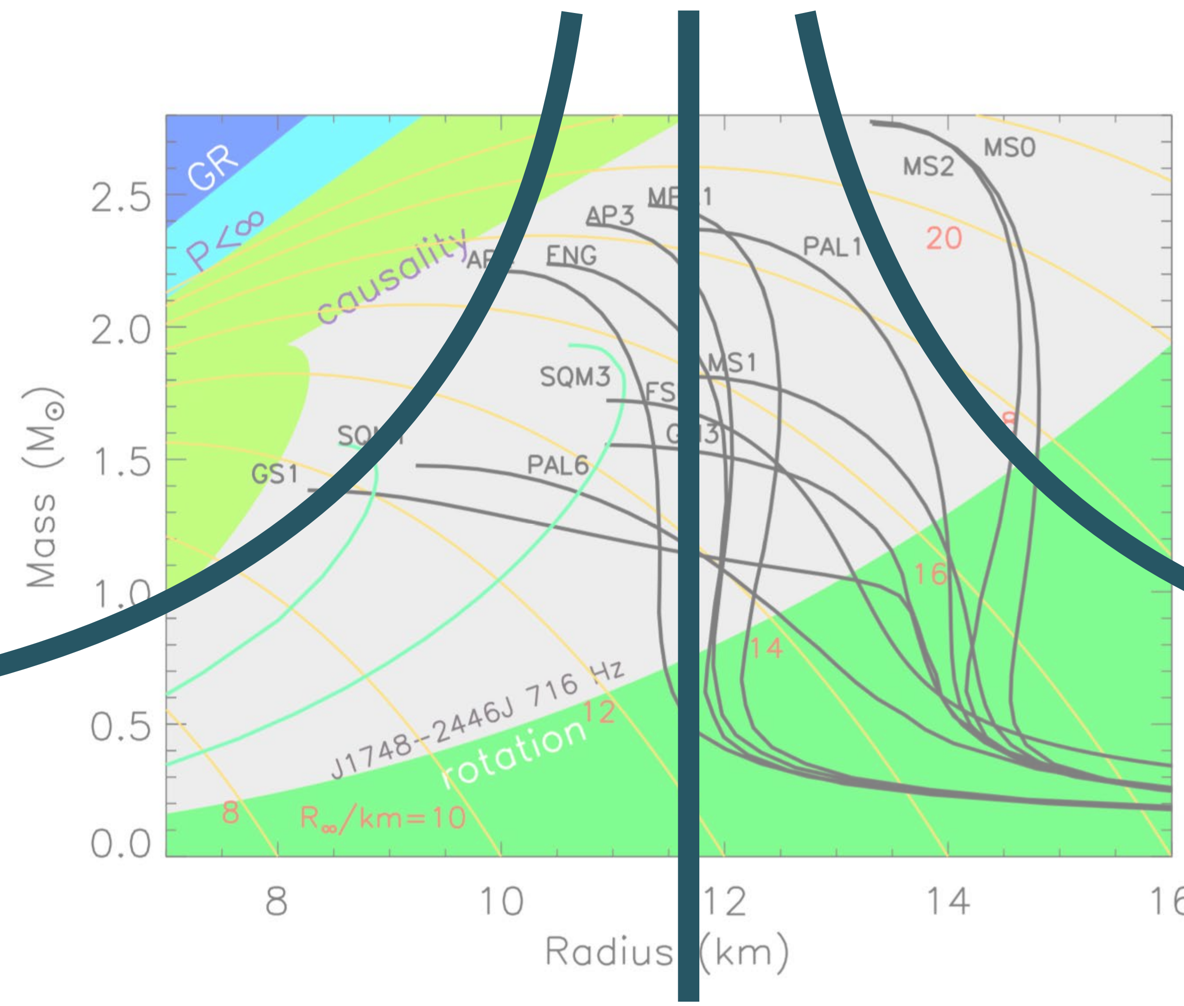
$$\gamma = (n+1)/n$$

$$M \propto K^{1/(3\gamma-4)} R^{(\gamma-2)/(3\gamma-4)}$$

$$\gamma \approx 2 \; (n \approx 1)$$

$$\gamma > 2$$

$$3/4 < \gamma < 2$$



Piecewise Polytopes: Numerical Relativity

Nuclear matter is not an ideal gas

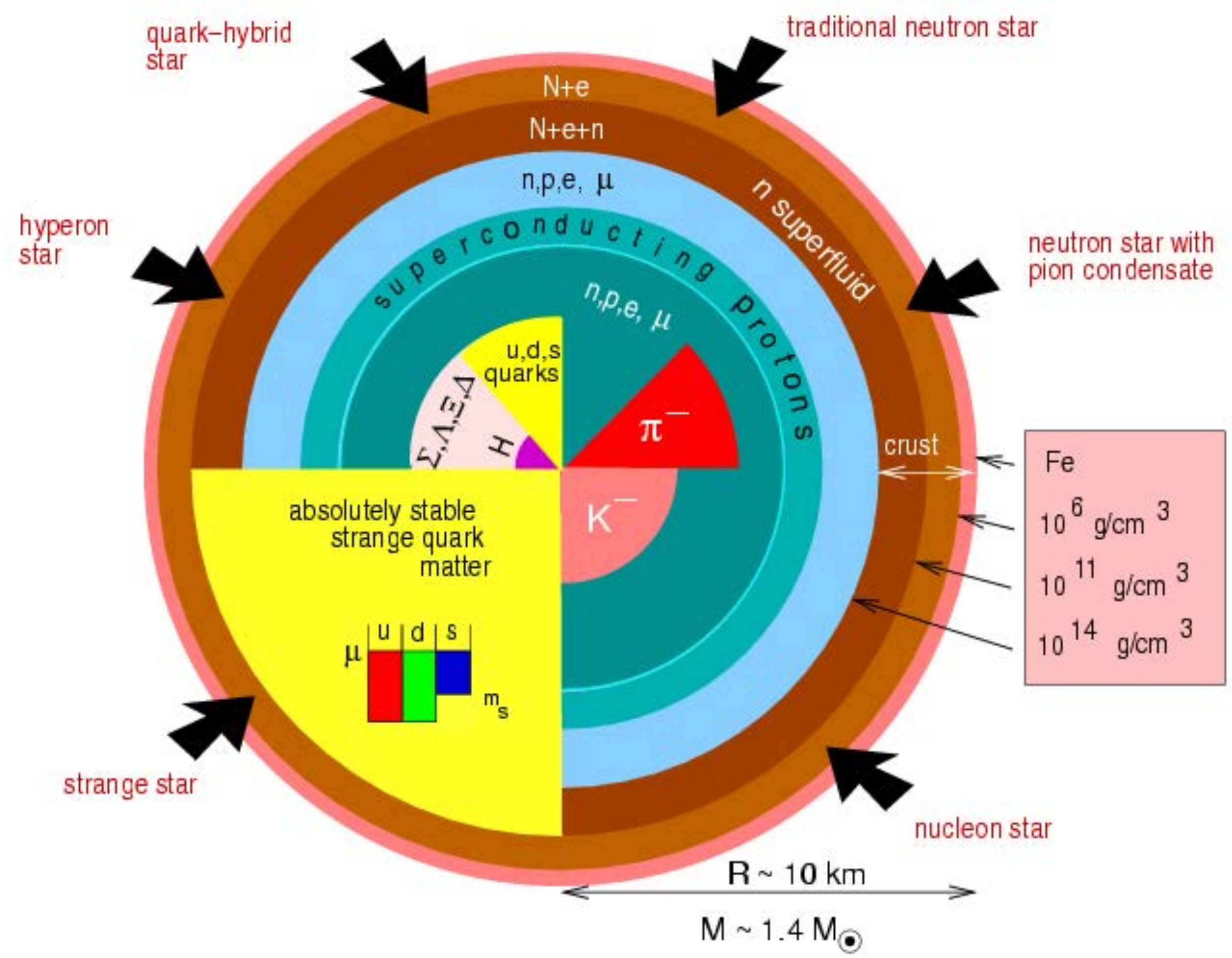
F. Weber 2005

nonrelativistic $\gamma = \frac{5}{3}$

ultrarelativistic $\gamma = \frac{4}{3}$

NS eos includes

$$\gamma \approx 2 \ (n \approx 1)$$



- **still uncertain due to the nature of strong interactions**
- **introduction of 3 body forces**
- **exotic states with strangeness**
- **… …**

Neutron Star Observations

- Radio pulsars
- Cooling
- Low-mass X-ray binaries
- Particle abundances
- Gravitational Waves

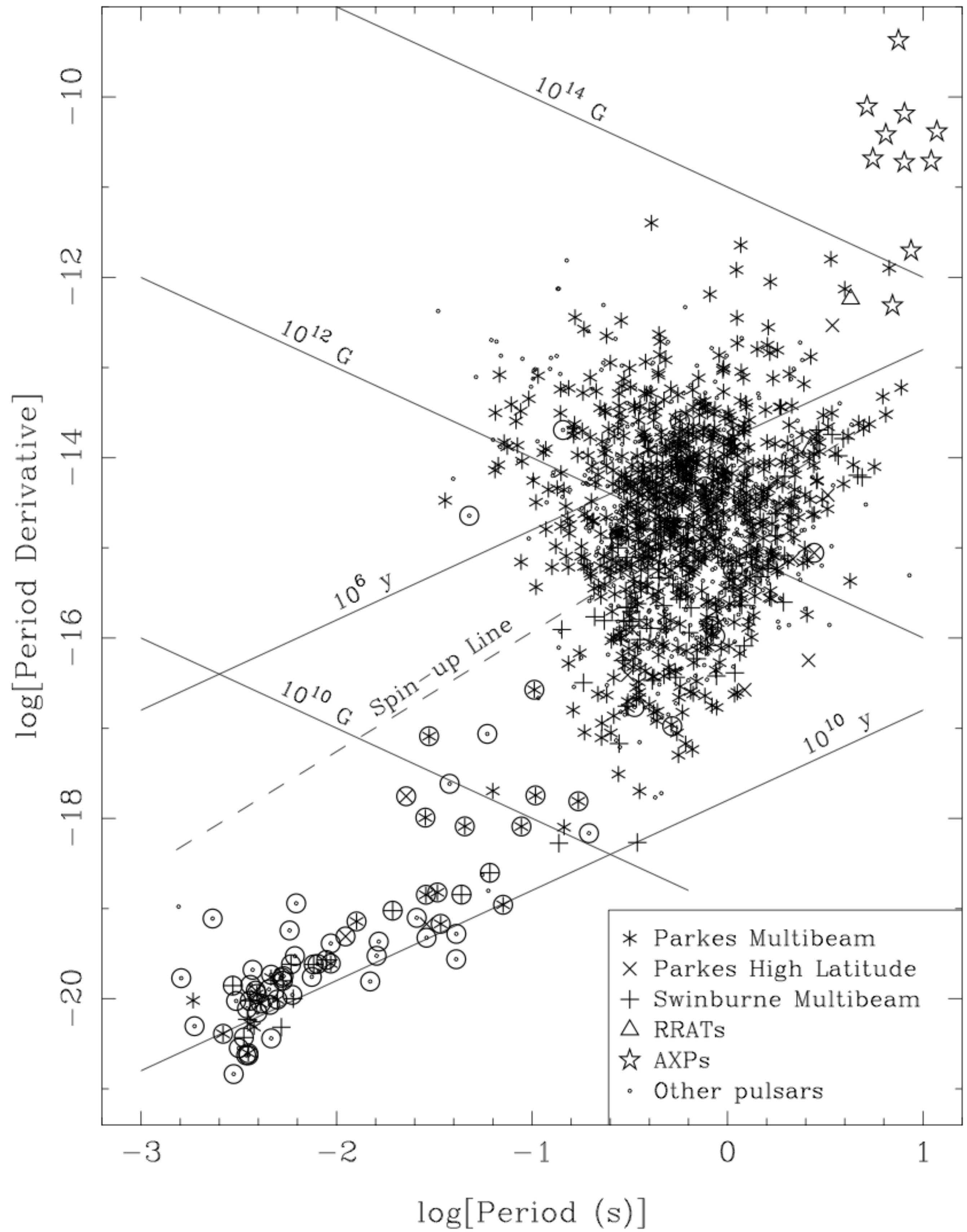
Millisecond Pulsars

Dipole Radiation

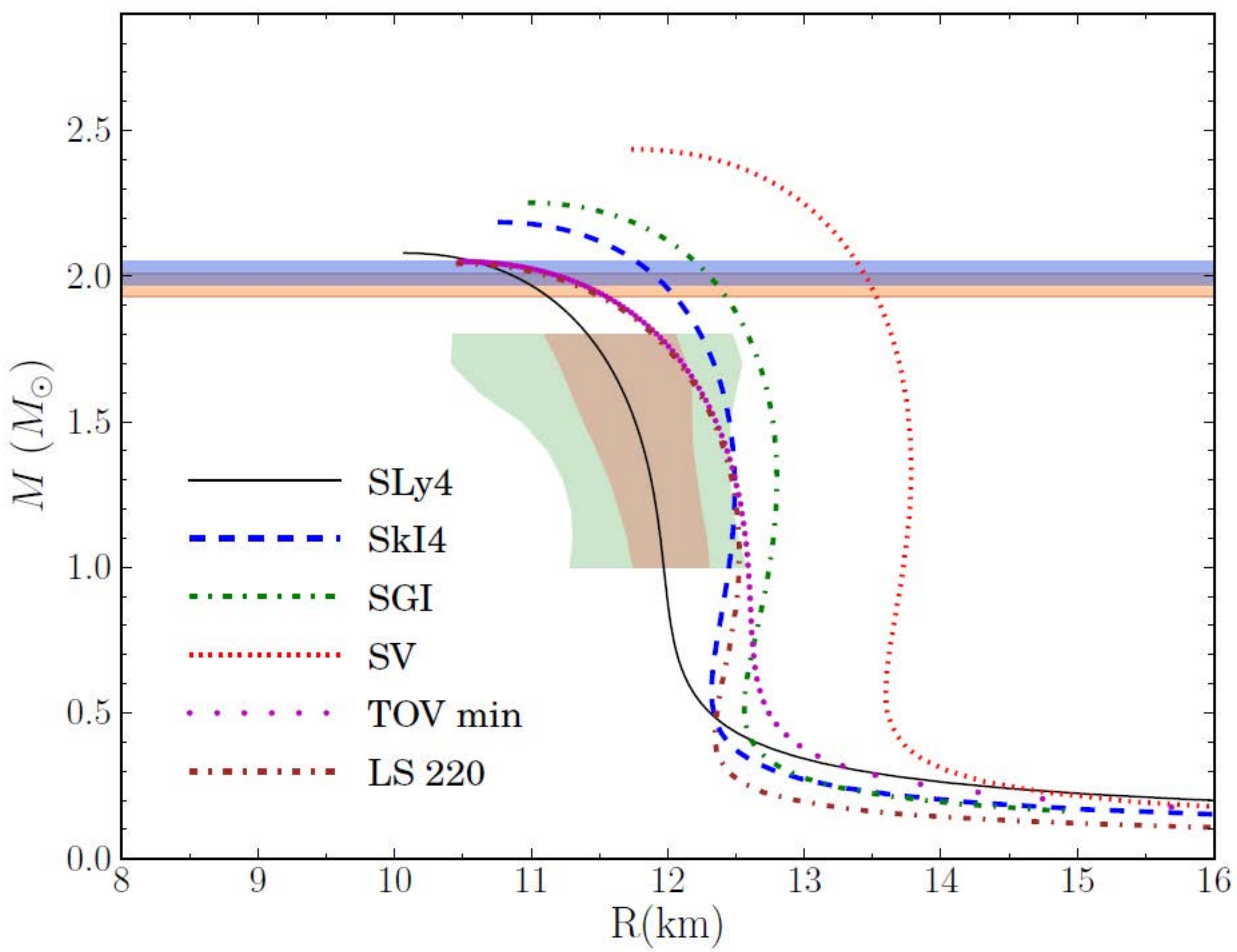
$$\dot{E}_{\rm rot} = I\Omega\dot{\Omega}$$

$$\dot{E}_{\rm dipole} = -\frac{B_\perp^2 R^6 \Omega^4}{6c^3}$$

$$\dot{\Omega} = -\frac{B_\perp^2 R^6 \Omega^3}{6Ic^3}$$



Maximum Mass of Neutron Stars



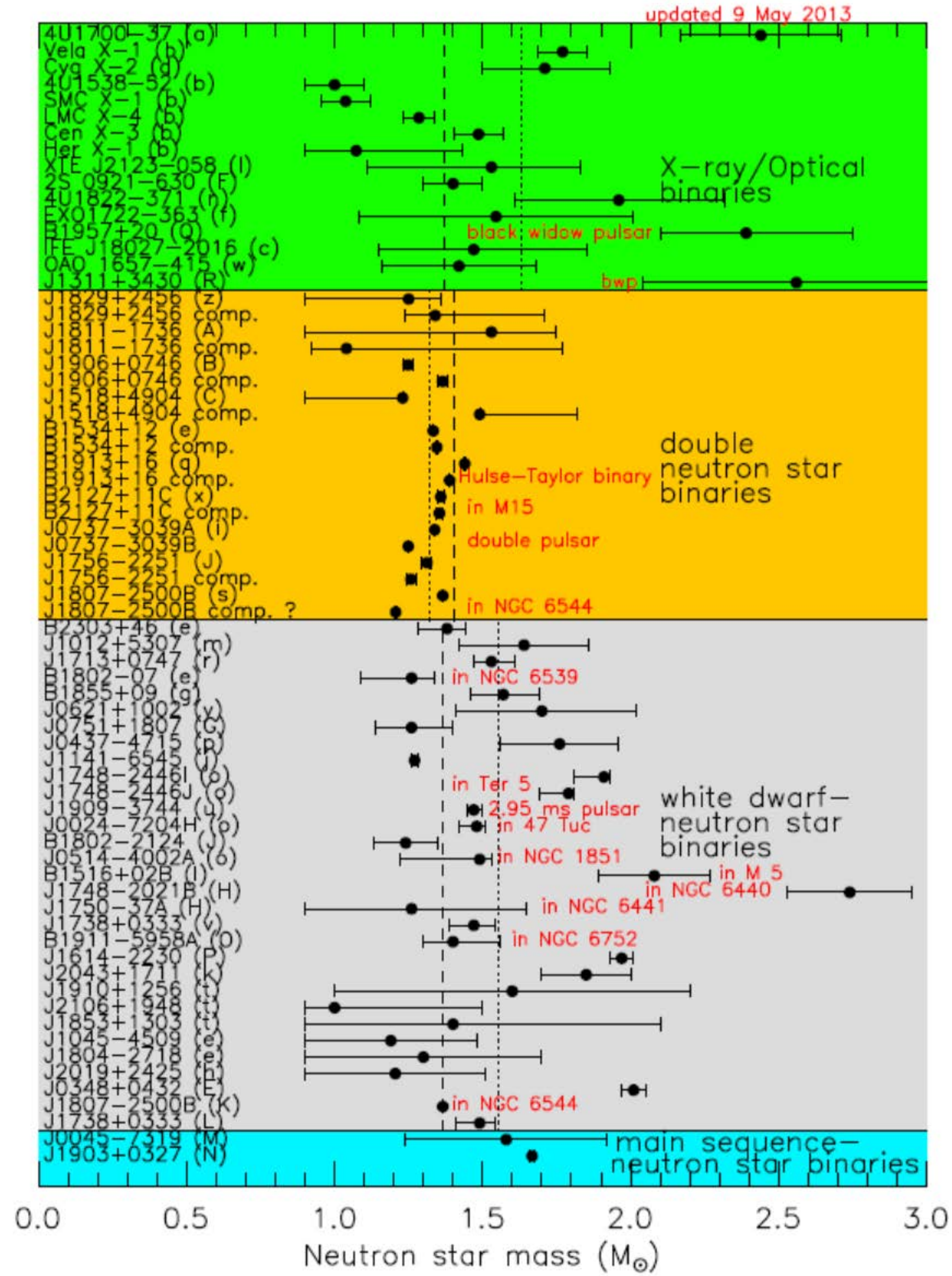
Neutron Star-White Dwarf Binaries

1.97 solar mass NS : Nature 467 (2010) 1081

2.01 solar mass NS : Science 340 (2013) 6131

Masses

- High-mass neutron stars in X-ray binaries & white dwarf-NS binaries (2010 & 2013)
- Less than 1.5 solar mass in double NS binaries
- Maximum NS mass is still uncertain



Prakash 2013

Moment of Inertia / Glitches

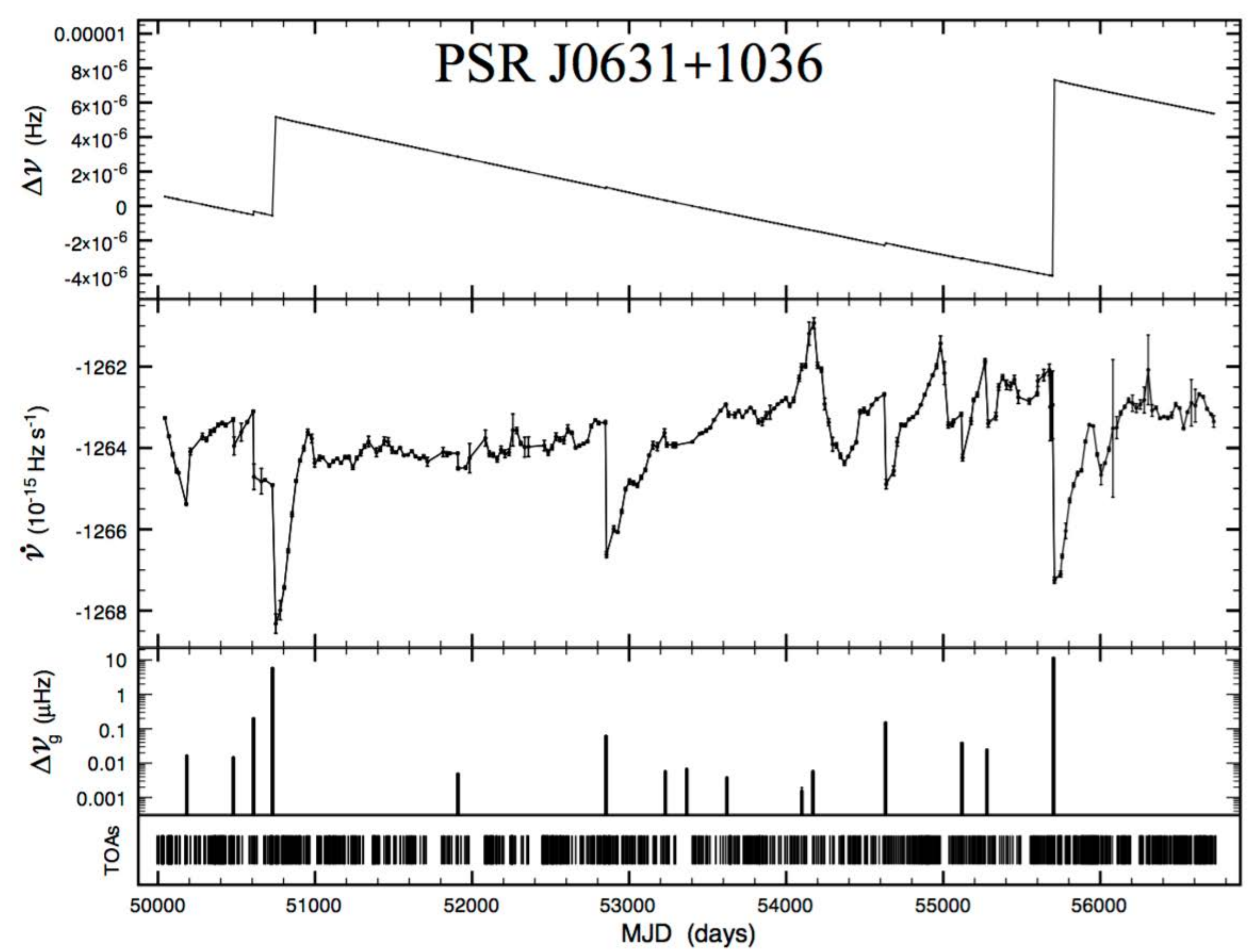
$$\dot{E}_{\rm rot} = I\Omega\dot{\Omega}$$

$$\dot{E}_{\rm dipole} = -\frac{B_\perp^2 R^6 \Omega^4}{6c^3}$$

$$\dot{\Omega} = -\frac{B_\perp^2 R^6 \Omega^3}{6Ic^3}$$

$$\dot{\Omega} \propto -\Omega^n$$

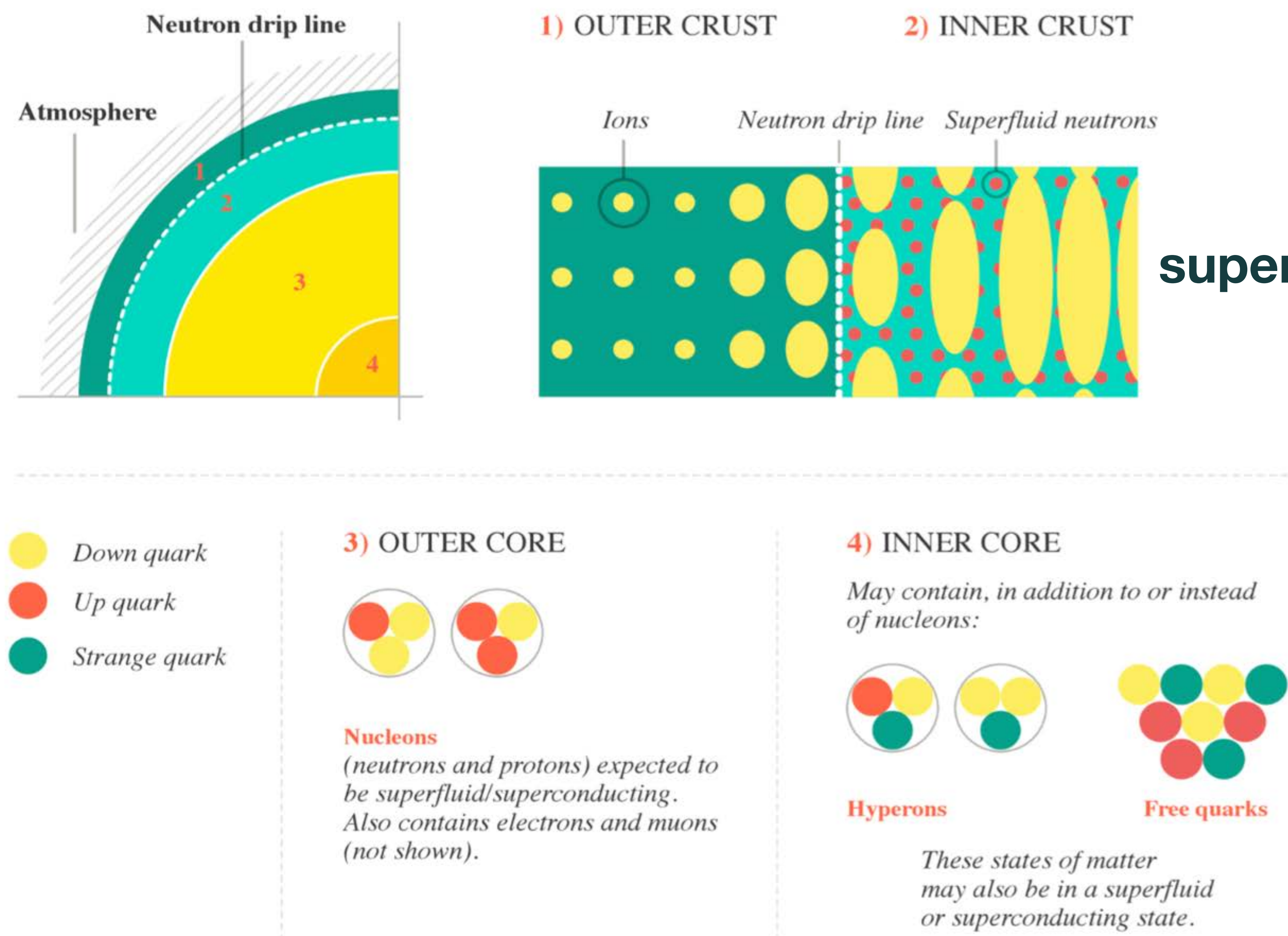
$$\tau_{\rm pulsar} = \frac{\Omega}{(1-n)\dot{\Omega}}$$



D. Antonopoulou (U. Amsterdam, 2015)

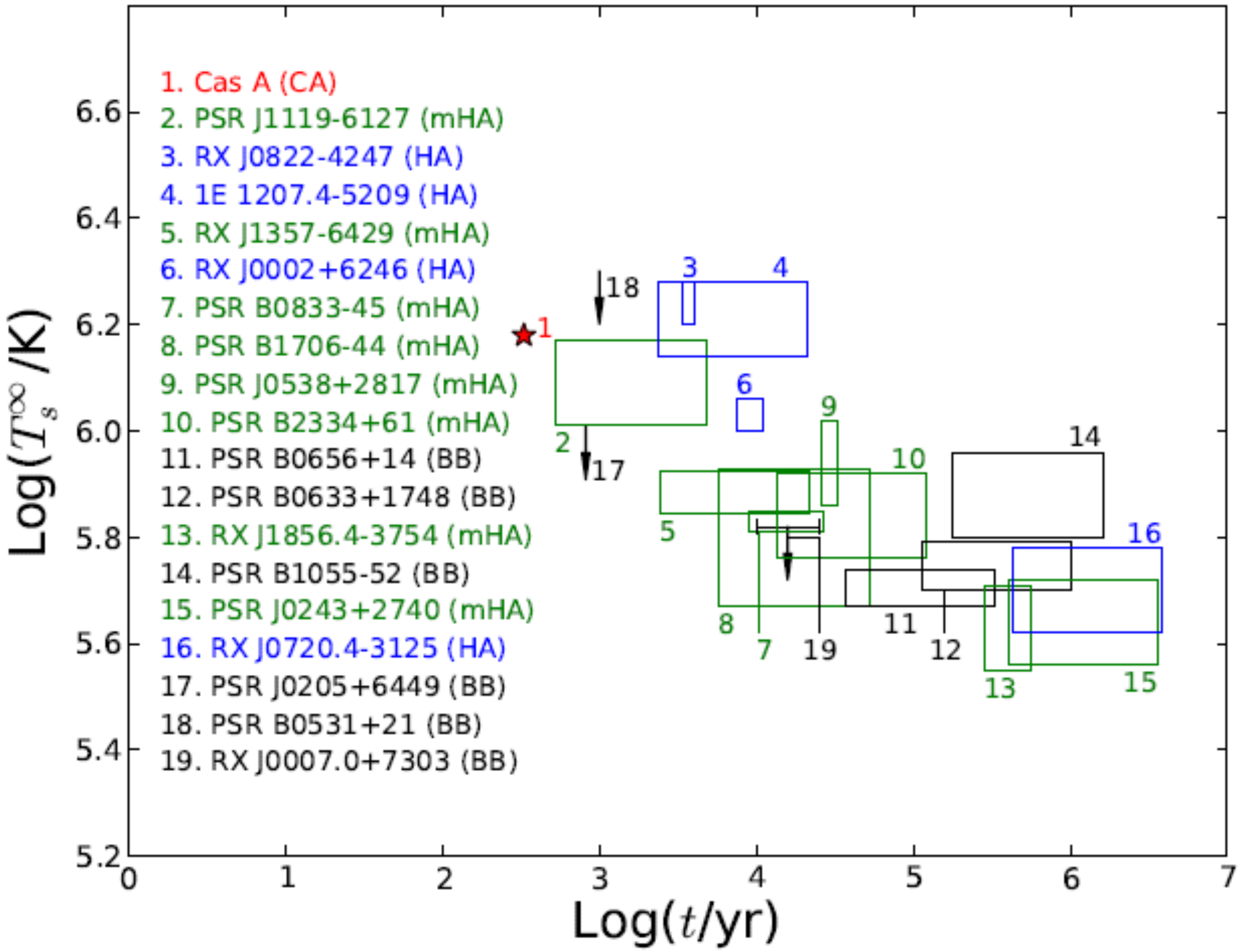
Superfluid Neutrons

D. Antonopoulou (U. Amsterdam, 2015)



superfluid

Neutron Star Cooling

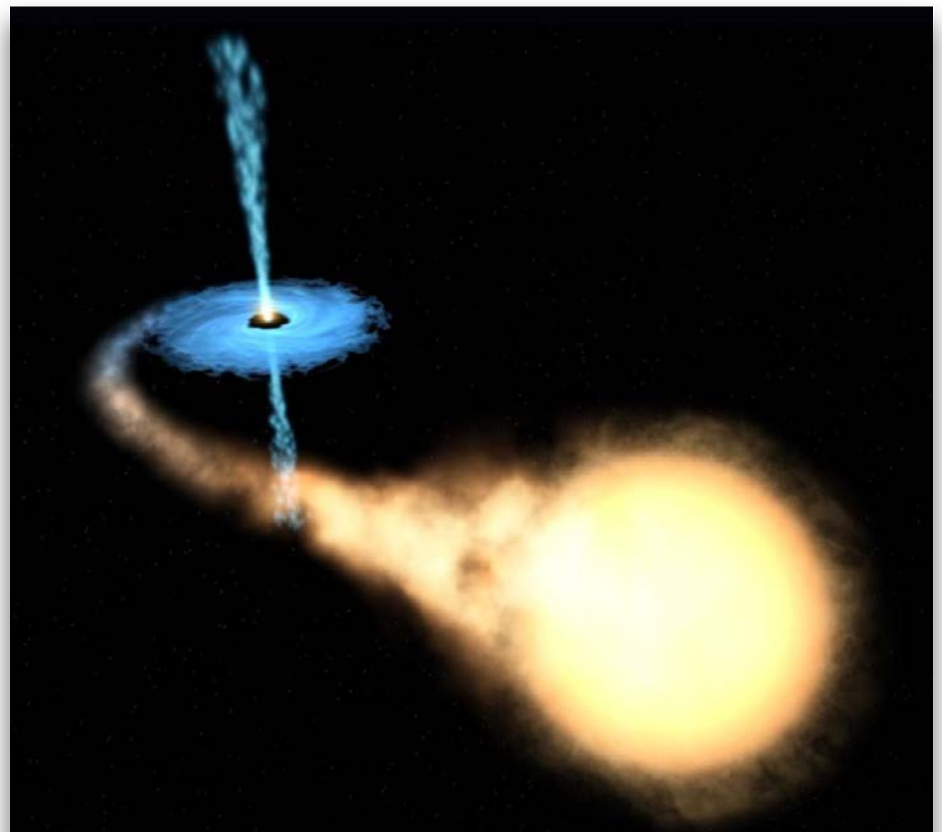


depends on

- particle fraction
- elements in the envelope
- nuclear superfluidity
- … …

arXiv:1501.04397

Low-Mass X-ray Binaries (LMXB)



Accreting Object: NS or BH

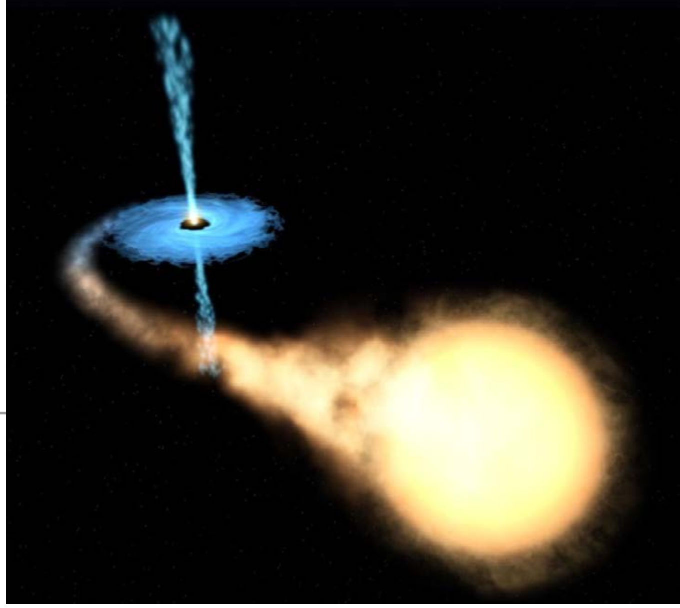
Companion: Low-Mass Main Sequence

Age: Old (> 10^9 year)

Accretion timescale: 10^7 - 10^9 year

X-ray energy: Soft (< 10 keV)

low-mass & high-mass X-ray binaries



Properties	LMXBs	HMXBs
Accreting object	Low B-field NS or BH	High B-field NS or BH
Companion	Low-mass main sequence	High-mass (O or B type) main sequence
	$(L_{opt}/L_x \ll 0.1)$	$(L_{opt}/L_x > 1)$
Stellar population	Old$(> 10^9$ yr)	Young$(< 10^7$ yr)
Mechanism	Roche-lobe overflow	Stellar wind
Accretion timescale	$10^7 - 10^9$ yr	10^5 yr
Variability	X-ray bursts, Transient behavior	Regular X-ray pulsation
X-ray spectra	Soft (≤ 10 keV)	Hard (≥ 15 keV)

Table 1: Summary of LMXBs and HMXBs (Rosswog et al. 2011)

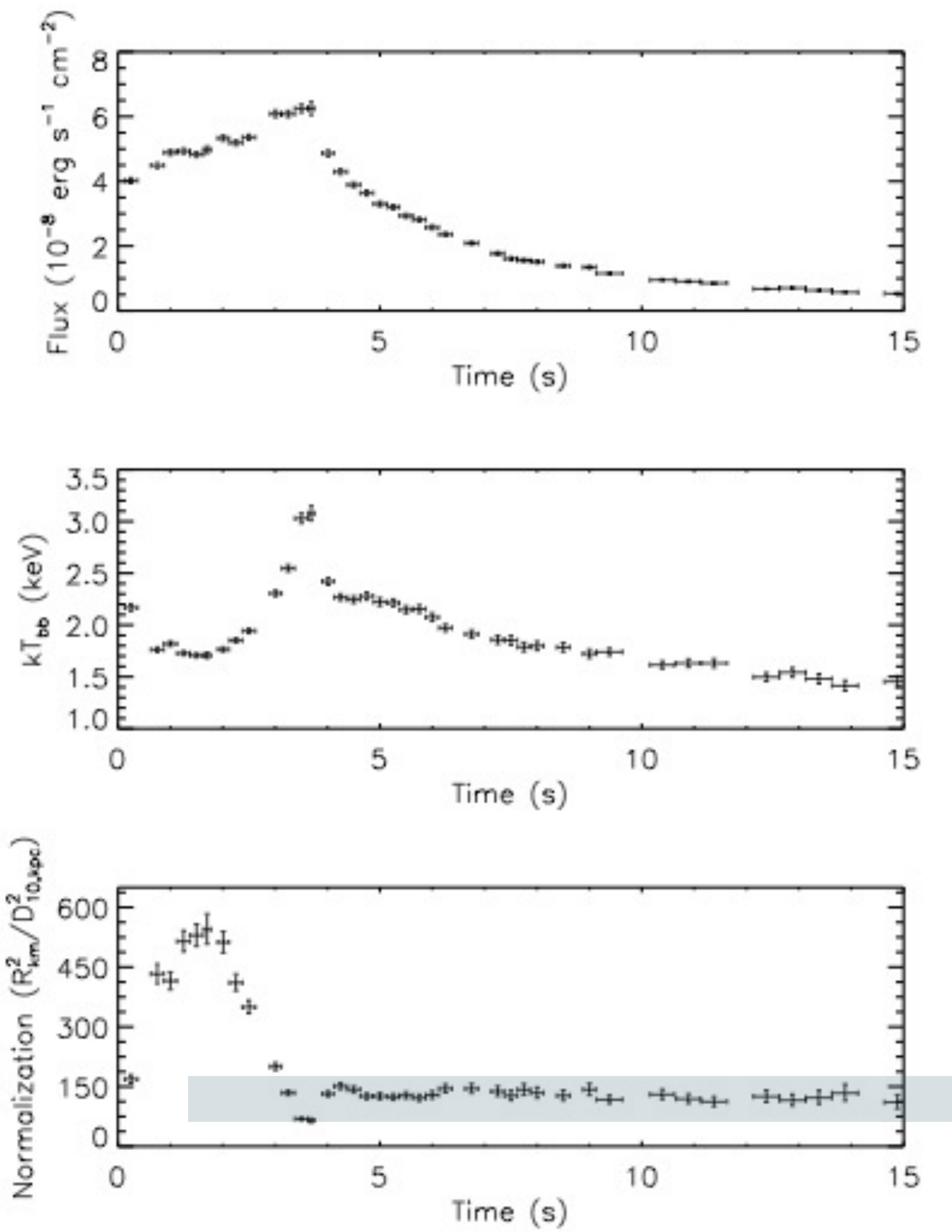
M & R from LMXB

with Myungkuk Kim, Young-Min Kim, Kyujin Kwak

flux

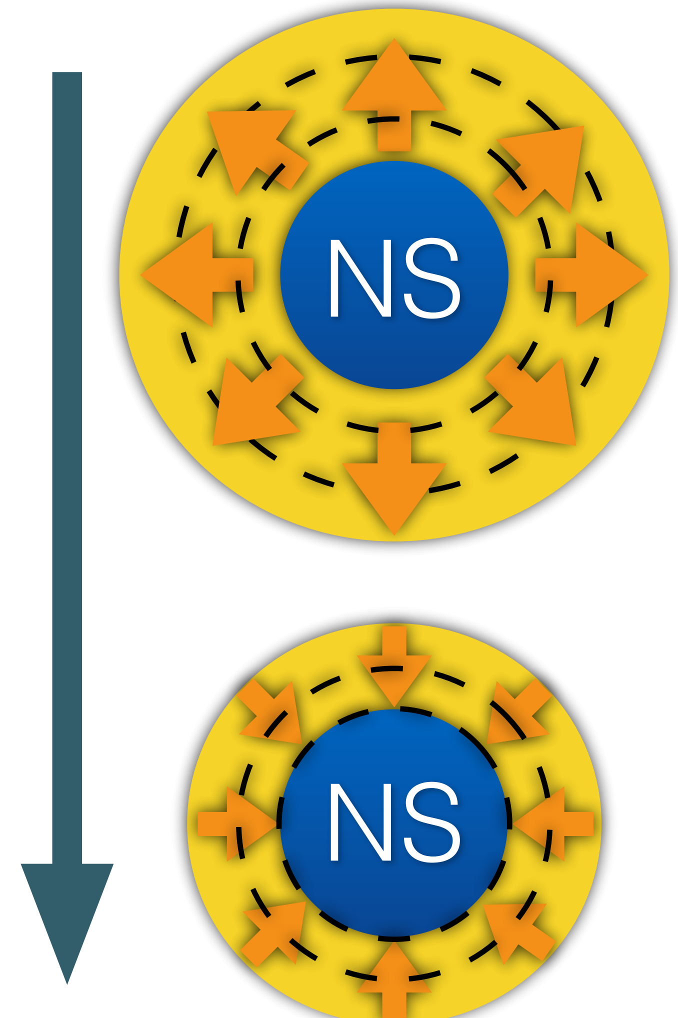
temperature

radius



Ozel et al. 2009

expansion



touchdown

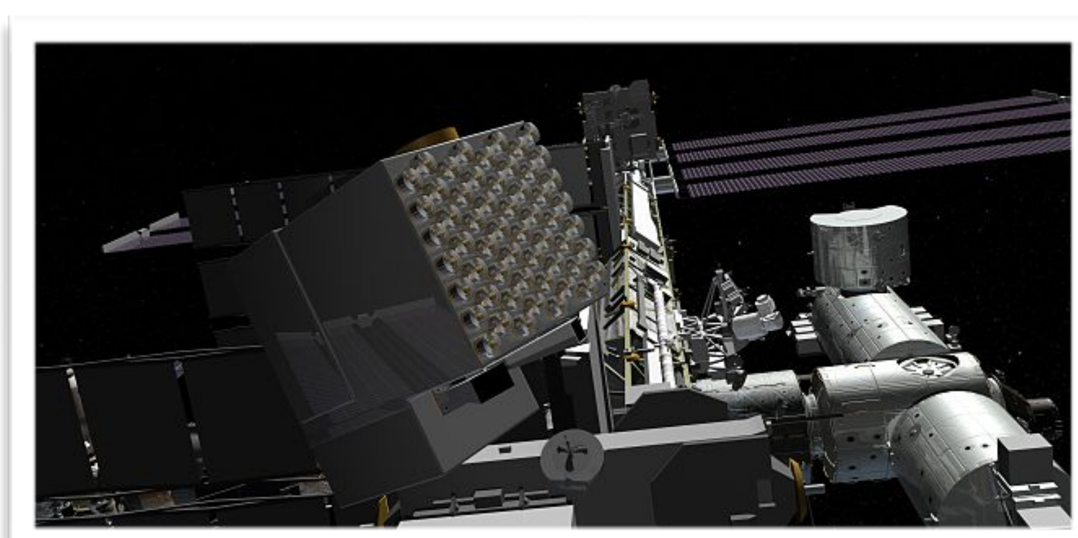
Low-Mass X-ray Binaries (LMXB)

Steiner, Lattimer, Brown, ApJ, 2010

Object	M ($M_\odot$)	R (km)	M ($M_\odot$)	R (km)
	$r_{\rm ph} = R$		$r_{\rm ph} \gg R$	
4U 1608–522	$1.52^{+0.22}_{-0.18}$	$11.04^{+0.53}_{-1.50}$	$1.64^{+0.34}_{-0.41}$	$11.82^{+0.42}_{-0.89}$
EXO 1745–248	$1.55^{+0.12}_{-0.36}$	$10.91^{+0.86}_{-0.65}$	$1.34^{+0.450}_{-0.28}$	$11.82^{+0.47}_{-0.72}$
4U 1820–30	$1.57^{+0.13}_{-0.15}$	$10.91^{+0.39}_{-0.92}$	$1.57^{+0.37}_{-0.31}$	$11.82^{+0.42}_{-0.82}$
M13	$1.48^{+0.21}_{-0.64}$	$11.04^{+1.00}_{-1.28}$	$0.901^{+0.28}_{-0.12}$	$12.21^{+0.18}_{-0.62}$
ω Cen	$1.43^{+0.26}_{-0.61}$	$11.18^{+1.14}_{-1.27}$	$0.994^{+0.51}_{-0.21}$	$12.09^{+0.27}_{-0.66}$
X7	$0.832^{+1.19}_{-0.051}$	$13.25^{+1.37}_{-3.50}$	$1.98^{+0.10}_{-0.36}$	$11.3^{+0.95}_{-1.03}$

NICER Neutron star Interior Composition ExploreR

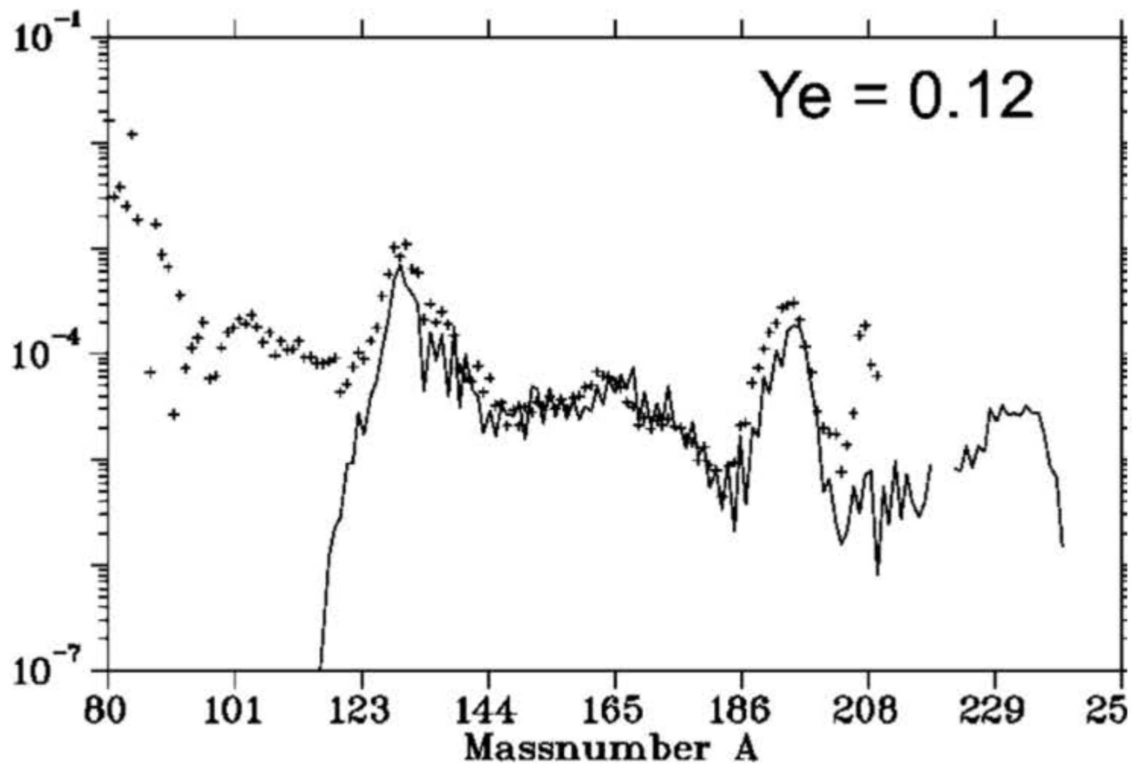
- launch: early 2017, SpaceX
- platform: ISS ELC (ExPRESS Logistics Carrier)
- instrument: X-ray (0.2-12 keV)
- objective
  - structure: neutron star radii to 5%, cooling timescales
  - dynamics: stability of pulsars as clocks, properties of outbursts, oscillations, and precession
  - energetics: intrinsic radiation patterns, spectra, and luminosities



Neutron Star Mergers

- **Gravitational Waves**
- **Sources of Heavy Elements**

S Rosswog 2015

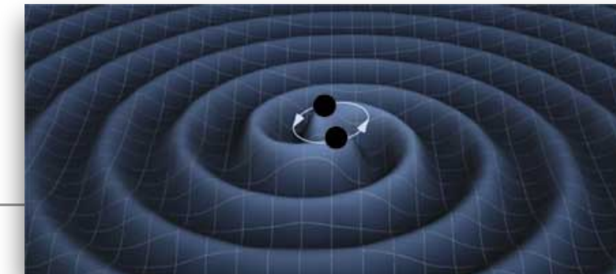


solar pattern vs NS-merger

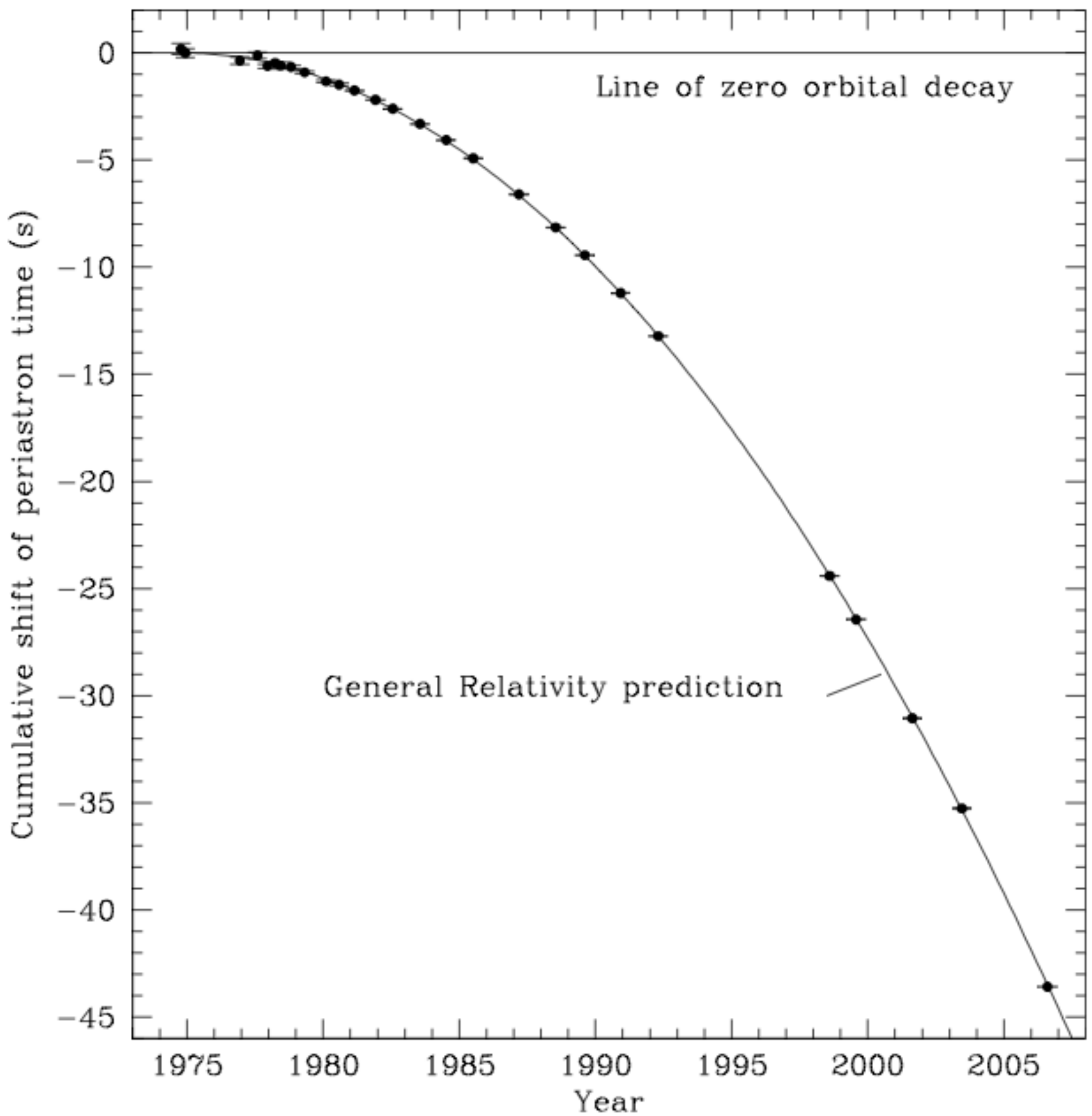
- **Supernovae:**
  neutrino-driven wind
  r-process **upto A~130**

- **NS mergers:**
  r-process **upto A~195**

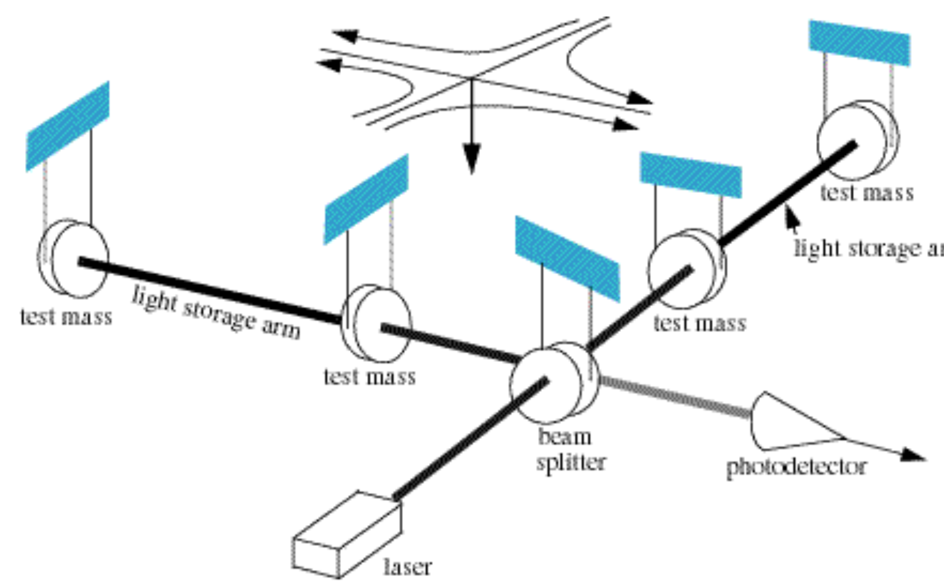
Gravitational Waves



B1913+16 Hulse & Taylor (1975)



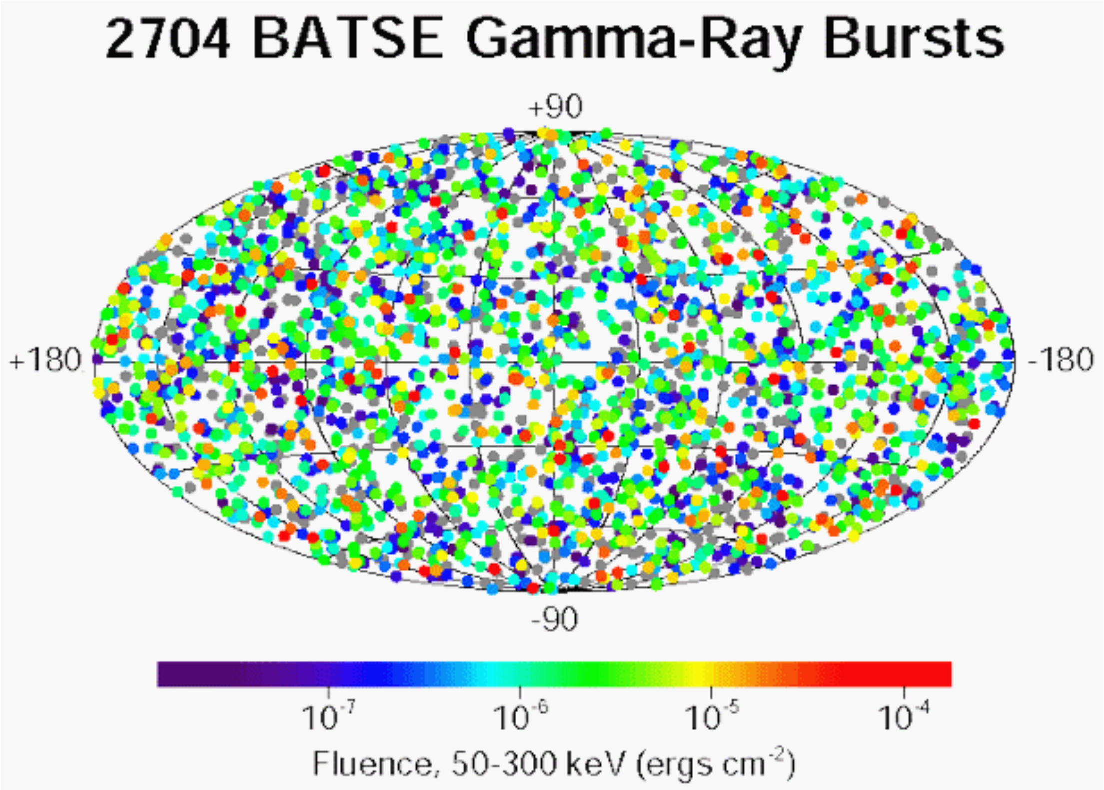
Weisberg, Nice, Taylor, ApJ (2010)

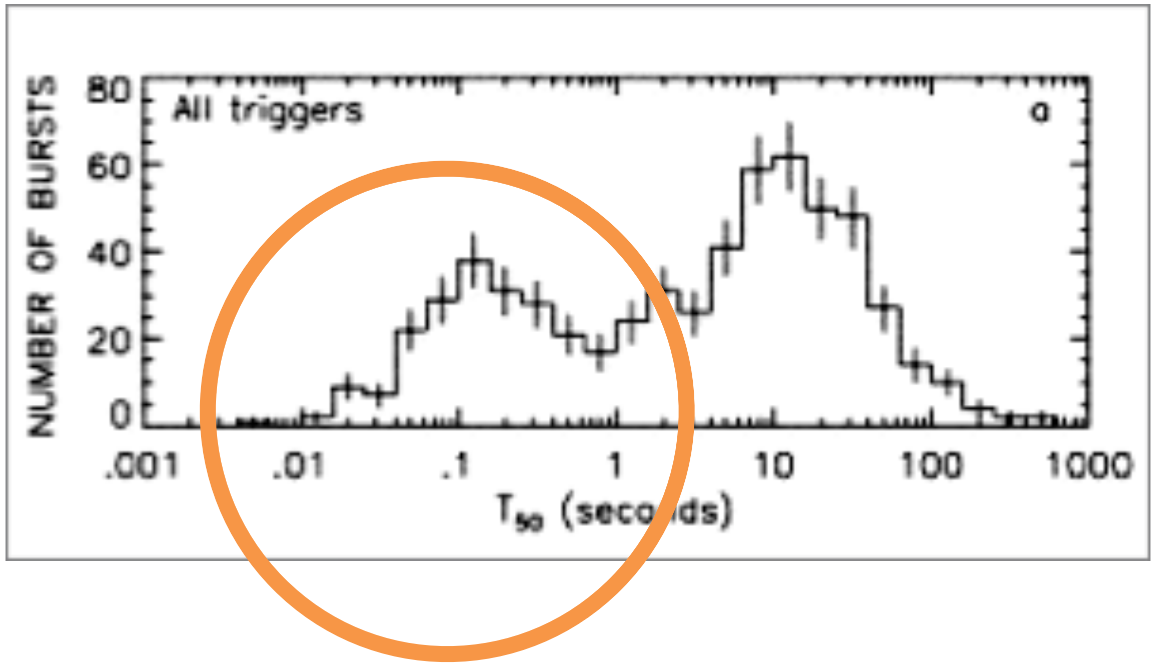


NS (radio pulsar) which will coalesce within Hubble time

PSR	P (ms)	P_b (hr)	e	Total Mass $M_\odot$	τ_c (Myr)	$\tau_{\rm GW}$ (Myr)	
J0737−3039A	22.70	2.45	0.088	2.58	210	87	(2003)
J0737−3039B	2773	2.45	0.088	2.58	50	87	(2004)
B1534+12	37.90	10.10	0.274	2.75	248	2690	(1990)
J1756−2251	28.46	7.67	0.181	2.57	444	1690	(2004)
B1913+16	59.03	7.75	0.617	2.83	108	310	(1975)
B2127+11C	30.53	8.04	0.681	2.71	969	220	(1990)
J1141−6545†	393.90	4.74	0.172	2.30	1.4	590	(2000)

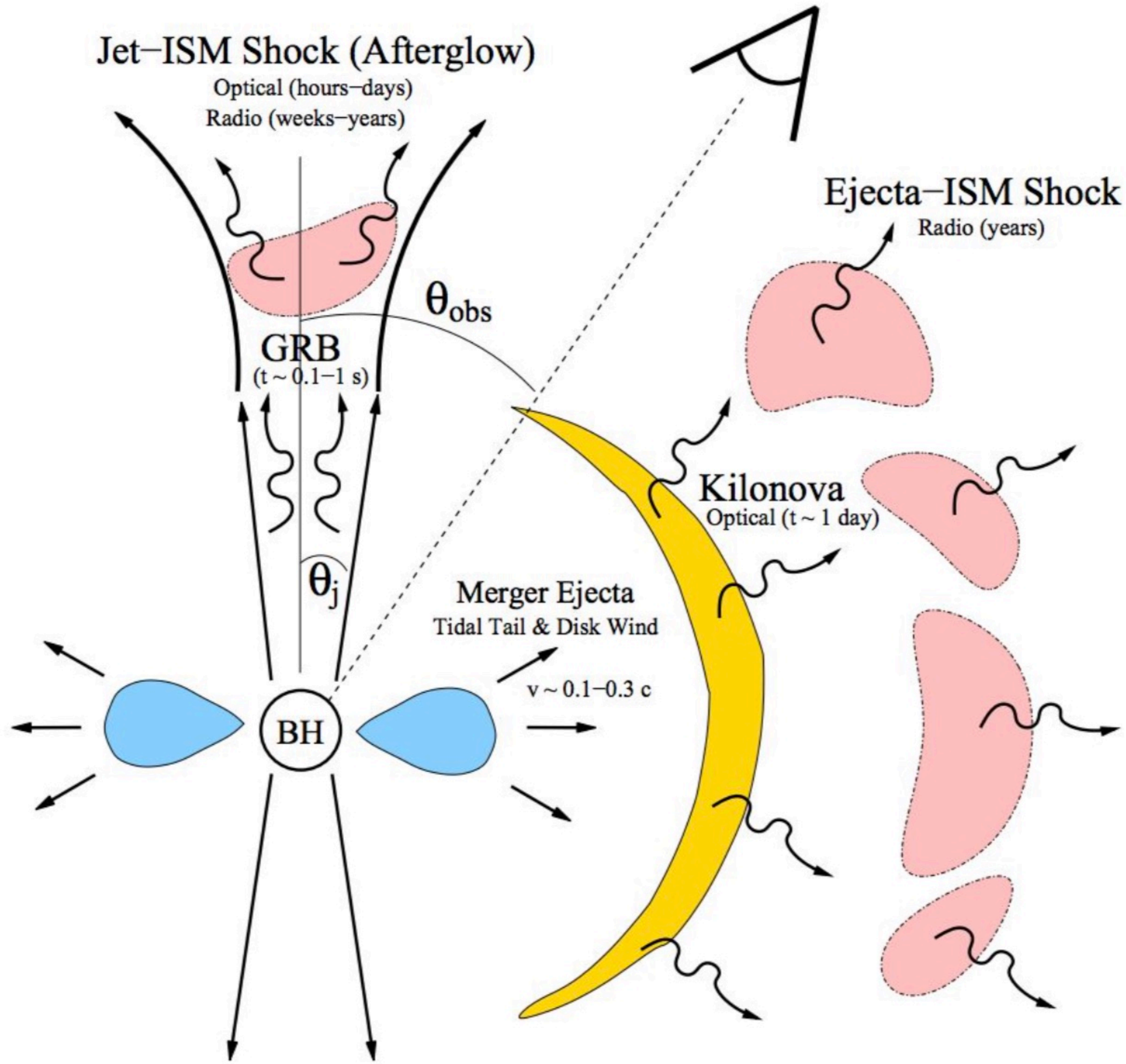
sGRB short-hard gamma-ray bursts from NS mergers





GRB / Kilonova

Binary NS Mergers

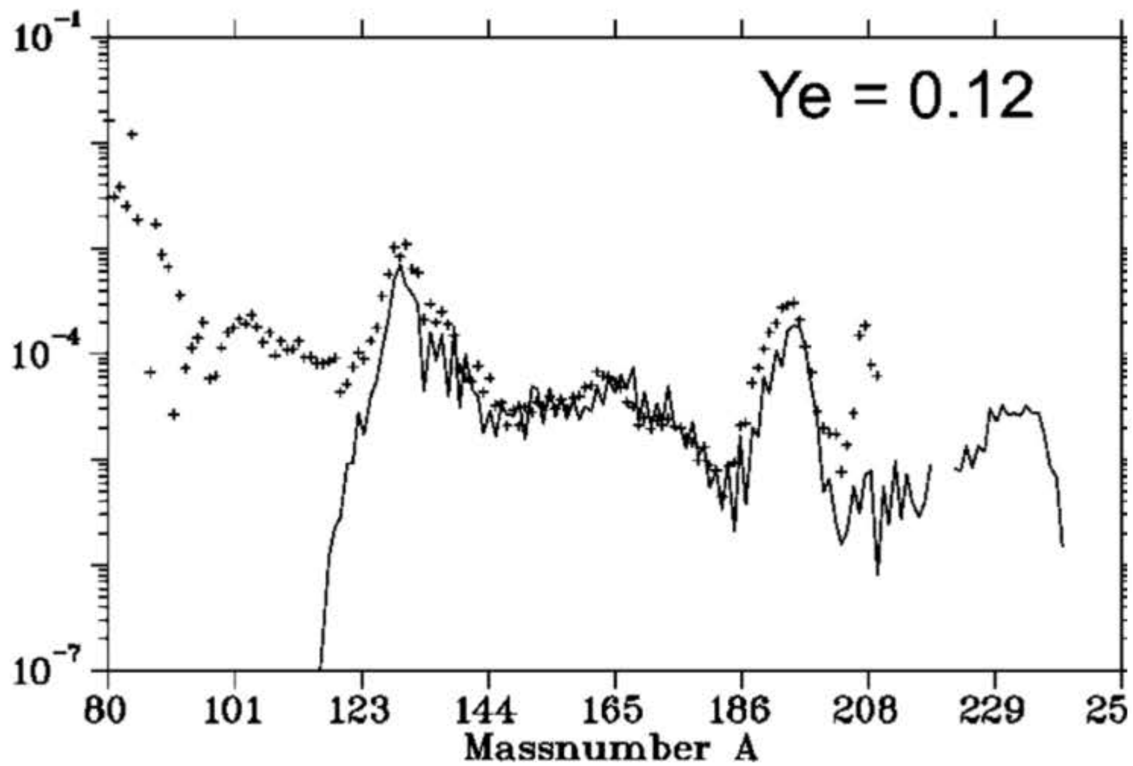


Metzger and Berger 2012

Neutron Star Mergers / Kilonova

- **Gravitational Waves**
- **Sources of Heavy Elements**

S Rosswog 2015



solar pattern vs NS-merger

- **Supernovae:**
  neutrino-driven wind
  r-process **upto A~130**

- **NS mergers:**
  r-process **upto A~195**

Neutron Star Constraints

Equation of States

Nuclear Equation of States

$$x = \frac{\rho_p}{\rho_p + \rho_n}$$

$$E(\rho, x) = -B + \frac{K_0}{18}\left(\frac{\rho}{\rho_0} - 1\right)^2 + \frac{K_0'}{162}\left(\frac{\rho}{\rho_0} - 1\right)^3 + E_{\rm sym}(\rho)(1-2x)^2 + \cdots$$

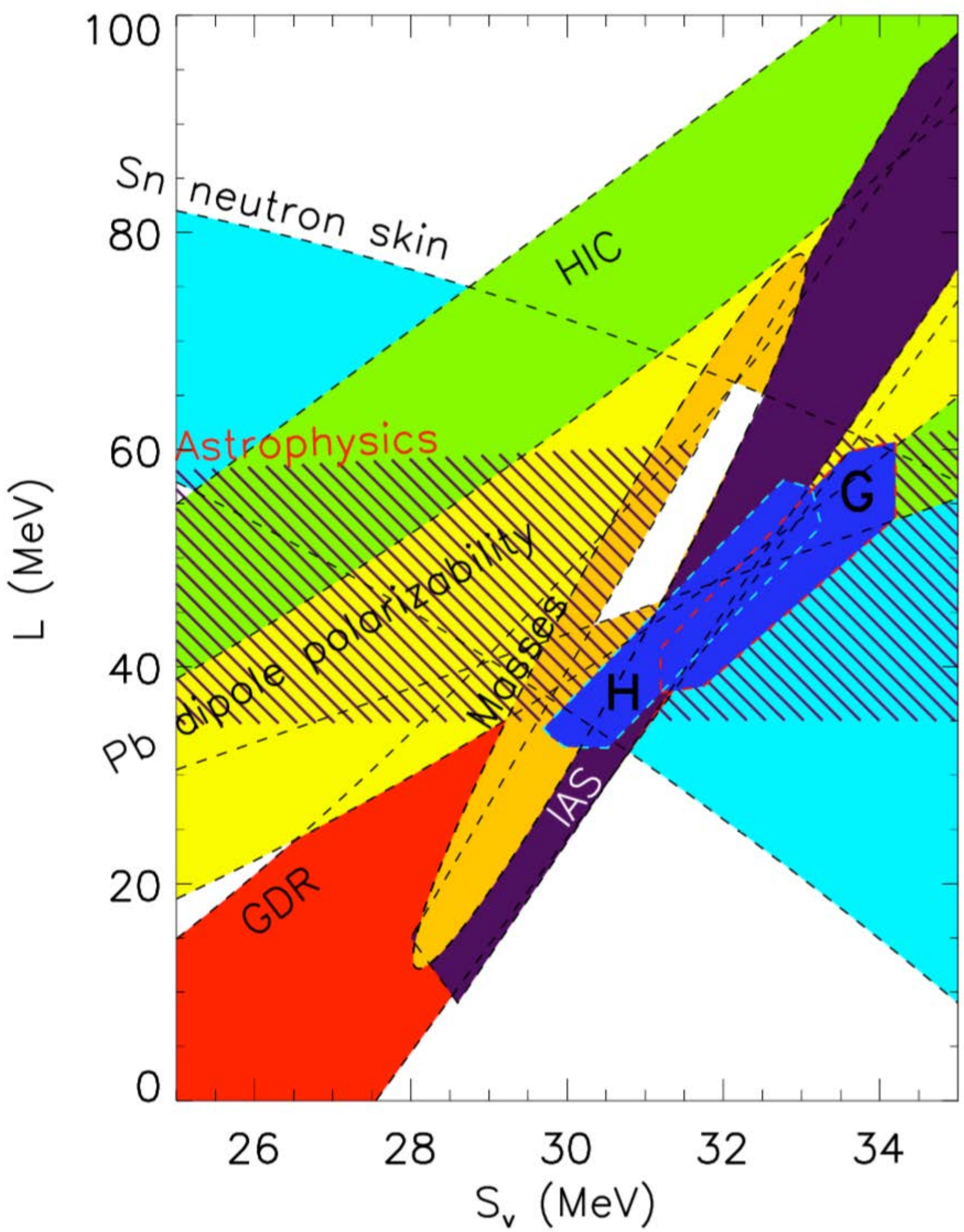
Incompressibility $K_0 \simeq 230\ \mathrm{MeV}$

Skewness $K_0' \sim -2000\ \mathrm{MeV}$

$$S_0 \equiv E_{\rm sym}(\rho_0)$$ **Symmetry Energy**

$$L \equiv 3\rho \left.\frac{\partial E_{\rm sym}}{\partial \rho}\right|_{\rho_0}$$

Symmetry Energy



$$S_0 \equiv E_{\rm sym}(\rho_0)$$

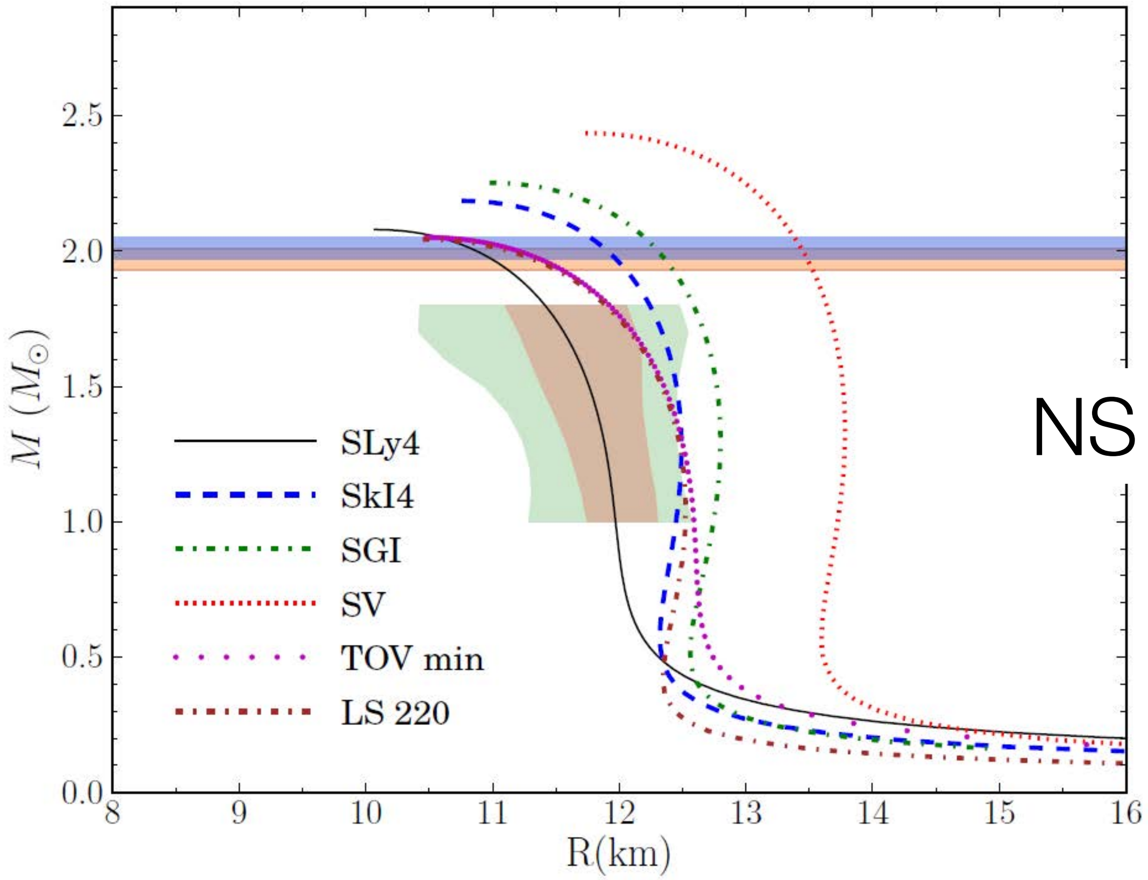
$$L \equiv 3\rho \left.\frac{\partial E_{\rm sym}}{\partial \rho}\right|_{\rho_0}$$

J.M. Latter (2016)

Strangeness

Role of strangeness

- dense matter with u & d quarks: p(uud) + n(udd)
- **s-quark can reduce pressure of dense matter**
- **reduce maximum mass of neutron stars**



NS EOS without strangeness

There are possibilities of NS EOS with strangeness

Hyperons in Skyrme force models

Y Lim, C H Hyun, K Kwan, CHL, IJMPE 12, 1550100 (2015)

Skyrme-type forces : NN interaction

$$v_{NN}(\boldsymbol{r}_{ij}) = t_0(1+x_0P_\sigma)\delta(\boldsymbol{r}_{ij}) + \frac{1}{2}t_1(1+x_1P_\sigma)[\boldsymbol{k}'^2_{ij}\delta(\boldsymbol{r}_{ij}) + \delta(\boldsymbol{r}_{ij})\boldsymbol{k}^2_{ij}]$$
$$+ t_2(1+x_2P_\sigma)\boldsymbol{k}'_{ij}\cdot\delta(\boldsymbol{r}_{ij})\boldsymbol{k}_{ij} + \frac{1}{6}t_3(1+x_3P_\sigma)\rho_N^\epsilon(\boldsymbol{R})\delta(\boldsymbol{r}_{ij})$$
$$+ iW_0\boldsymbol{k}'_{ij}\cdot\delta(\boldsymbol{r}_{ij})(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)\times\boldsymbol{k}_{ij},$$

$$\boldsymbol{r}_{ij} = \boldsymbol{r}_i - \boldsymbol{r}_j,\ \boldsymbol{R} = (\boldsymbol{r}_i+\boldsymbol{r}_j)/2,\ \boldsymbol{k}_{ij} = -i(\overrightarrow{\nabla}_i - \overrightarrow{\nabla}_j)/2,\ \boldsymbol{k}'_{ij} = i(\overleftarrow{\nabla}_i - \overleftarrow{\nabla}_j)/2,$$

$$P_\sigma = (1+\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)/2,$$

NN model	ρ_0	B	S_v	L	K	m_N^*/m_N	$M_{\max}/M_\odot$
SLy4	0.160	16.0	32.0	45.9	230	0.694	2.07
SkI4	0.160	16.0	29.5	60.4	248	0.649	2.19
SGI	0.155	15.9	28.3	63.9	262	0.608	2.25

N-Lambda interaction

$$v_{N\Lambda} = u_0(1 + y_0 P_\sigma)\delta(\boldsymbol{r}_{N\Lambda}) + \frac{1}{2}u_1[\boldsymbol{k}'^2_{N\Lambda}\delta(\boldsymbol{r}_{N\Lambda}) + \delta(\boldsymbol{r}_{N\Lambda})\boldsymbol{k}^2_{N\Lambda}]$$

$$+ u_2\boldsymbol{k}'_{N\Lambda} \cdot \delta(\boldsymbol{r}_{N\Lambda})\boldsymbol{k}_{N\Lambda} + \frac{3}{8}u'_3(1 + y_3 P_\sigma)\rho_N^\gamma\left(\frac{\boldsymbol{r}_N + \boldsymbol{r}_\Lambda}{2}\right)\delta(\boldsymbol{r}_{N\Lambda}),$$

Table 2. Parameters for the $N\Lambda$ interactions. u_0 is in unit of MeV$\cdot$ fm^3, u_1 and u_2 in unit of MeV$\cdot$ fm^5, and u'_3 in unit of MeV$\cdot$ fm$^{3+3\gamma}$. y_0 and y_3 are dimensionless. The last column $U^{\rm opt}_\Lambda$ is the depth of the Λ-nucleus optical potential in unit of MeV at the saturation density.

$N\Lambda$ model	γ	u_0	u_1	u_2	u'_3	y_0	y_3	$U^{\rm opt}_\Lambda$
HPΛ2	1	−399.946	83.426	11.455	2046.818	−0.486	−0.660	−31.23
OΛ2	1/3	−417.7593	1.5460	−3.2671	1102.2221	−0.3854	−0.5645	−28.27
YBZ6	1	−372.2	100.4	79.60	2000	−0.107	0	−29.73

Lambda-Lambda interaction

$$v_{\Lambda\Lambda}(\boldsymbol{r}_{ij}) = \lambda_0\delta(\boldsymbol{r}_{ij}) + \frac{1}{2}\lambda_1[\boldsymbol{k}'^2_{ij}\delta(\boldsymbol{r}_{ij}) + \delta(\boldsymbol{r}_{ij})\boldsymbol{k}^2_{ij}] + \lambda_2\boldsymbol{k}'_{ij}\cdot\delta(\boldsymbol{r}_{ij})\boldsymbol{k}_{ij} + \lambda_3\rho_N^{\alpha}(\boldsymbol{R})\delta(\boldsymbol{r}_{ij})$$

$$v_{\Lambda\Lambda}^{\mathrm{FRF}}(r) = \sum_{i=1}^{3}(v_i + v_i^{\sigma}\boldsymbol{\sigma}_\Lambda\cdot\boldsymbol{\sigma}_\Lambda)e^{-\mu_i r^2}.$$

Table 3. Parameters of the $\Lambda\Lambda$ interactions in the Skyrme-type force. λ_0 is in unit of $\mathrm{MeV}\cdot\mathrm{fm}^3$, and λ_1 in unit of $\mathrm{MeV}\cdot\mathrm{fm}^5$. Note that all models considered here do not have λ_2 momentum interaction and λ_3, α density dependent interaction.

$\Lambda\Lambda$ model	λ_0	λ_1	λ_2	λ_3	α
S$\Lambda\Lambda$1	−312.6	57.5	0	0	—
S$\Lambda\Lambda$2	−437.7	240.7	0	0	—
S$\Lambda\Lambda$3	−831.8	922.9	0	0	—
S$\Lambda\Lambda$1′	−37.9	14.1	0	0	—
S$\Lambda\Lambda$3′	−156.4	347.2	0	0	—

Lambda-Lambda interaction / Binding energy

$$v_{\Lambda\Lambda}(\boldsymbol{r}_{ij}) = \lambda_0\delta(\boldsymbol{r}_{ij}) + \frac{1}{2}\lambda_1[\boldsymbol{k}'^2_{ij}\delta(\boldsymbol{r}_{ij}) + \delta(\boldsymbol{r}_{ij})\boldsymbol{k}^2_{ij}] + \lambda_2\boldsymbol{k}'_{ij}\cdot\delta(\boldsymbol{r}_{ij})\boldsymbol{k}_{ij} + \lambda_3\rho_N^\alpha(\boldsymbol{R})\delta(\boldsymbol{r}_{ij})$$

$$v_{\Lambda\Lambda}^{\mathrm{FRF}}(r) = \sum_{i=1}^{3}(v_i + v_i^\sigma\boldsymbol{\sigma}_\Lambda\cdot\boldsymbol{\sigma}_\Lambda)e^{-\mu_i r^2}.$$

Nuclei	$B_{\Lambda\Lambda}$(S$\Lambda\Lambda$1)	$B_{\Lambda\Lambda}$(S$\Lambda\Lambda$2)	$B_{\Lambda\Lambda}$(S$\Lambda\Lambda$3)	$B_{\Lambda\Lambda}$(Exp.)
$^{6}_{\Lambda\Lambda}$He	11.88	9.25	7.60	6.93 ± 0.16[22]
$^{10}_{\Lambda\Lambda}$Be	19.78	18.34	15.19	14.94 ± 0.13[23,a]
$^{11}_{\Lambda\Lambda}$Be	20.55	19.26	16.27	20.49 ± 1.15[22]
$^{12}_{\Lambda\Lambda}$Be	21.10	19.97	17.18	22.23 ± 1.15[22]
$^{13}_{\Lambda\Lambda}$B	21.21	20.26	17.76	23.30 ± 0.70[22]

Maximum mass of NS

NN model (without Λ)[†]	SLy4 (2.07)		SkI4 (2.19)			SGI (2.25)		
$N\Lambda$ model (no $\Lambda\Lambda$)[††]	HPΛ2* (1.51)	OΛ2 (1.08)	HPΛ2 (1.52)	OΛ2 (1.19)	YBZ6* (1.80)	HPΛ2 (1.52)	OΛ2 (1.22)	YBZ6* (1.79)
S$\Lambda\Lambda$1	*1.40*	1.00	1.41	1.12	*1.70*	1.42	1.16	*1.69*
S$\Lambda\Lambda$2	*1.58*	1.28	1.57	1.30	*1.79*	1.57	1.28	*1.77*
S$\Lambda\Lambda$3	*1.85*	1.57	1.87	1.62	*2.03*	1.88	1.65	*2.04*
S$\Lambda\Lambda$1′	*1.51*	1.08	1.51	1.18	*1.79*	1.51	1.21	*1.78*
S$\Lambda\Lambda$3′	*1.76*	1.43	1.76	1.47	*1.97*	1.77	1.49	*1.96*
FRF	*1.61*	1.22	1.60	1.25	*1.86*	1.59	1.26	*1.84*

*Numbers in italic correspond to three selected models, SLy4-HPΛ2, SkI4-YBZ6 and SGI-YBZ6, which give relatively large maximum NS mass.

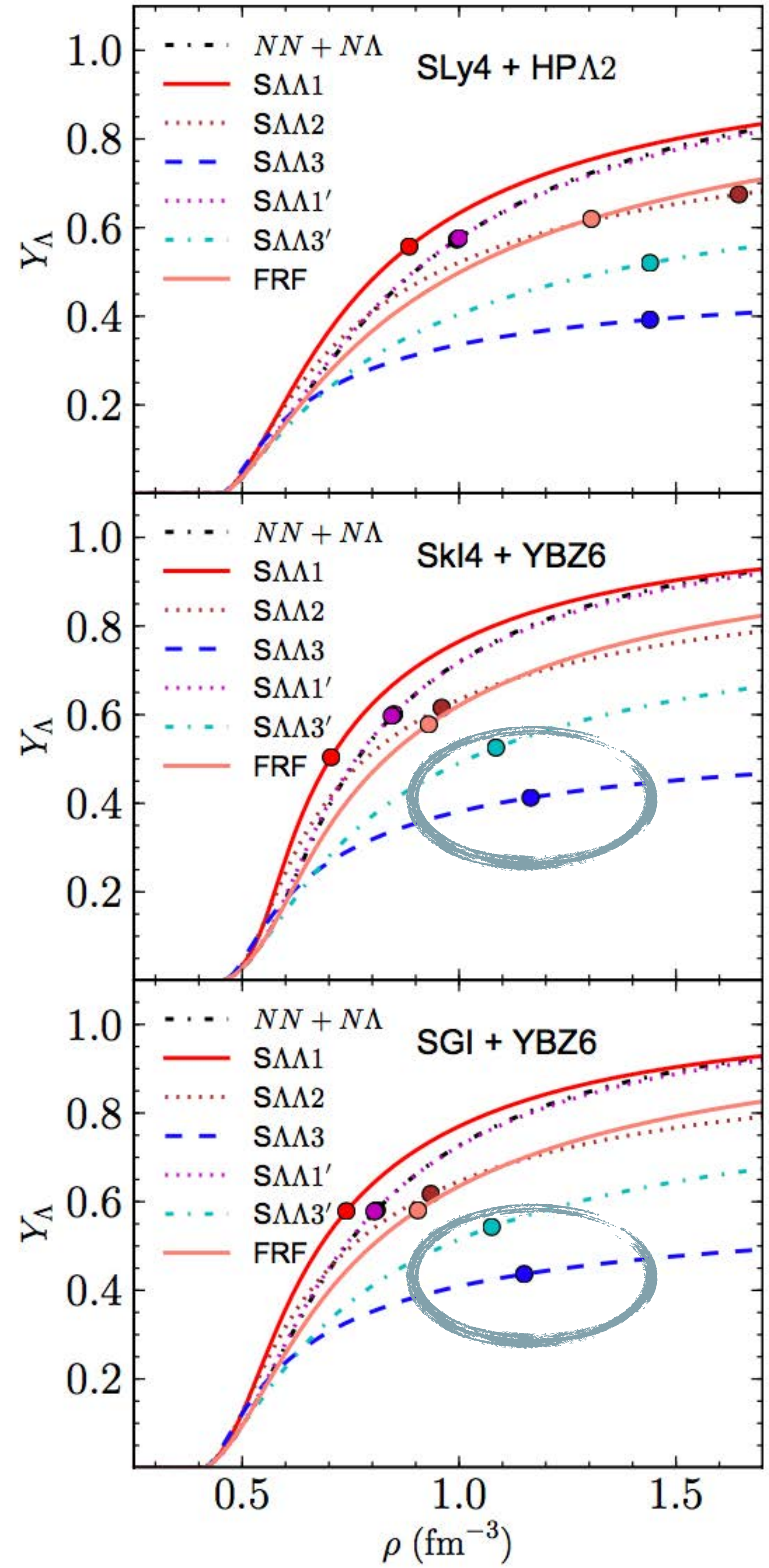
[†]Maximum NS mass without both $N\Lambda$ and $\Lambda\Lambda$ interactions.

[††]Maximum NS mass with $N\Lambda$ but without $\Lambda\Lambda$ interactions.

Central densities

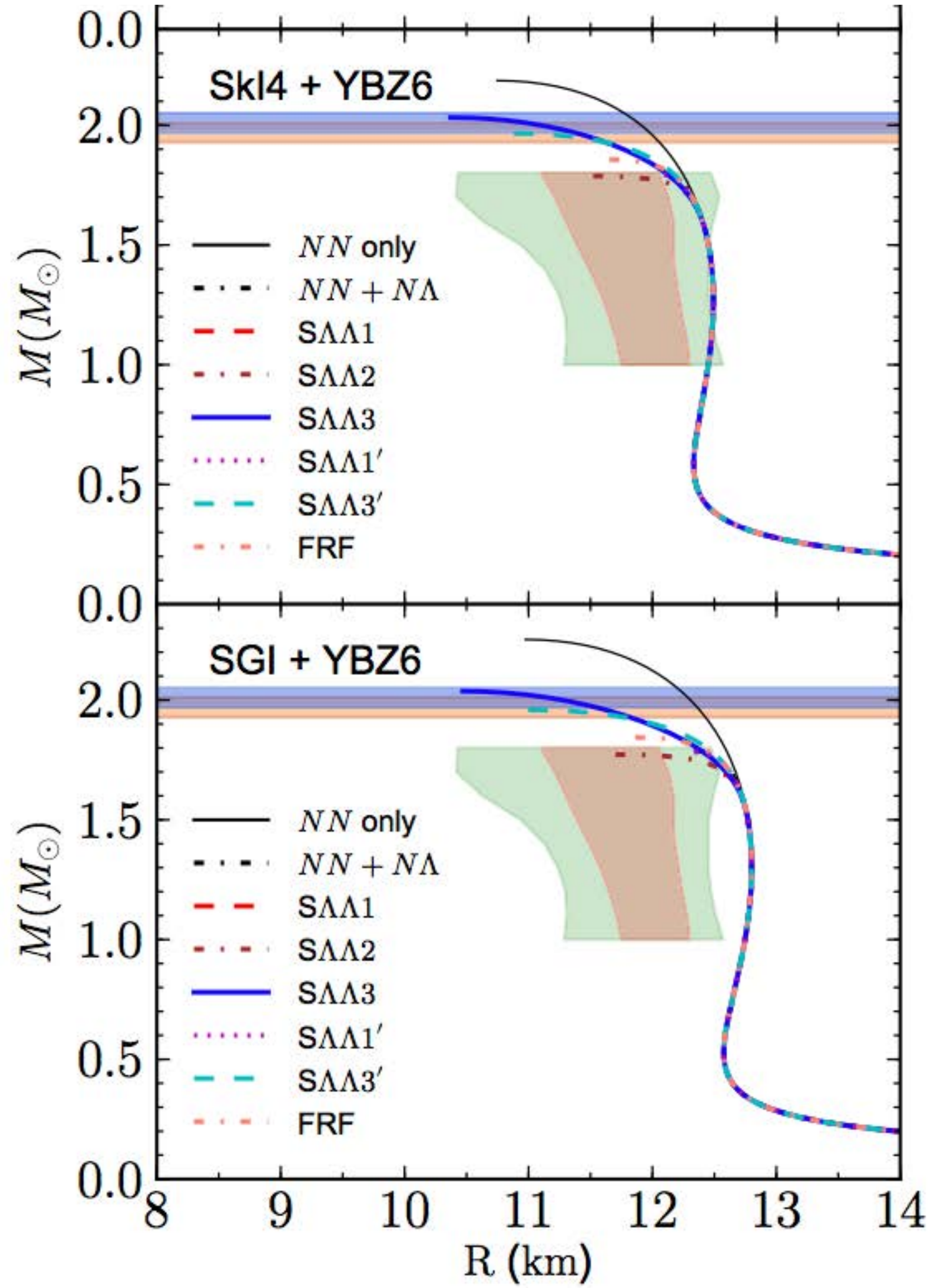
Table 6. Density in fm^{-3} at the center of stars corresponding to the locations of filled circles in Fig. 1.

	SLy4+HLΛ2	SkI4+YBZ6	SGI+YBZ6
Without ΛΛ	1.00	0.85	0.81
SΛΛ1	0.89	0.71	0.74
SΛΛ2	1.65	0.96	0.94
SΛΛ3	1.44	1.17	1.15
SΛΛ1′	1.00	0.85	0.81
SΛΛ3′	1.44	1.09	1.08
FRF	1.31	0.93	0.91



O : central densities for maximum mass

Maximum NS mass



Kaons with Skyrme-type models

Y Lim, K Kwan, C H Hyun, C-H Lee, PRC 89, 055804 (2014)

Kaons

$$
\begin{aligned}
\mathcal{L} = & \frac{f_\pi^2}{4}\mathrm{Tr}\partial_\mu U \partial^\mu U^\dagger + c\mathrm{Tr}[m_q(U + U^\dagger - 2)] \\
& + i\mathrm{Tr}\bar{B}\gamma^\mu\partial_\mu B + i\mathrm{Tr}B^\dagger[V_0, B] - D\mathrm{Tr}B^\dagger\boldsymbol{\sigma}\cdot\{\mathbf{A}, B\} \\
& - F\mathrm{Tr}B^\dagger\boldsymbol{\sigma}\cdot[\mathbf{A}, B] \\
& + a_1\mathrm{Tr}B^\dagger(\xi m_q \xi + \mathrm{H.c.})B + a_2\mathrm{Tr}B^\dagger B(\xi m_q \xi + \mathrm{H.c.}) \\
& + a_3\mathrm{Tr}B^\dagger B\mathrm{Tr}(m_q U + \mathrm{H.c.})
\end{aligned}
$$

$$
M = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}
$$

$$
B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}}\Lambda \end{pmatrix}
$$

Kaons

$$\langle K^- \rangle = v_K e^{-i\mu_K t}$$

$$\begin{aligned}\mathcal{L}_K = {} & f_\pi^2 \frac{\mu_K^2}{2} \sin^2\theta - 2m_K^2 f_\pi^2 \sin^2\frac{\theta}{2} \\ & - n^\dagger n[-\mu_K + (2a_2 + 4a_3)m_s] \sin^2\frac{\theta}{2} \\ & - p^\dagger p[-2\mu_K + (2a_1 + 2a_2 + 4a_3)m_s] \sin^2\frac{\theta}{2}\end{aligned}$$

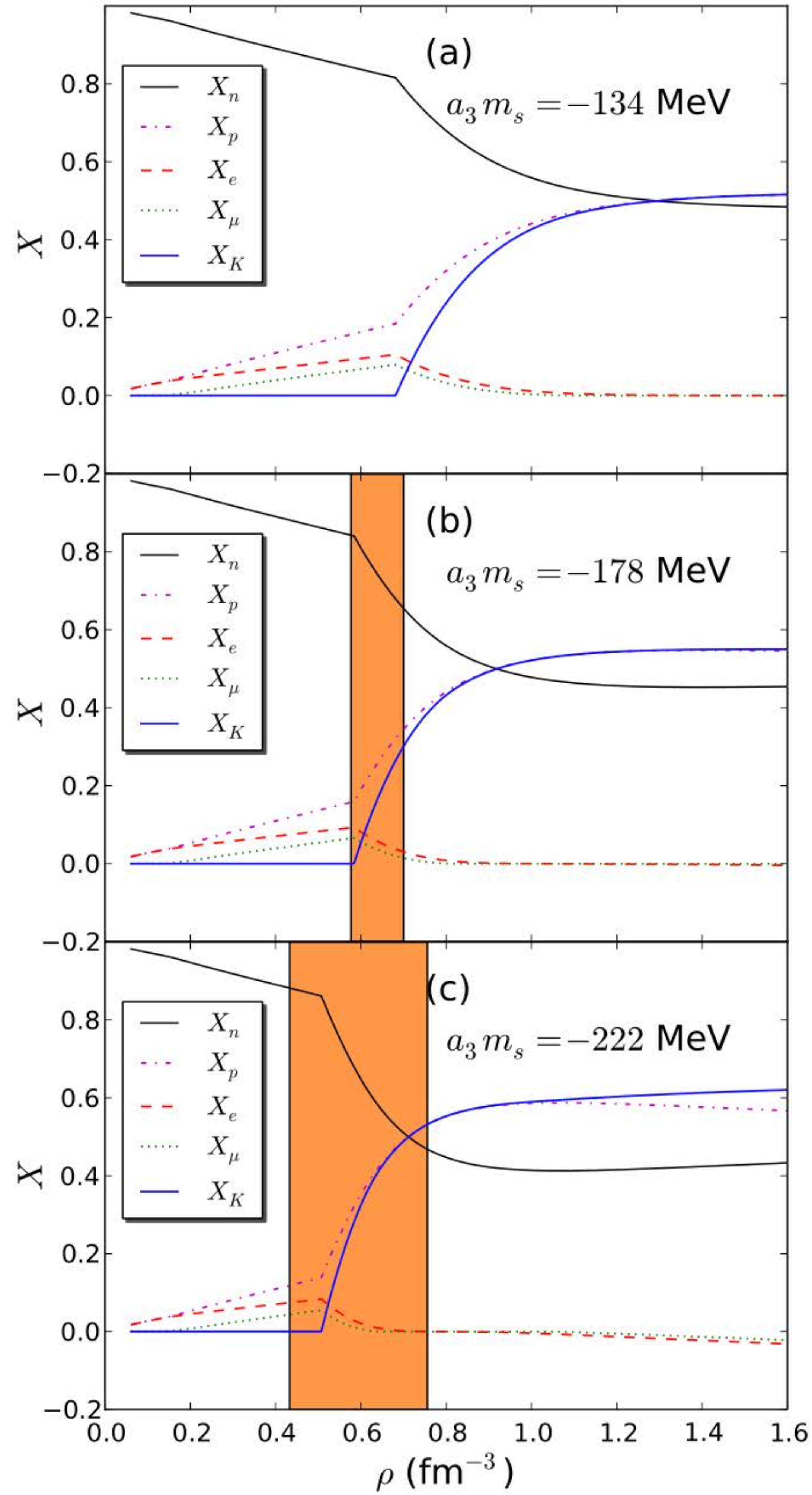
$$\begin{aligned}\mathcal{H}_K = {} & -f_\pi^2 \frac{\mu_K^2}{2} \sin^2\theta + 2m_K^2 f_\pi^2 \sin^2\frac{\theta}{2} + \mu_K \rho_p \\ & -\mu_K(\rho + \rho_p) \sin^2\frac{\theta}{2} + a_{K1}\rho_p \sin^2\frac{\theta}{2} + a_{K2}\rho \sin^2\frac{\theta}{2}\end{aligned}$$

Particle fraction

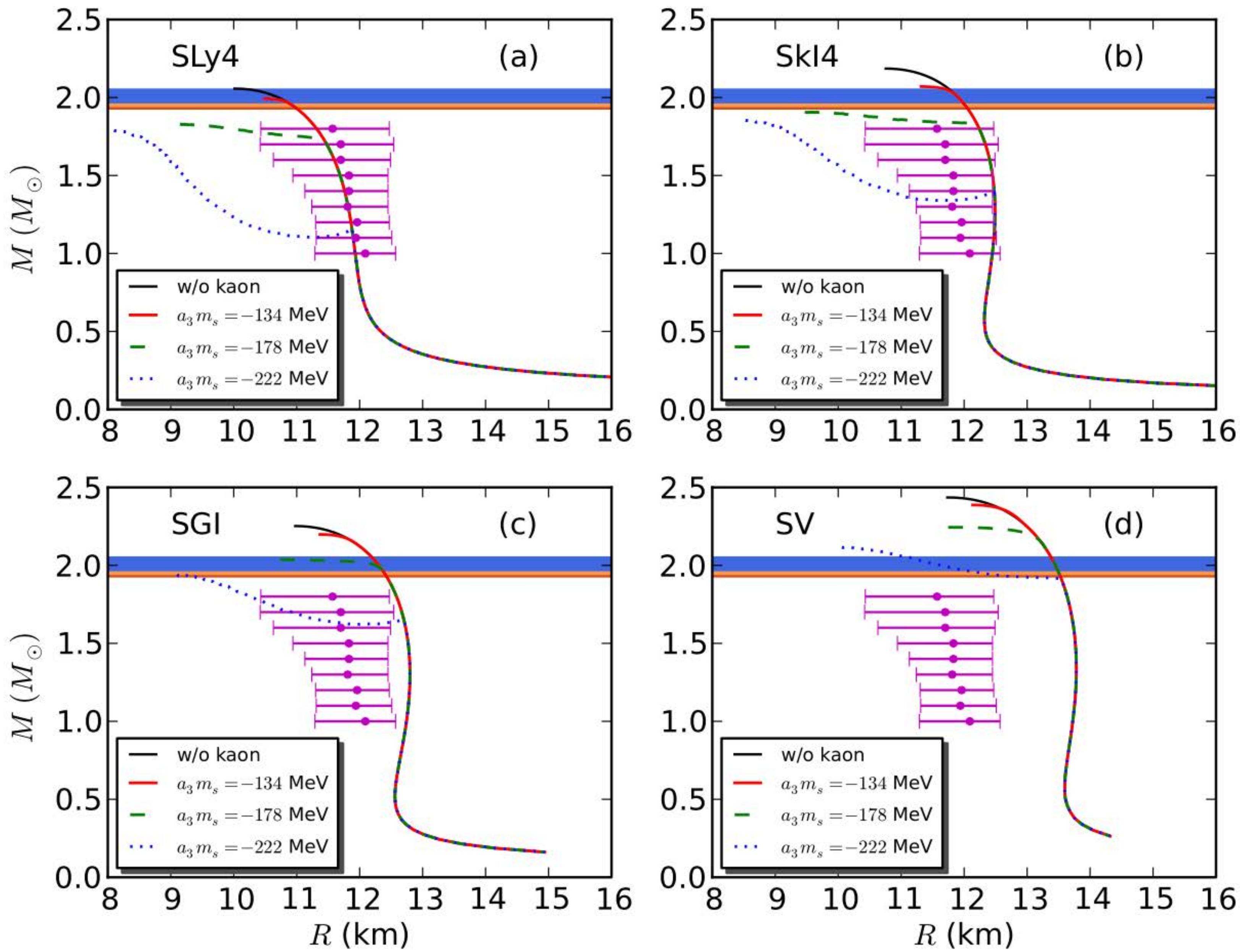
$$m_s \langle \bar{s}s \rangle_p = -2(a_2 + a_3)m_s,$$
$$\Sigma^{KN} = -\tfrac{1}{2}(a_1 + 2a_2 + 4a_3)m_s$$

$$a_1 m_s = -67\,\text{MeV},$$
$$a_2 m_s = 134\,\text{MeV}.$$

Orange region: maxwell construction



NS mass with kaons



Data: Steiner et al. ApJ 722, 33 (2010)

NS with strangeness

- **In general, strangeness reduces maximum NS mass**
- **But, there still remain possibilities for the strangeness**

masses with Lambda hyperons

SLy4	SkI4	SGI
$1.15 \sim 1.85$	$1.47 \sim 1.97$	$1.44 \sim 2.04$

masses with kaon condensation

consistent with 2 Msun NS

$a_3 m_s$	SLy4	SkI4	SGI
-134	$1.94 \sim 1.99$	$2.00 \sim 2.06$	$2.12 \sim 2.19$
-178	$1.74 \sim 1.83$	$1.84 \sim 1.90$	$1.95 \sim 2.03$
-222	$1.49 \sim 1.79$	$1.64 \sim 1.85$	$1.75 \sim 1.93$

NS maximum mass with strangeness

and SGI, and $S\Lambda\Lambda 3'$ with SkI4. The lower limit is the mass at which Λ hyperons start to contribute and the upper limit is the maximum mass of neutron stars in the presence of Λ hyperons.

SLy4	SkI4	SGI
$1.15 \sim 1.85$	$1.47 \sim 1.97$	$1.44 \sim 2.04$

given in units of MeV. The lower limit is the mass at which the kaon condensation starts to appear in the core, and the upper limit is the maximum mass of neutron stars with kaon condensation.

$a_3 m_s$	SLy4	SkI4	SGI
-134	$1.94 \sim 1.99$	$2.00 \sim 2.06$	$2.12 \sim 2.19$
-178	$1.74 \sim 1.83$	$1.84 \sim 1.90$	$1.95 \sim 2.03$
-222	$1.49 \sim 1.79$	$1.64 \sim 1.85$	$1.75 \sim 1.93$

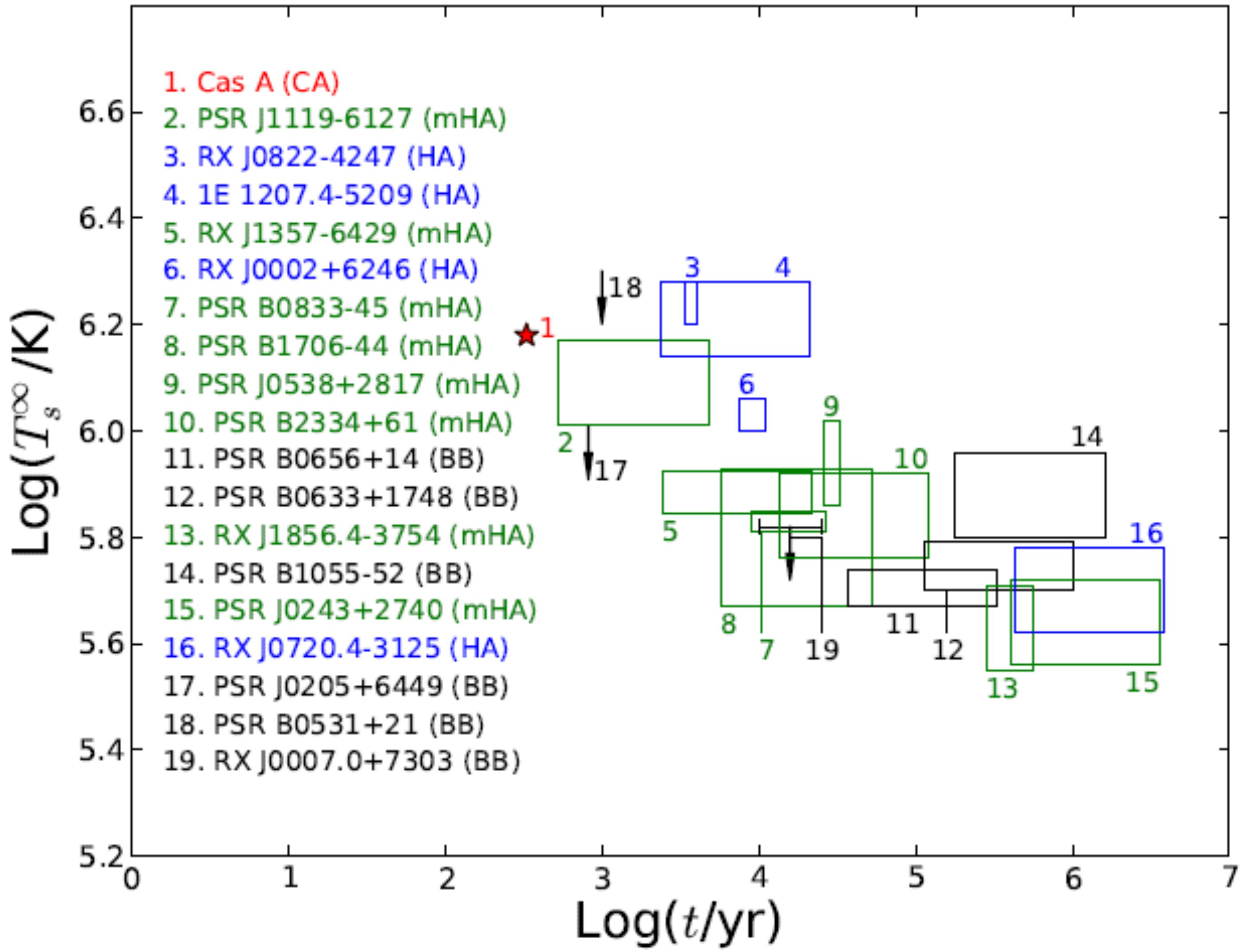
NS cooling with Strangeness

Q) Can **NS EOS with strangeness** be consistent with
both **NS maximum mass & NS cooling**?

arXiv:1501.04397

No.	Source	$\text{Log}(t_{sd}/\text{yr})$	$\text{Log}(t_{kin}/\text{yr})$	$\text{Log}(T_s^\infty/\text{K})$	$\text{Log}(L^\infty/\text{erg s}^{-1})$
1	Cas A	$2.518^{+0.007}_{-0.007}$	-	$6.18^{+0.01}_{-0.01}$	33.83 - 33.88
2	PSR J1119-6127	3.20	-	$6.08^{+0.09}_{-0.07}$	32.88 - 33.66
3	RX J0822-4247†	3.90	$3.57^{+0.04}_{-0.04}$	$6.24^{+0.04}_{-0.04}$	33.85 - 34.00
				$6.65^{+0.04}_{-0.04}$	33.60 - 33.90
4	1E 1207.4-5209	$5.53^{+0.44}_{-0.19}$	$3.85^{+0.48}_{-0.48}$	$6.21^{+0.07}_{-0.07}$	33.27 - 33.74
				$6.48^{+0.01}_{-0.01}$	33.27 - 33.74
5	PSR J1357-6429	3.86	-	$5.88^{+0.04}_{-0.04}$	32.46 - 32.80
				$6.23^{+0.05}_{-0.05}$	32.35 - 32.76
6	RX J0002+6246	-	$3.96^{+0.08}_{-0.08}$	$6.03^{+0.03}_{-0.03}$	33.08 - 33.33
				$6.15^{+0.11}_{-0.11}$	32.18 - 32.81
7	PSR B0833-45†	4.05	$4.26^{+0.17}_{-0.31}$	$5.83^{+0.02}_{-0.02}$	32.41 - 32.70
				$6.18^{+0.02}_{-0.02}$	32.04 - 32.32
8	PSR B1706-44	4.24	-	$5.80^{+0.13}_{-0.13}$	31.81 - 32.93
				$6.22^{+0.04}_{-0.04}$	32.48 - 33.08
9	PSR J0538+2817	$4.47^{+0.05}_{-0.06}$	-	$5.94^{+0.08}_{-0.08}$	32.32 - 33.33
10	PSR B2334+61	4.61	-	$5.84^{+0.08}_{-0.08}$	31.93 - 32.96
11	PSR B0656+14	5.04	-	$5.71^{+0.03}_{-0.04}$	32.18 - 32.97
12	PSR B0633+1748†	5.53	-	$5.75^{+0.04}_{-0.05}$	30.85 - 31.51
13	RX J1856.4-3754	-	$5.70^{+0.05}_{-0.25}$	$5.63^{+0.08}_{-0.08}$	31.32 - 32.35
14	PSR B1055-52	5.73	-	$5.88^{+0.08}_{-0.08}$	32.05 - 33.08
15	PSR J0243+2740	6.08	-	$5.64^{+0.08}_{-0.08}$	29.10 - 30.13
16	RX J0720.4-3125	6.11	-	$5.70^{+0.08}_{-0.08}$	31.37 - 32.40
17	PSR J0205+6449††	-	2.91	< 6.01	< 33.29
18	PSR B0531+21††	-	3.0	< 6.30	< 34.45
19	RX J0007.0+7303††	-	4.0-4.4	< 5.82	< 32.54

Neutron Star Cooling



depends on

- particle fraction
- elements in the envelope
- nuclear superfluidity
- … …

arXiv:1501.04397

Cooling Mechanism

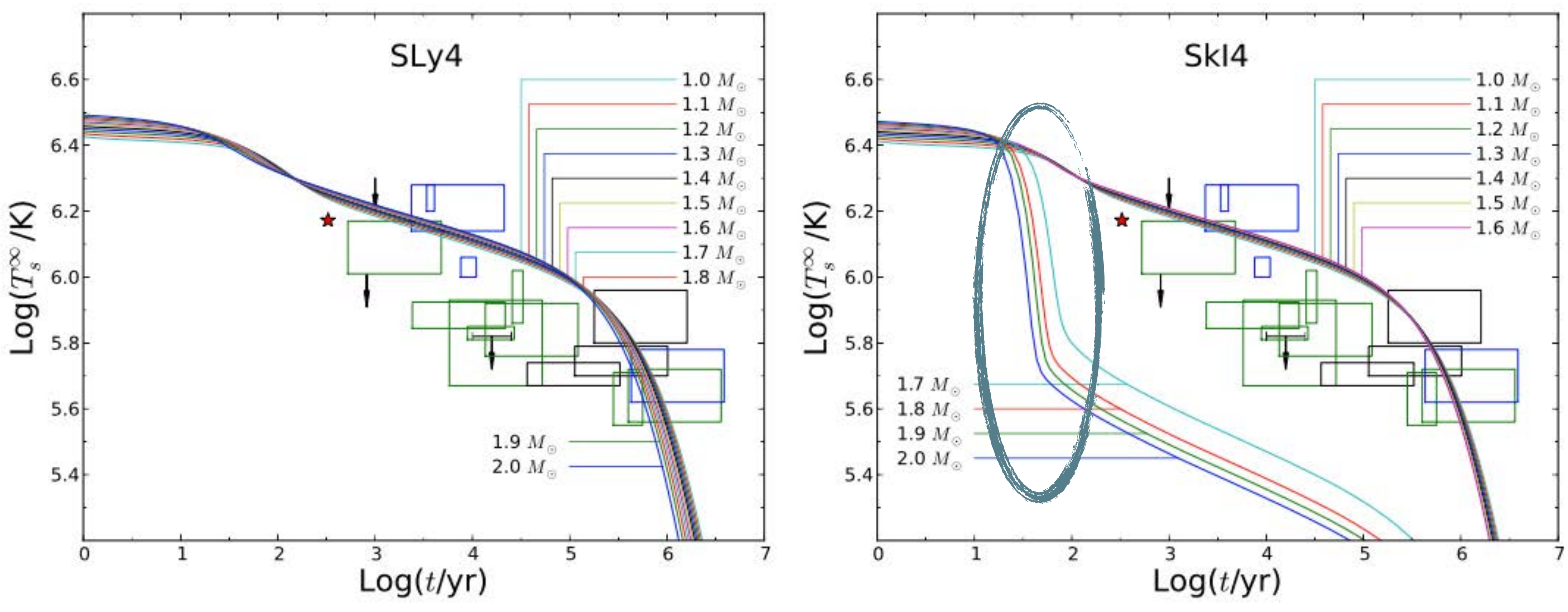
- **Photon emission** : mostly on the surface
- **Neutrino emission** : entire region, major energy loss

Name	Process	Emissivity[b] (erg cm^{-3} s^{-1})	
Modified Urca (neutron branch)	$n+n \to n+p+e^- + \bar{\nu}_e$ $n+p+e^- \to n+n+\nu_e$	$\sim 2\times 10^{21}\, \mathcal{R}\, T_9^8$	Slow
Modified Urca (proton branch)	$p+n \to p+p+e^- + \bar{\nu}_e$ $p+p+e^- \to p+n+\nu_e$	$\sim 10^{21}\, \mathcal{R}\, T_9^8$	Slow
Bremsstrahlung	$n+n \to n+n+\nu\bar{\nu}$ $n+p \to n+p+\nu\bar{\nu}$ $p+p \to p+p+\nu\bar{\nu}$	$\sim 10^{19}\, \mathcal{R}\, T_9^8$	Slow
Cooper pair formations	$n+n \to [nn]+\nu\bar{\nu}$ $p+p \to [pp]+\nu\bar{\nu}$	$\sim 5\times 10^{21}\, \mathcal{R}\, T_9^7$ $\sim 5\times 10^{19}\, \mathcal{R}\, T_9^7$	
Direct Urca	$n \to p+e^- + \bar{\nu}_e$ $p+e^- \to n+\nu_e$	$\sim 10^{27}\, \mathcal{R}\, T_9^6$	Fast
π^- condensate	$n + <\pi^-> \to n+e^- + \bar{\nu}_e$	$\sim 10^{26}\, \mathcal{R}\, T_9^6$	Fast
K^- condensate	$n + <K^-> \to n+e^- + \bar{\nu}_e$	$\sim 10^{25}\, \mathcal{R}\, T_9^6$	Fast

Role of nucleon direct Urca

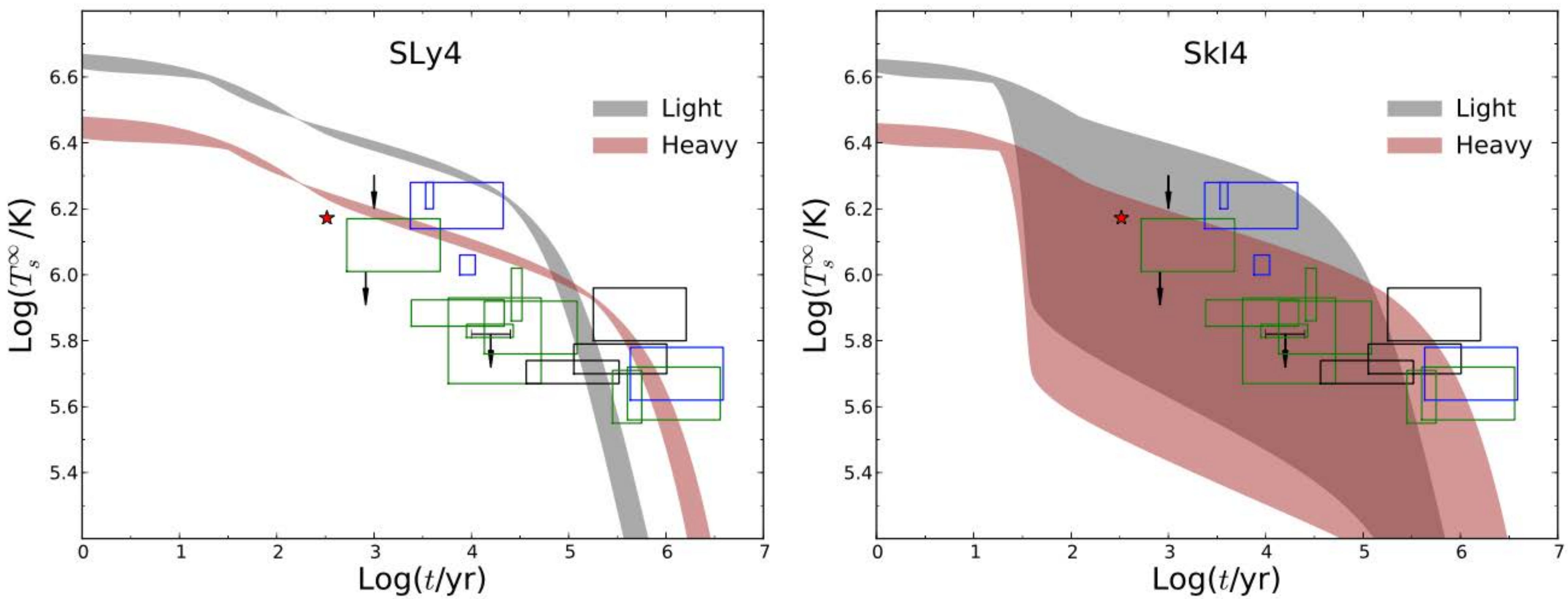
	SLy4	SkI4	SGI
$M_{\rm max}$	2.07 (-)	2.19 (1.63)	2.25 (1.72)

maximum mass without strangeness (critical mass for nucleon direct Urca)



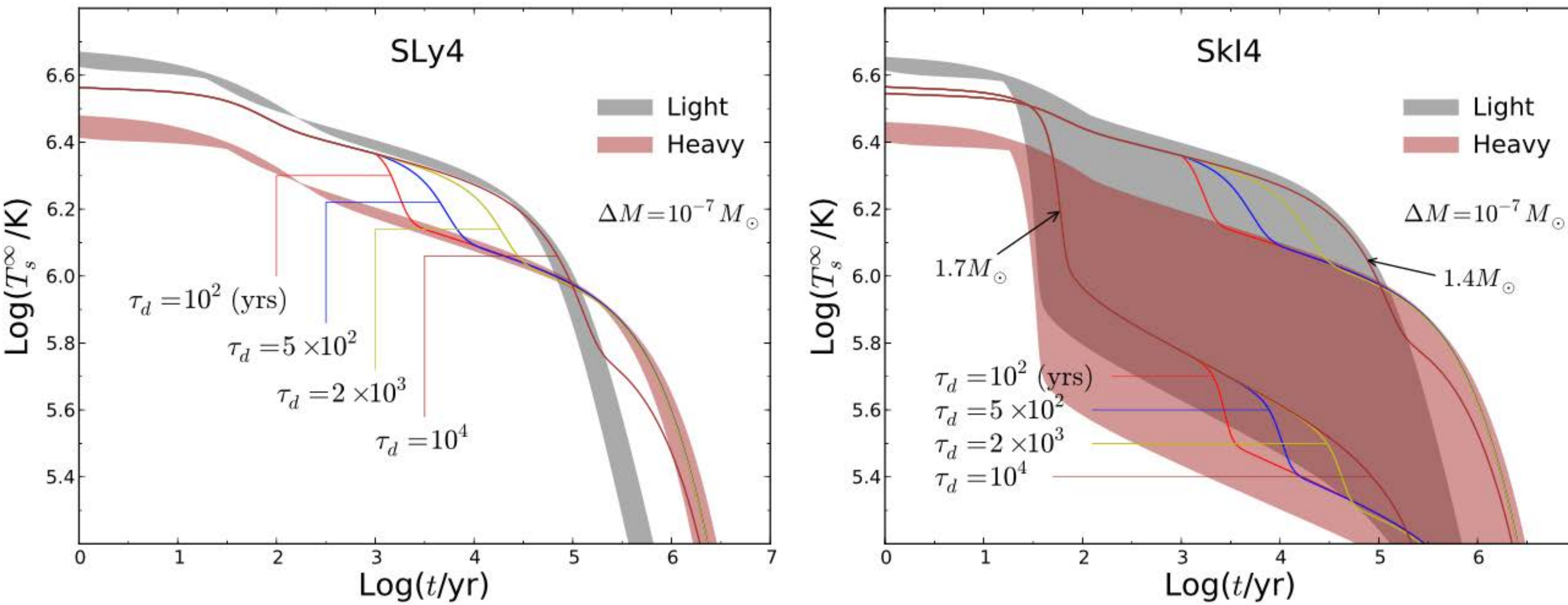
effect of direct Urca

effect of surface element



light element : H, He, C, O

effect of surface element



light elements burn into heavier elements

Cooling with strangeness

arXiv:1608.02078

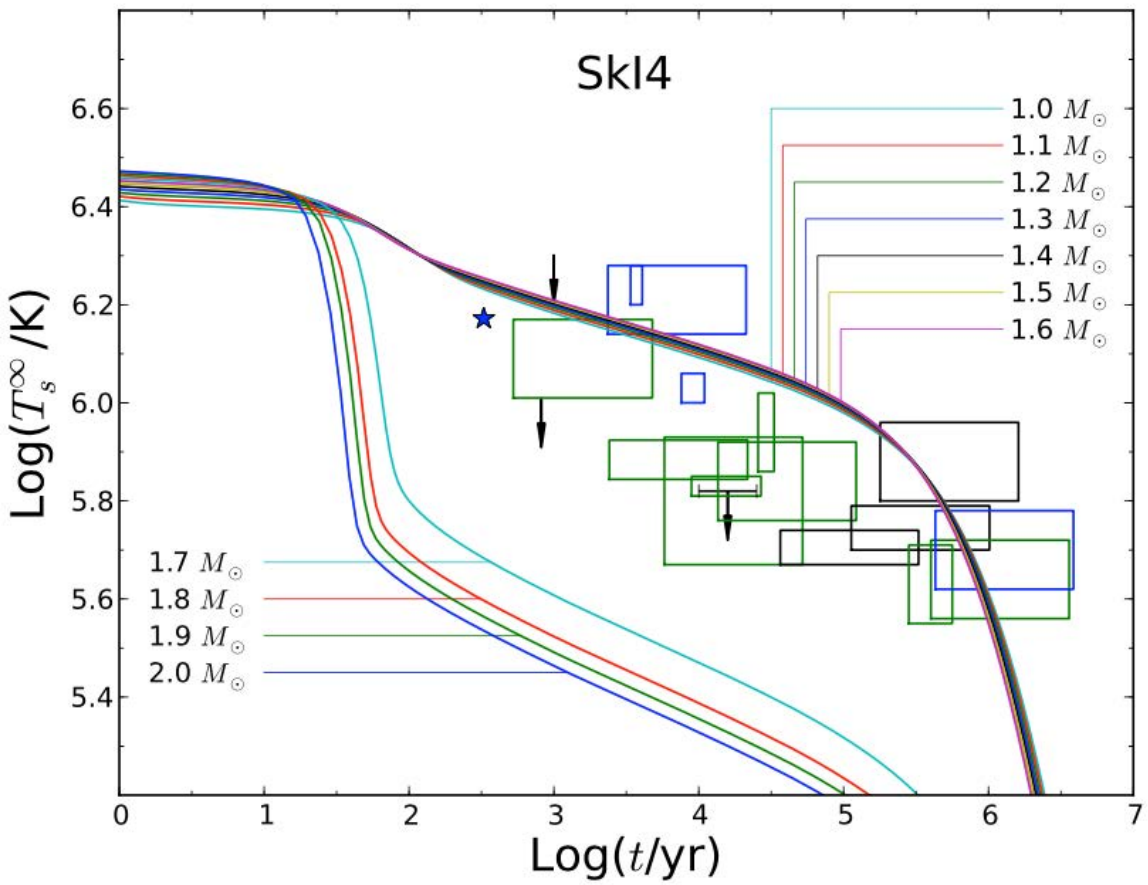
$$\Lambda \to p + l + \bar{\nu}_l$$
$$p + l \to \Lambda + \nu_l \,,$$

$$Q_\Lambda = 4.0 \times 10^{27} \frac{m_\Lambda^* m_p^*}{m_\Lambda m_p} \left(\frac{n_e}{n_0}\right)^{1/3} R\, T_9^6 \times \Theta_t \,\mathrm{erg\,cm^{-3}\,s^{-1}} \,,$$

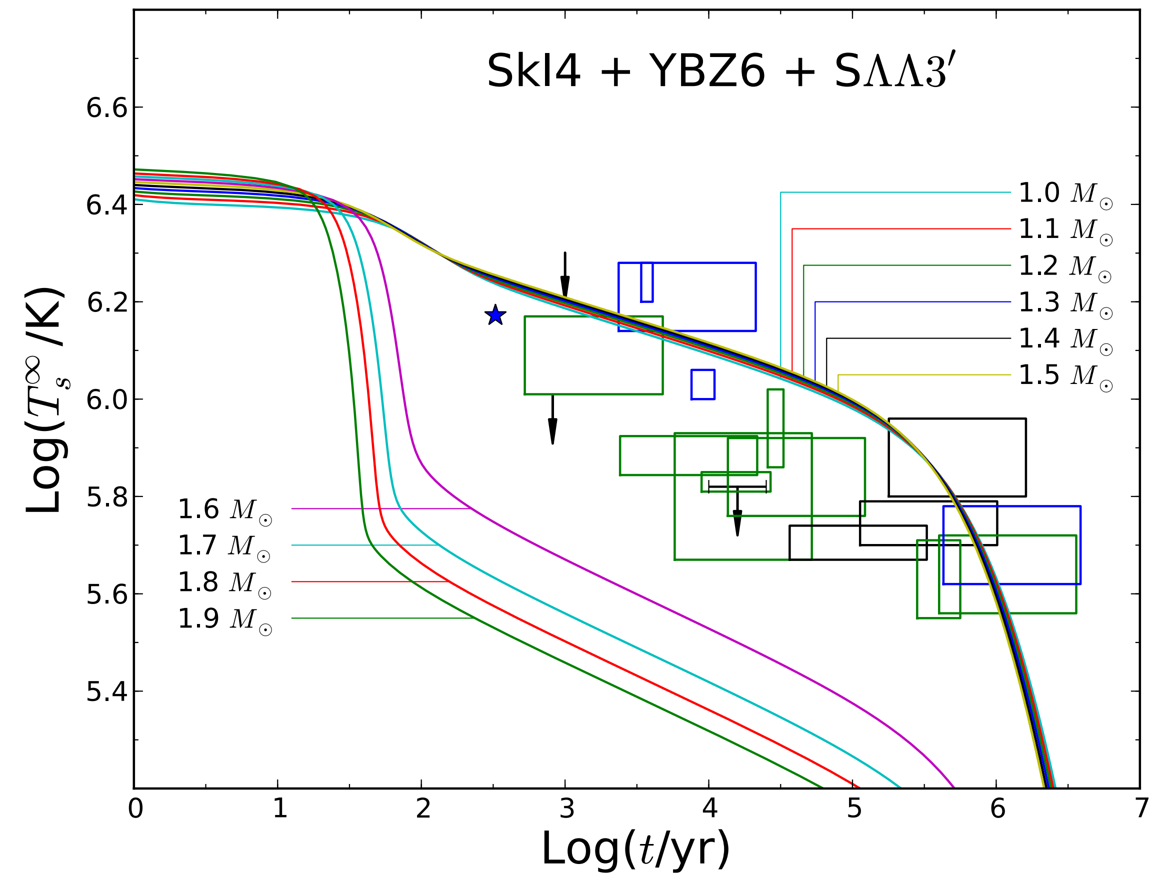
$$n + \langle K^- \rangle \to n + l + \bar{\nu}_l$$

$$Q_K = 2.5 \times 10^{26} \frac{m_n^{*2}}{m_n^2} \left(\frac{n_e}{n_0}\right)^{1/3} T_9^6\, \theta_K^2 \times \tan^2\theta_C \,\mathrm{erg\,cm^{-3}\,s^{-1}} \,,$$

Cooling with hyperons (Skyrme force model **SkI4**)



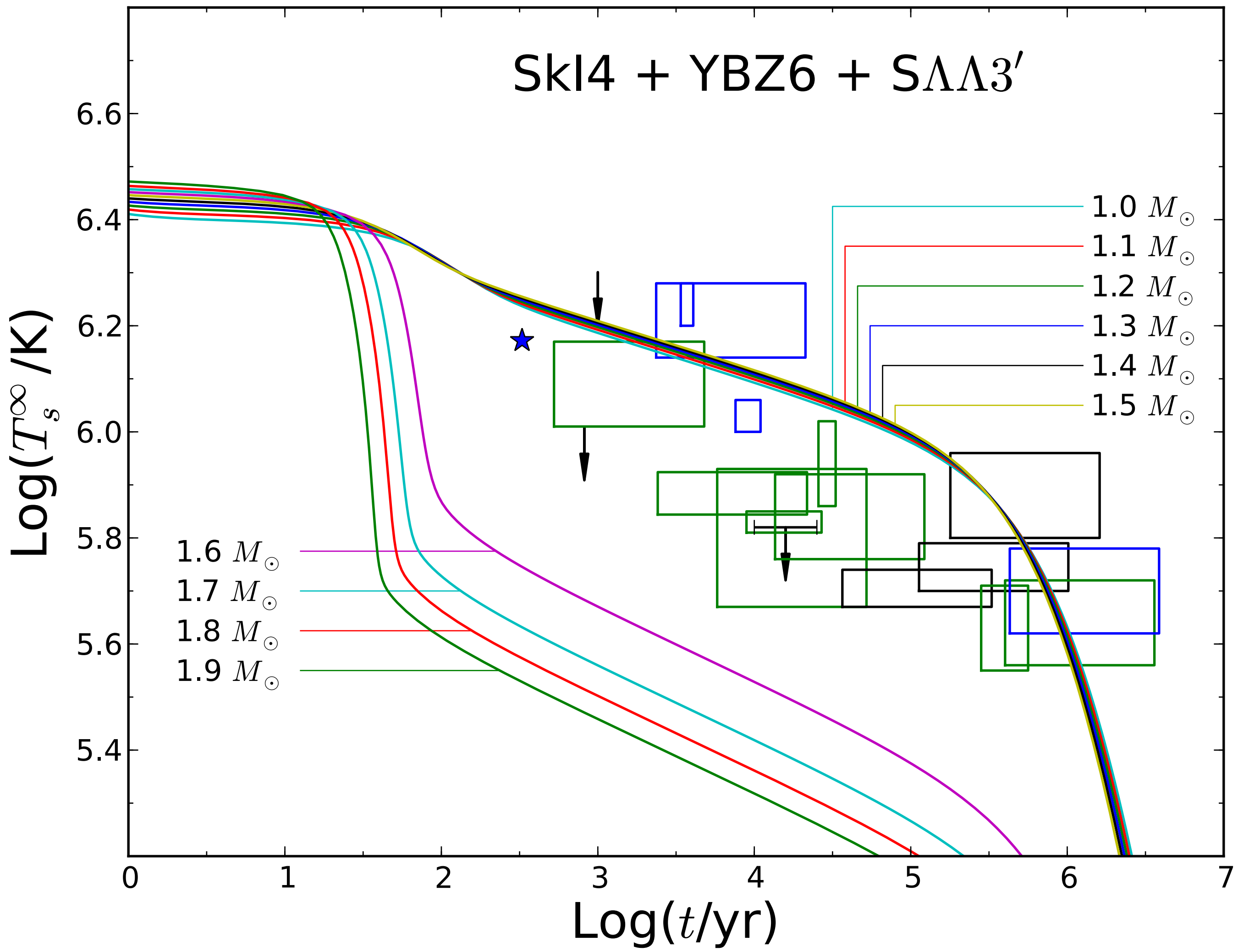
without hyperons
(sudden drop between 1.6~1.7)



with hyperons
(sudden drop between 1.5~1.6)

with Yeunhwan Lim, Chang Ho Hyun

Cooling with hyperons

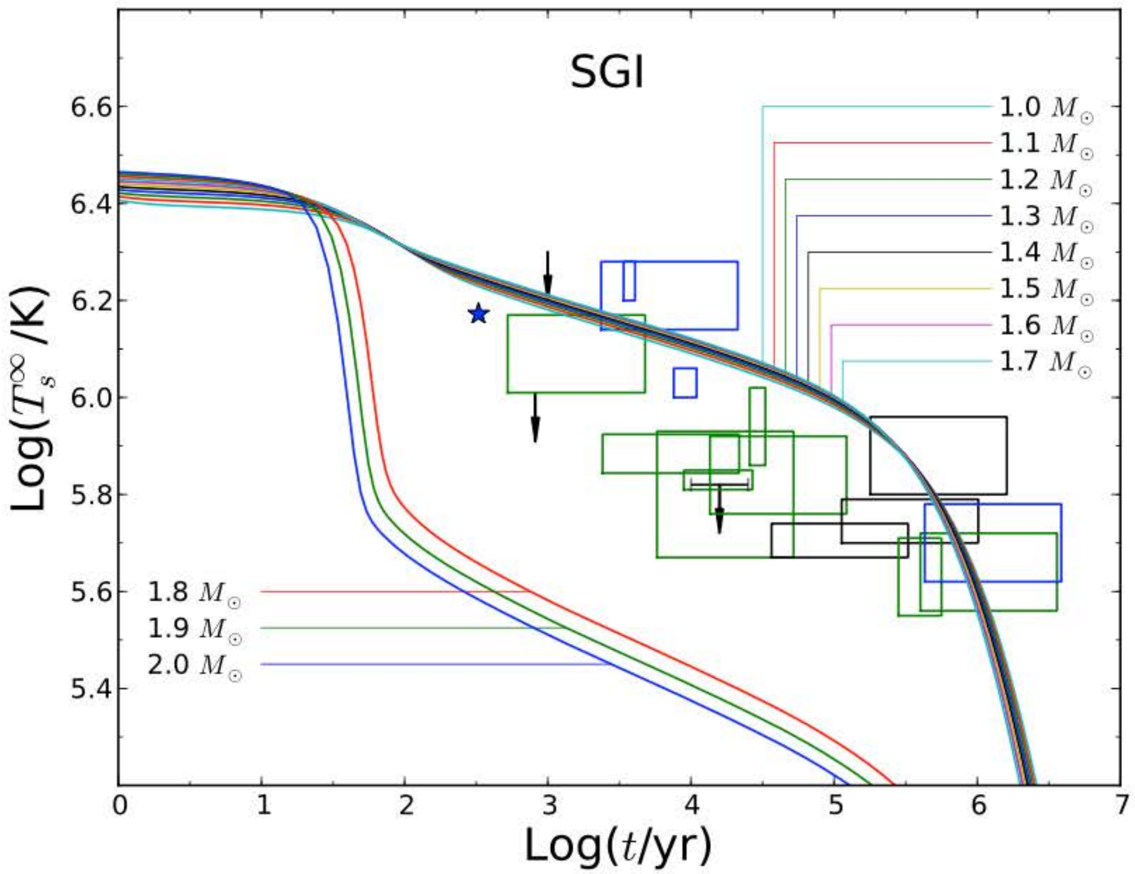


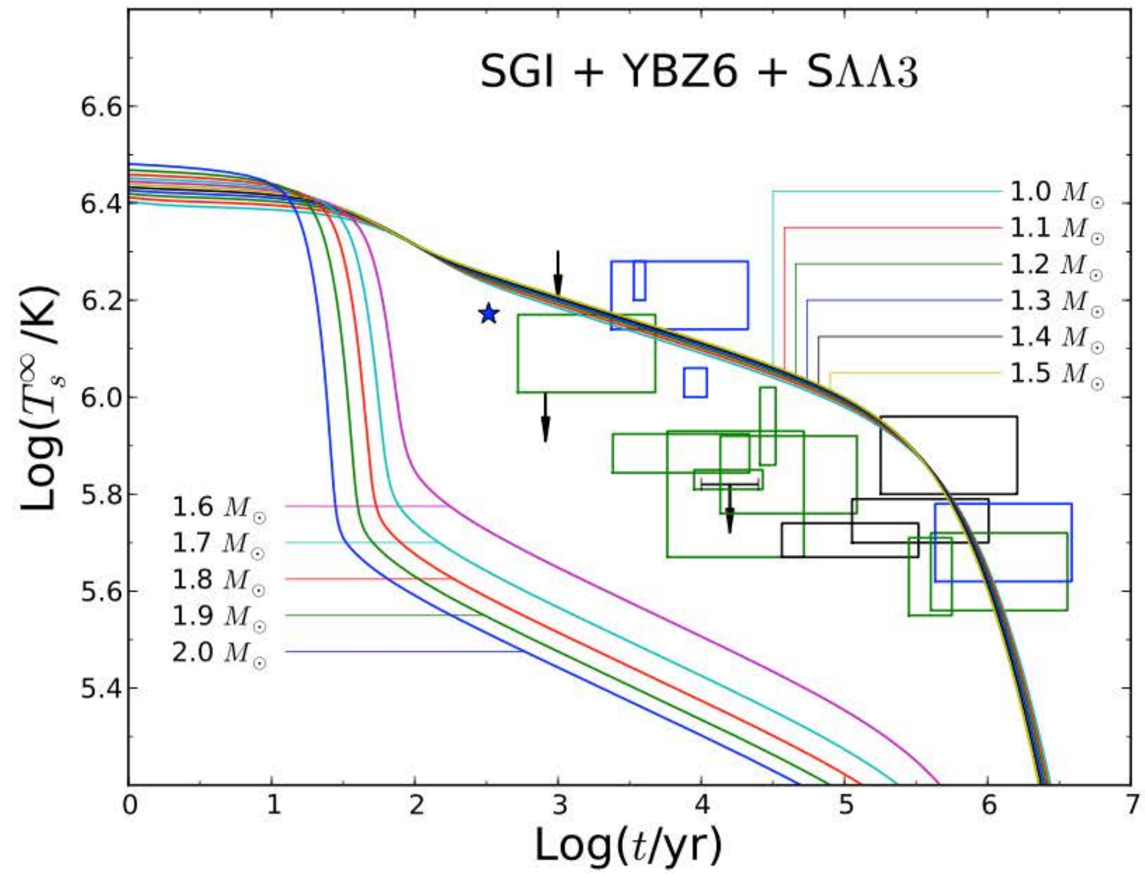
NS mass : 1.0 – 2.0 $M_\odot$

- abrupt drop: ingnition of direct URCA
- stiffer EoS allows early direct Urca
- no calculated-curve can explain middle-age data
- **require real fine-tuning**

with Yeunhwan Lim, Chang Ho Hyun

Cooling with hyperons (Skyrme force model **SGI**)



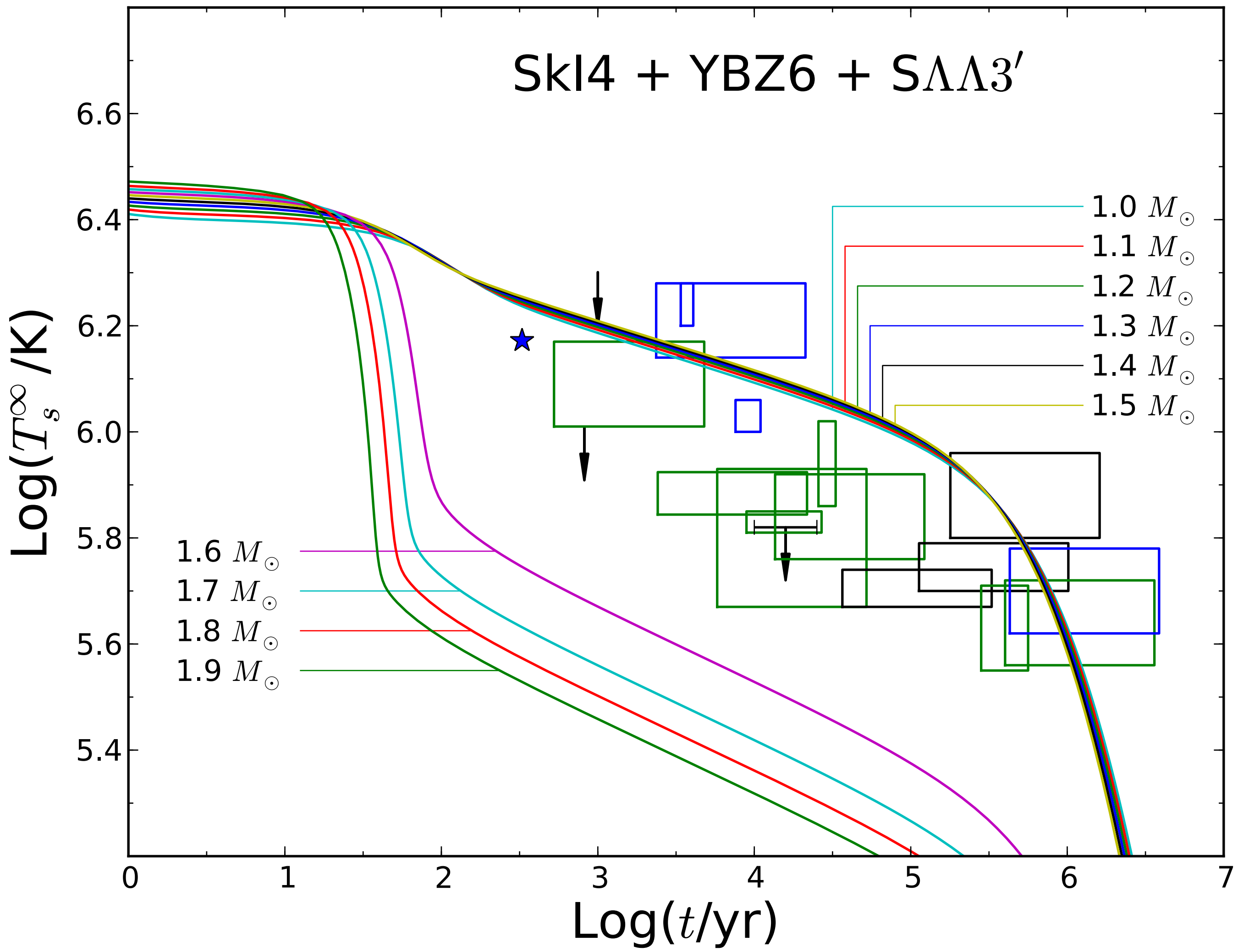


without hyperons
(sudden drop between 1.7~1.8)

with hyperons
(sudden drop between 1.5~1.6)

with Yeunhwan Lim, Chang Ho Hyun

Fine-tuning Problem



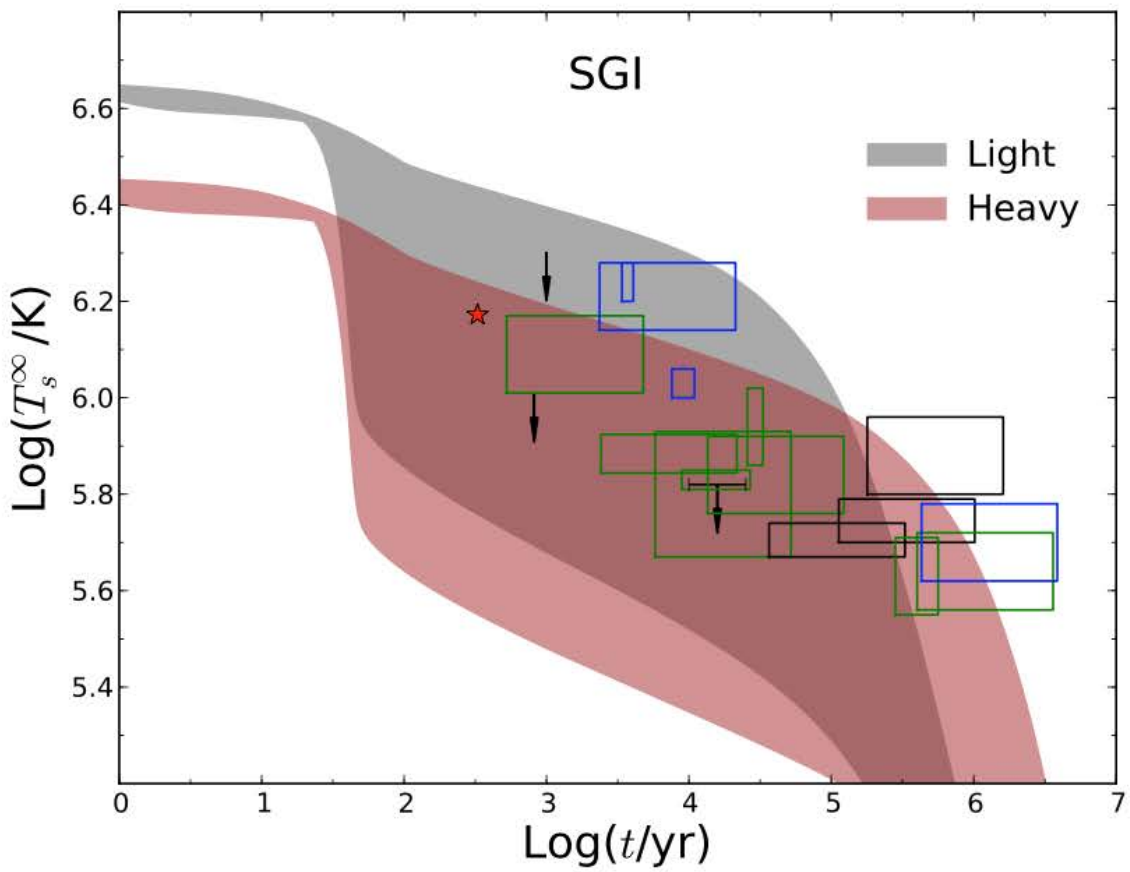
NS mass : 1.0 – 2.0 $M_\odot$

- abrupt drop: ingnition of direct URCA
- stiffer EoS allows early direct Urca
- no calculated-curve can explain middle-age data
- **require real fine-tuning**

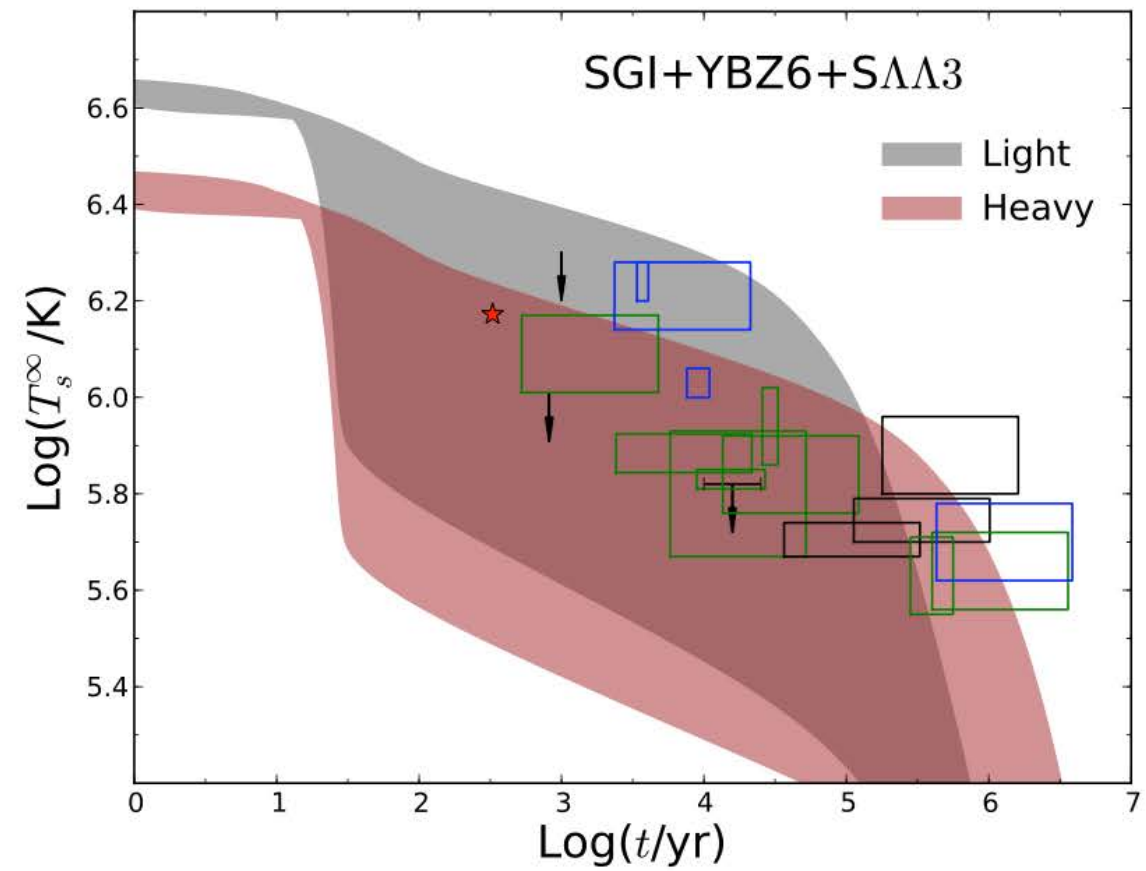
with Yeunhwan Lim, Chang Ho Hyun, Kyujin Kwak

Dependence on the surface elements

without hyperons



with hyperons

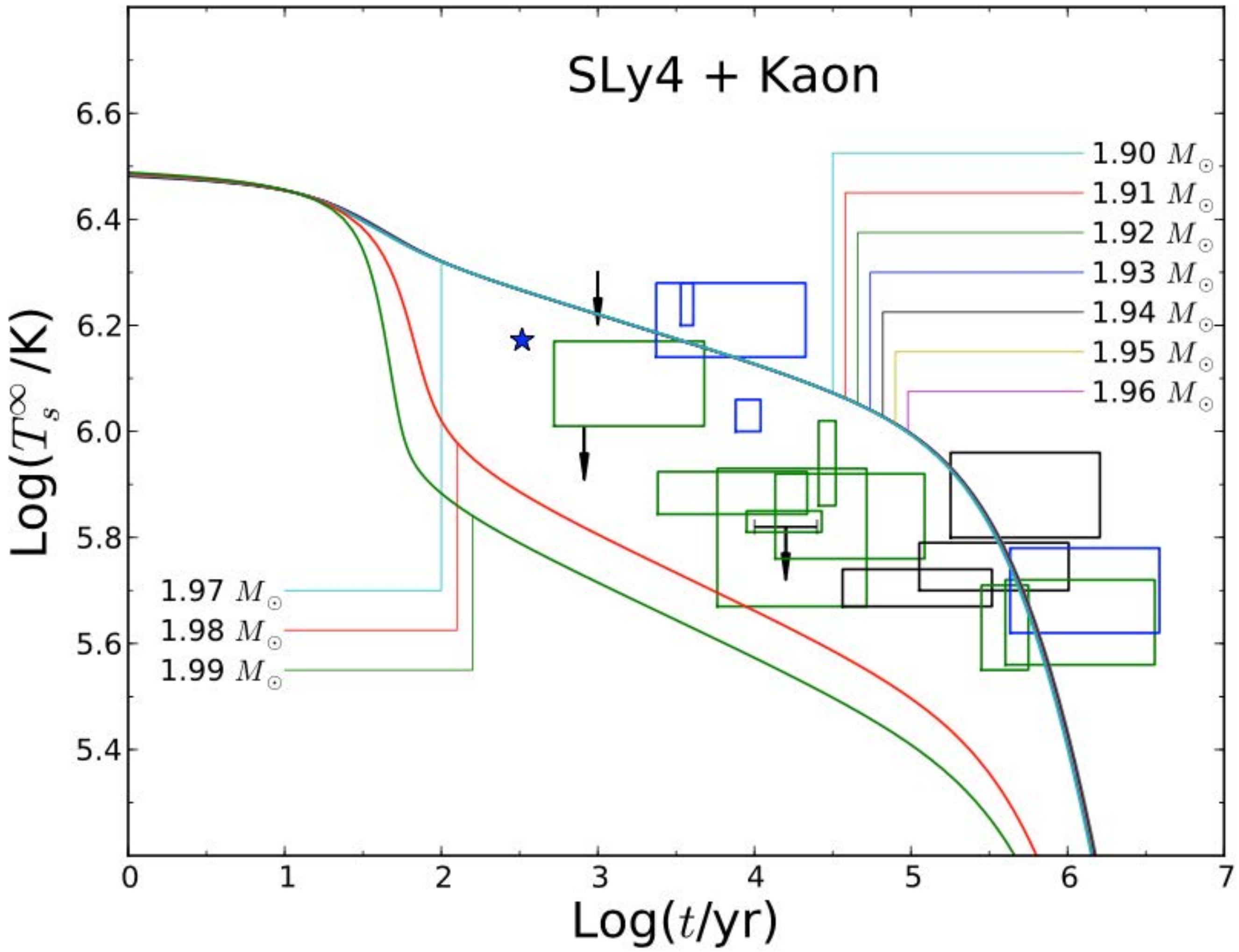


- **chemical evolution from light to heavy elements**
  (earlier plots are with heavy elements)
- **pulsar injection of light elements into magnetosphere**

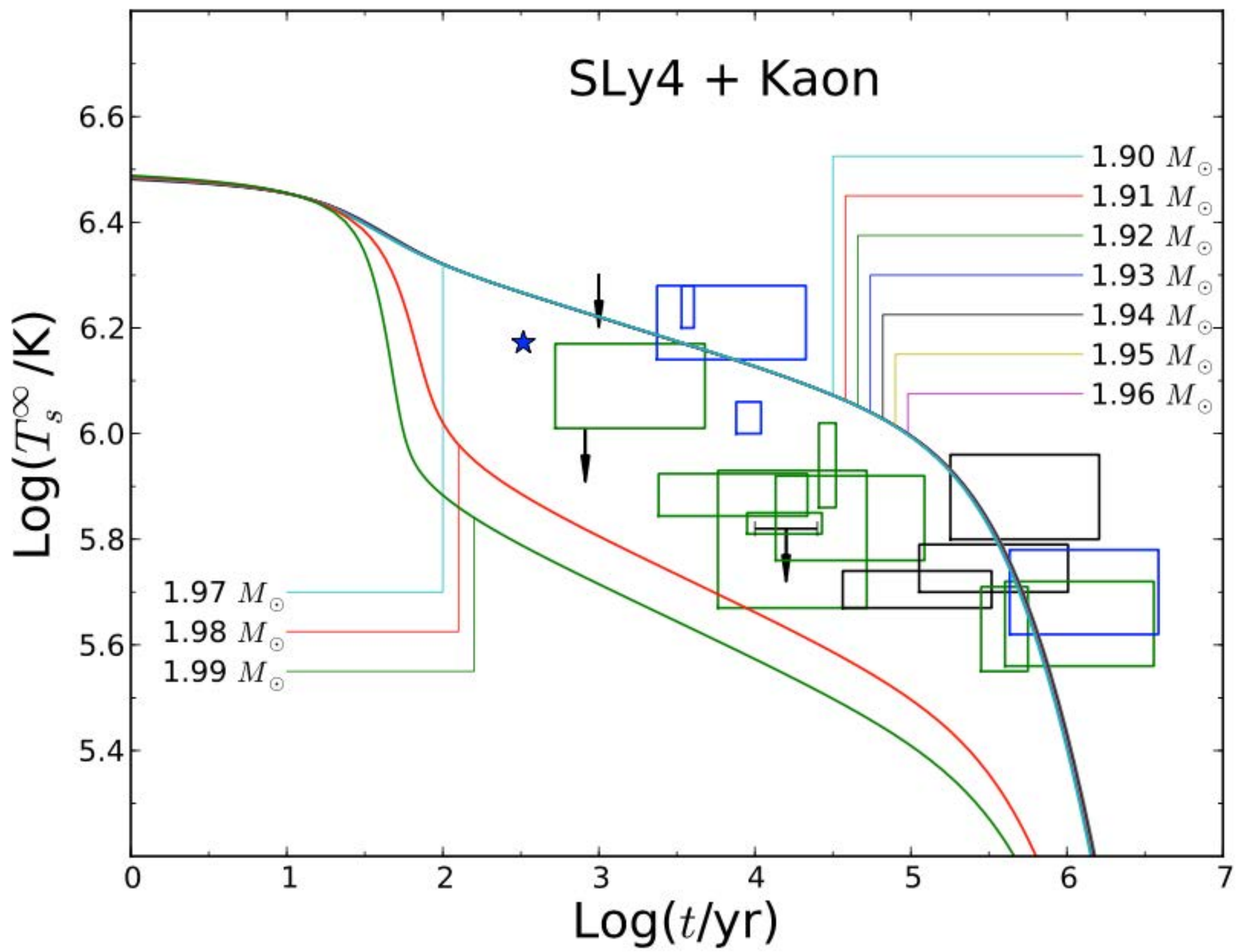
Negligible contribution with kaons

density for nucleon direct Urca < density for kaon condensation

Nucleon direct Urca is the dominant neutrino emission process



Negligible contribution with kaons



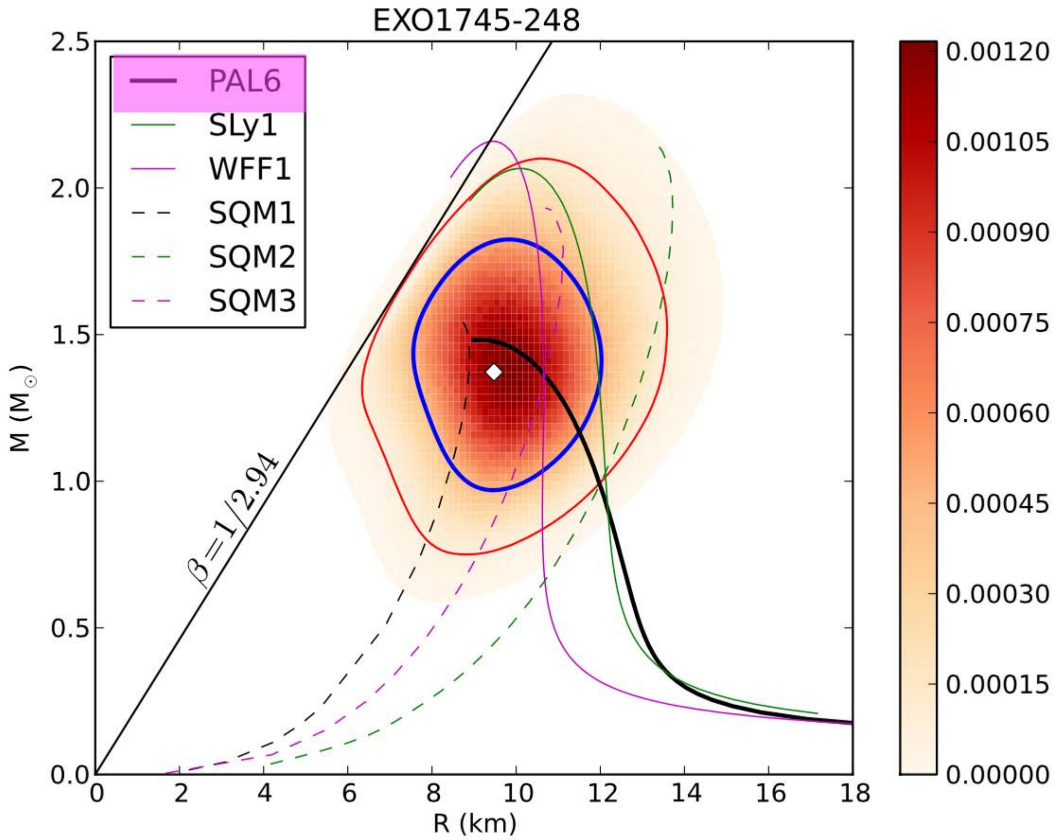
direct Urca comes before kaon condensation

Discussion

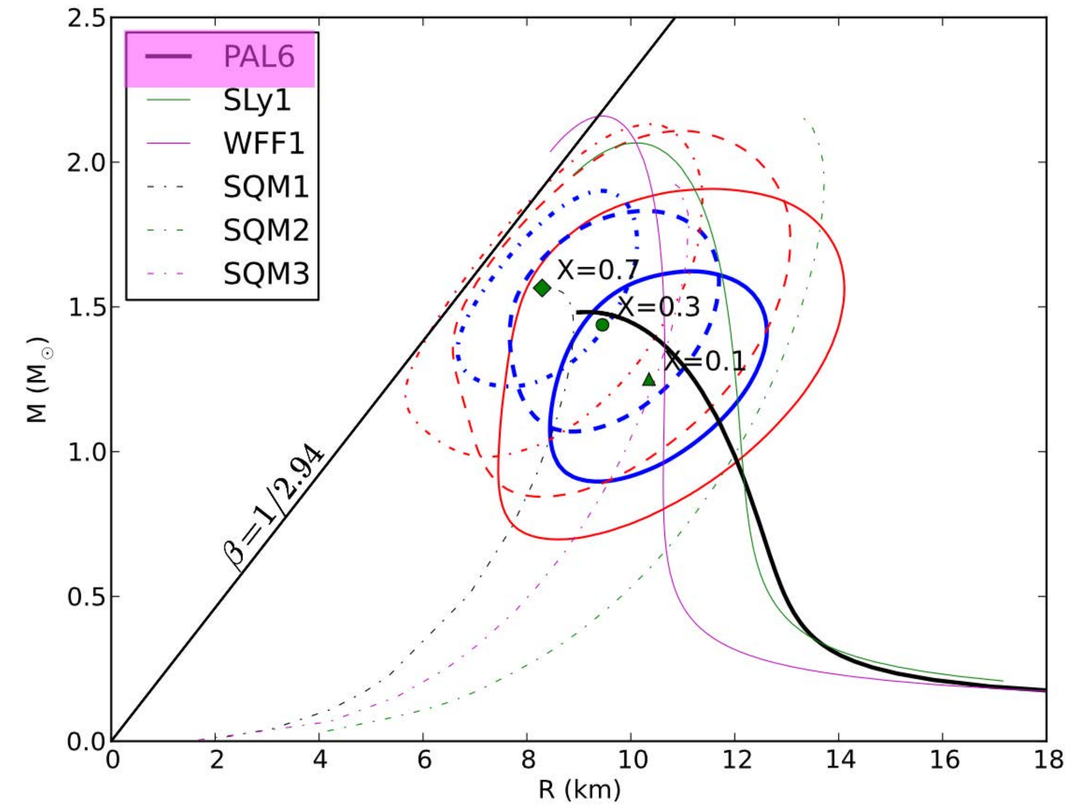
- **strangeness seems to be still alive**
- **but, cooling requires real fine tuning**
- **what is missing ?**

Neutron Star Binaries

Low-Mass X-ray Binaries (LMXB)

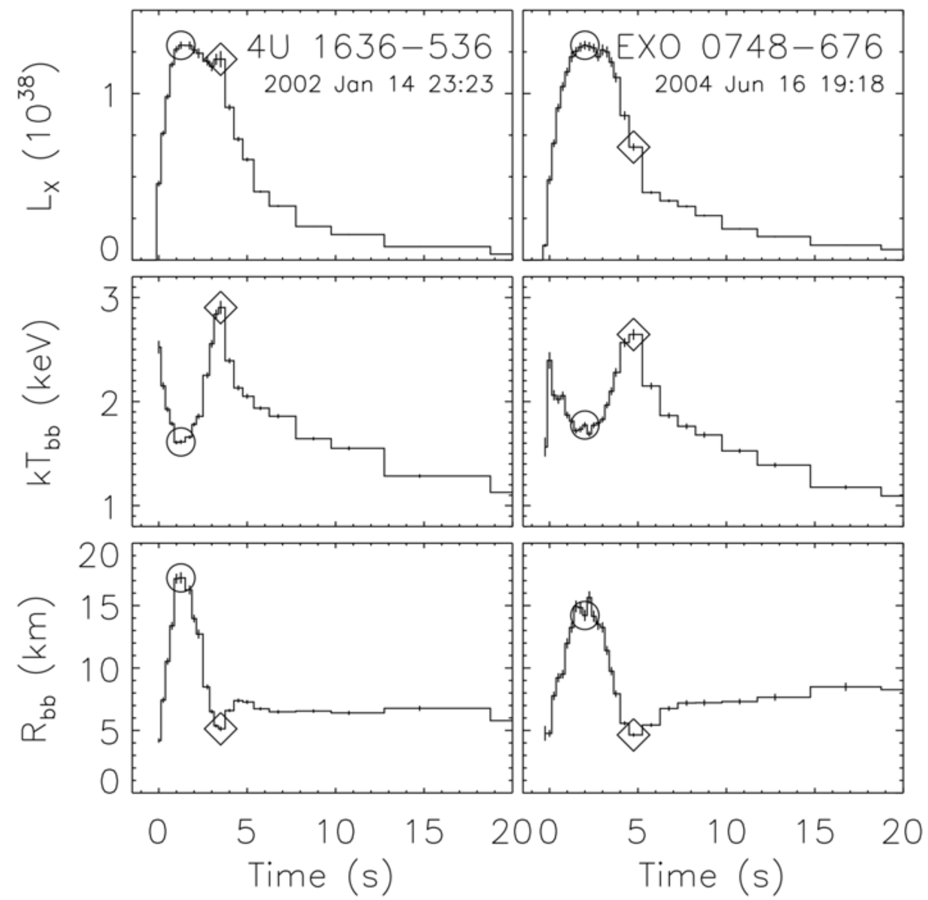


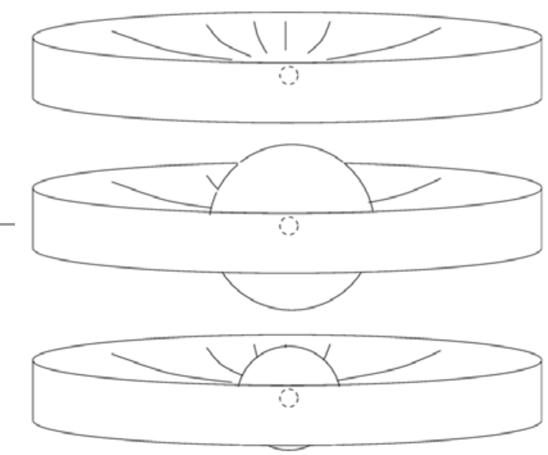
x : proton fraction



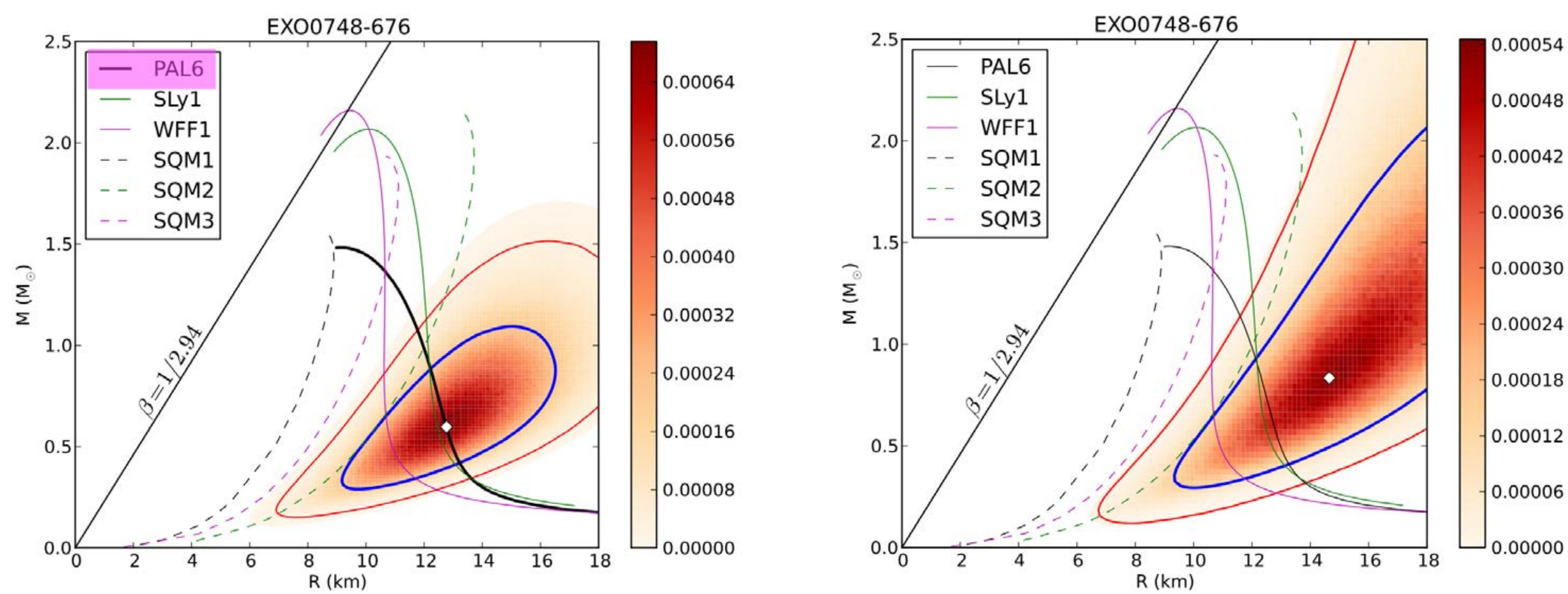
with Myungkuk Kim, Young-Min Kim, Kyujin Kwak

Disk Effect





Galloway et al. 2008



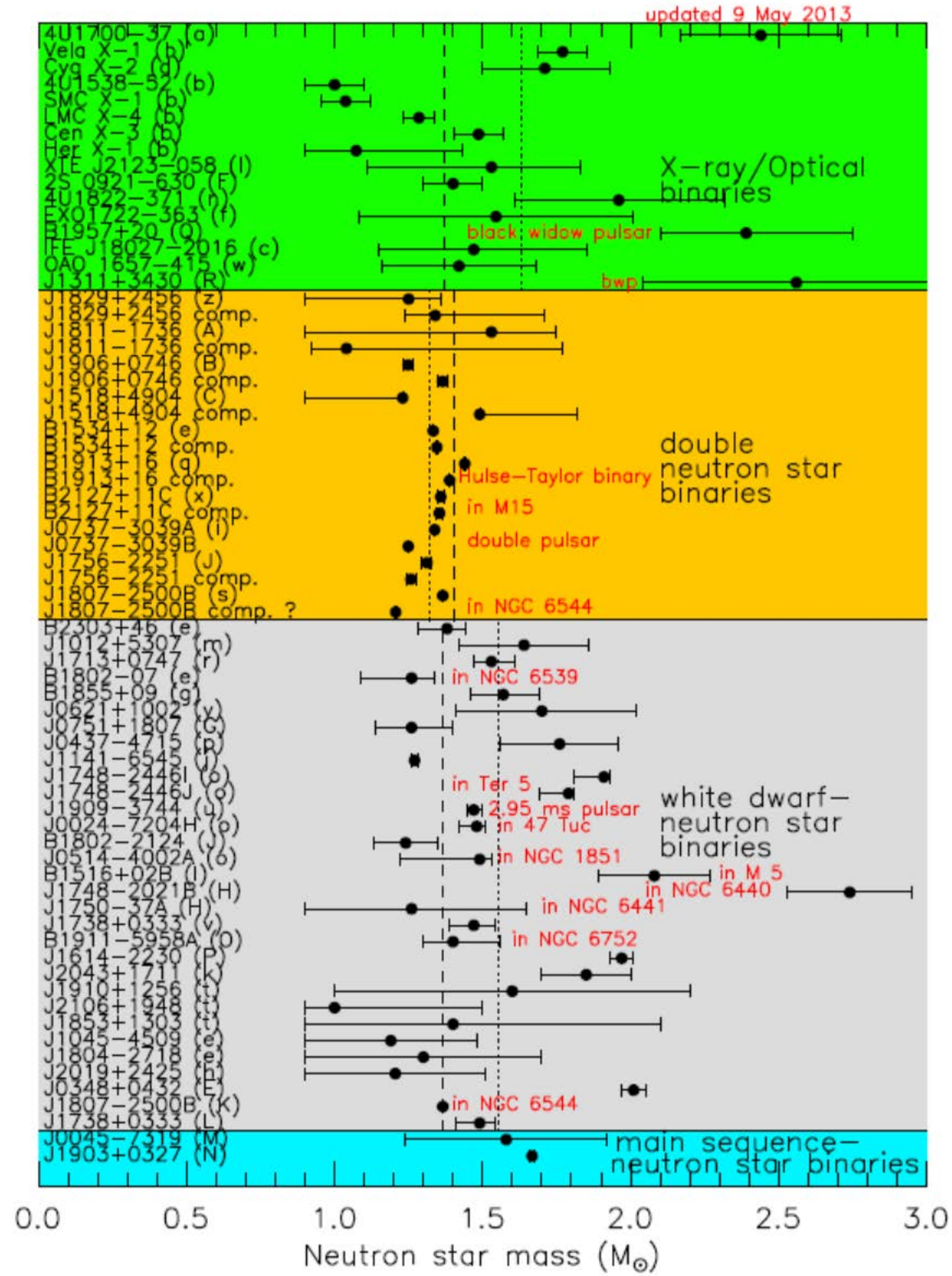
with Myungkuk Kim, Young-Min Kim, Kyujin Kwak

Black Hole Binaries

ApJ 575, 996 (2002)
ApJ 670, 741 (2007)
NPA 928, 296 (2014)

Masses

- High-mass neutron stars in X-ray binaries & white dwarf-NS binaries (2010 & 2013)
- Less than 1.5 solar mass in double NS binaries
- Maximum NS mass is still uncertain

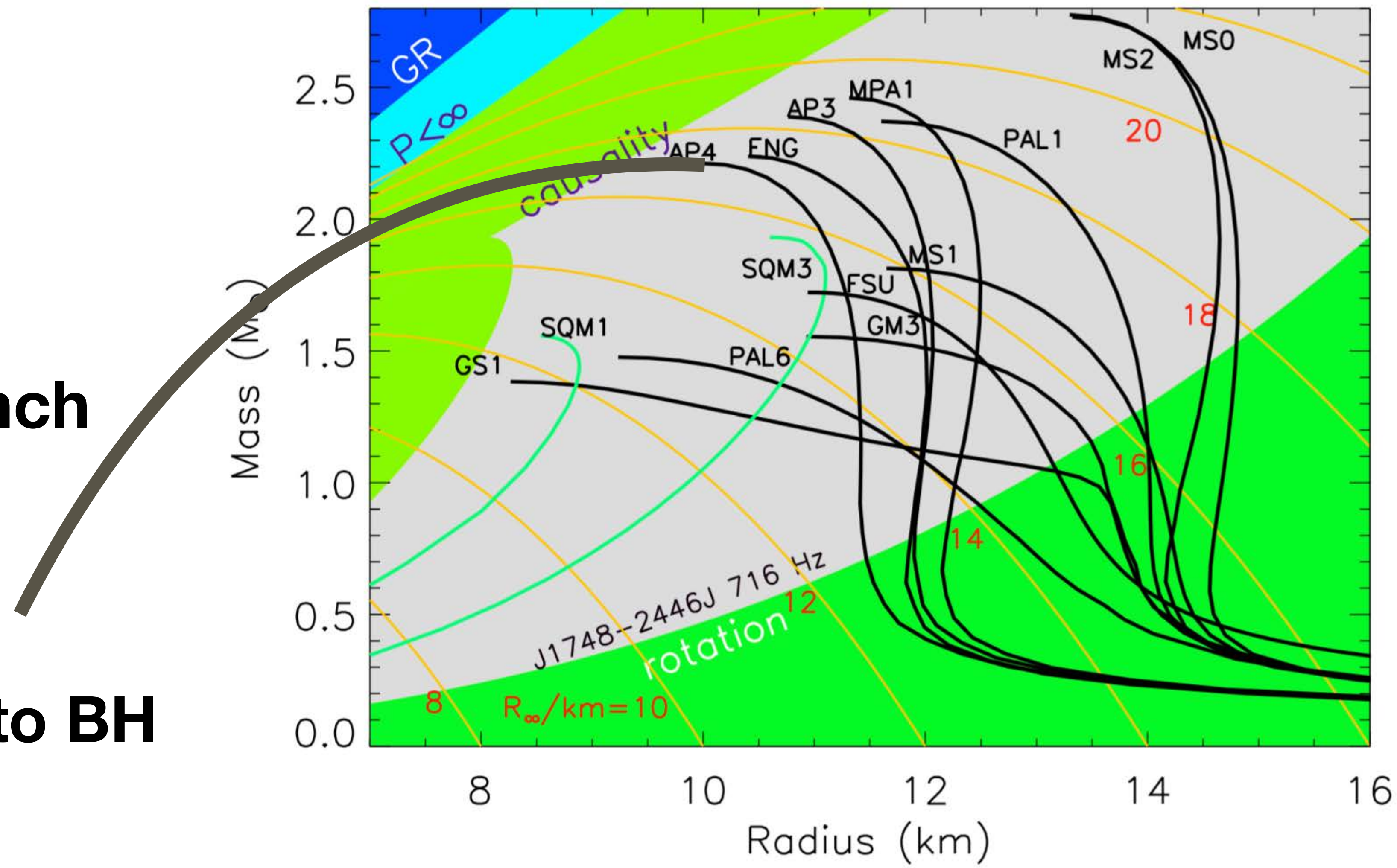


Prakash 2013

Black Hole Formation from Core Collapse

unstable branch

collapse to BH

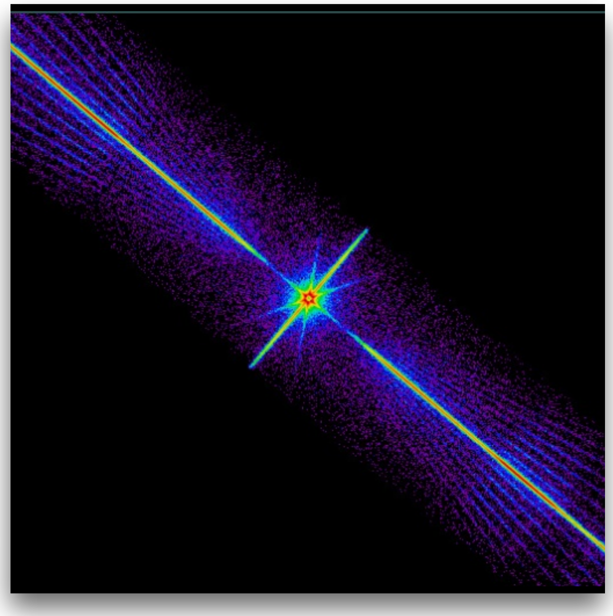


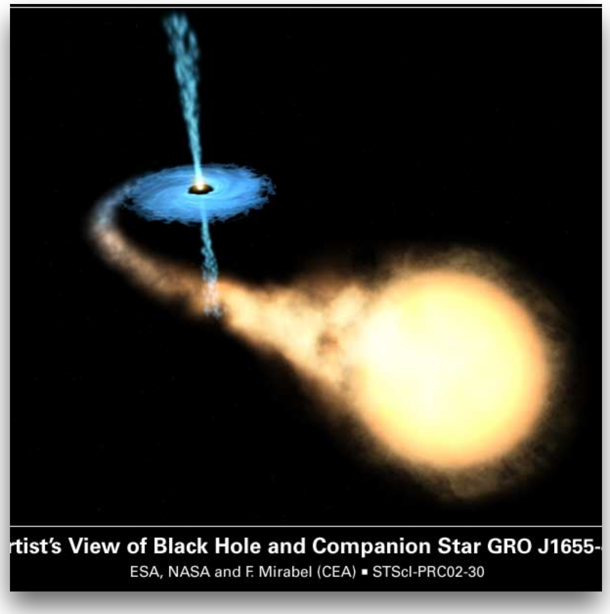
Black Hole Observation

Stellar Mass Black Holes

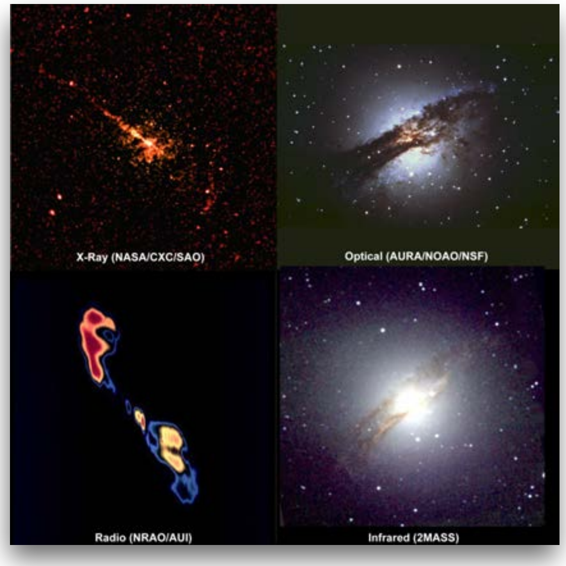
X-ray Binaries

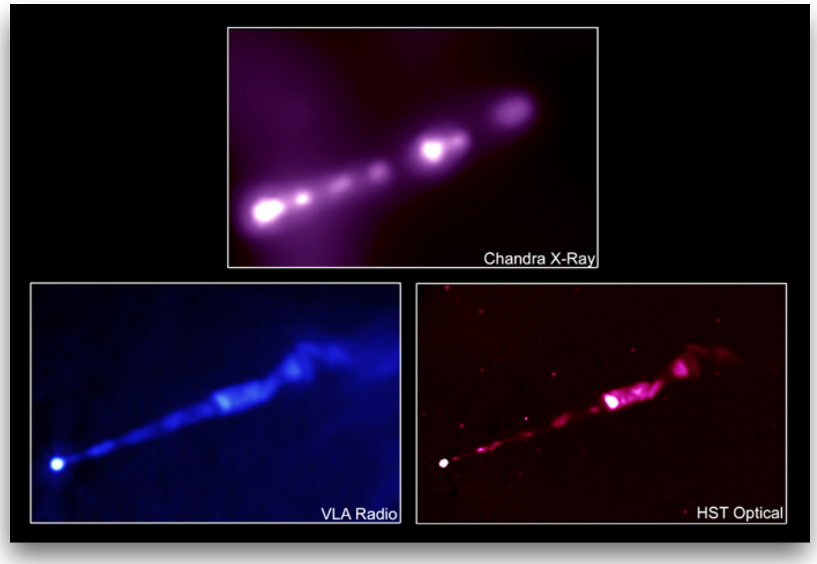
XTEJ1118+480

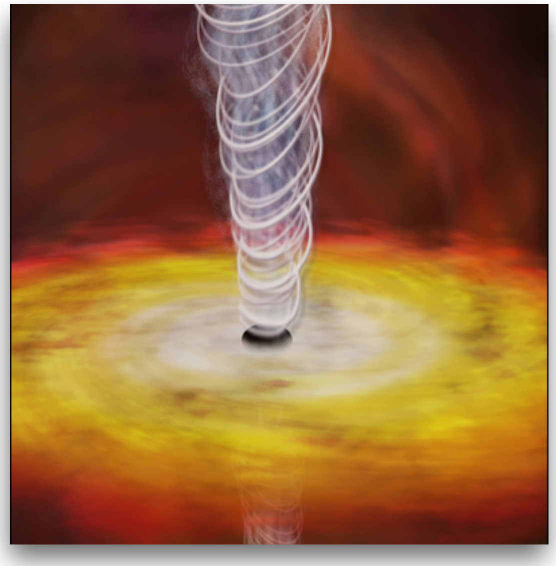




Galactic Center



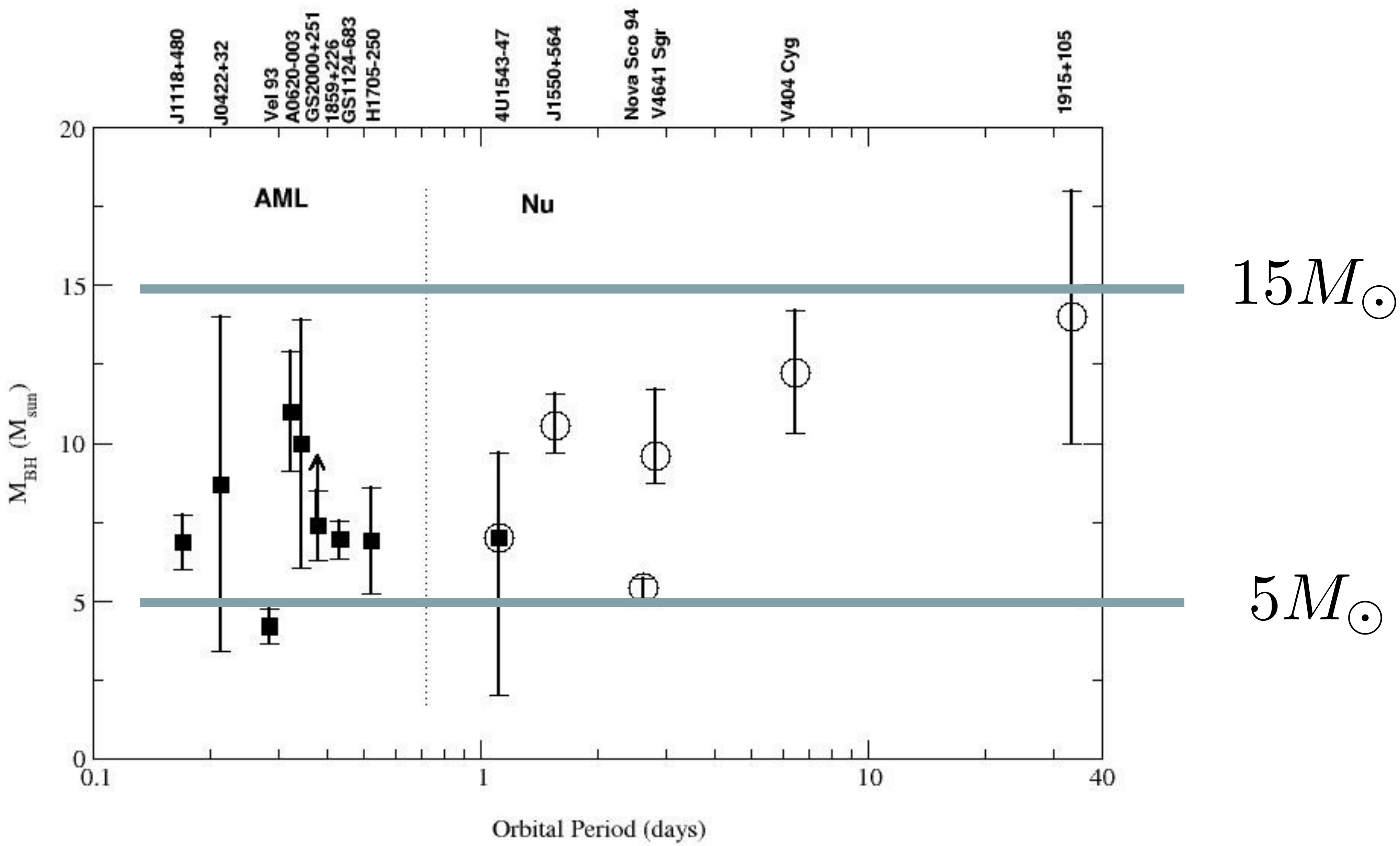




Centaurus A

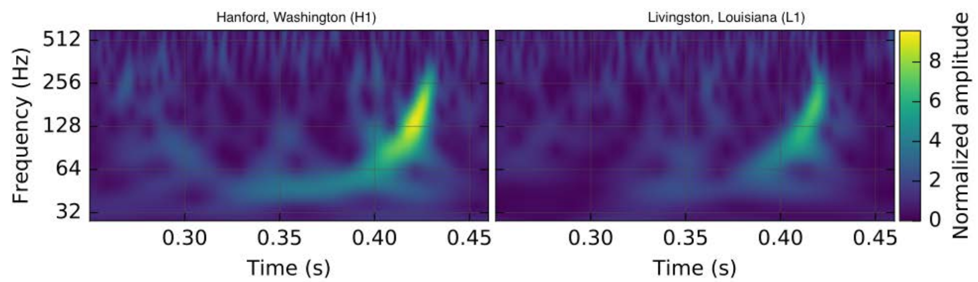
M87

Soft X-ray Transients



Main Sequence Companion | **Evolved Companion**

New Class of Black Hole Binaries



$$36M_\odot + 29M_\odot$$

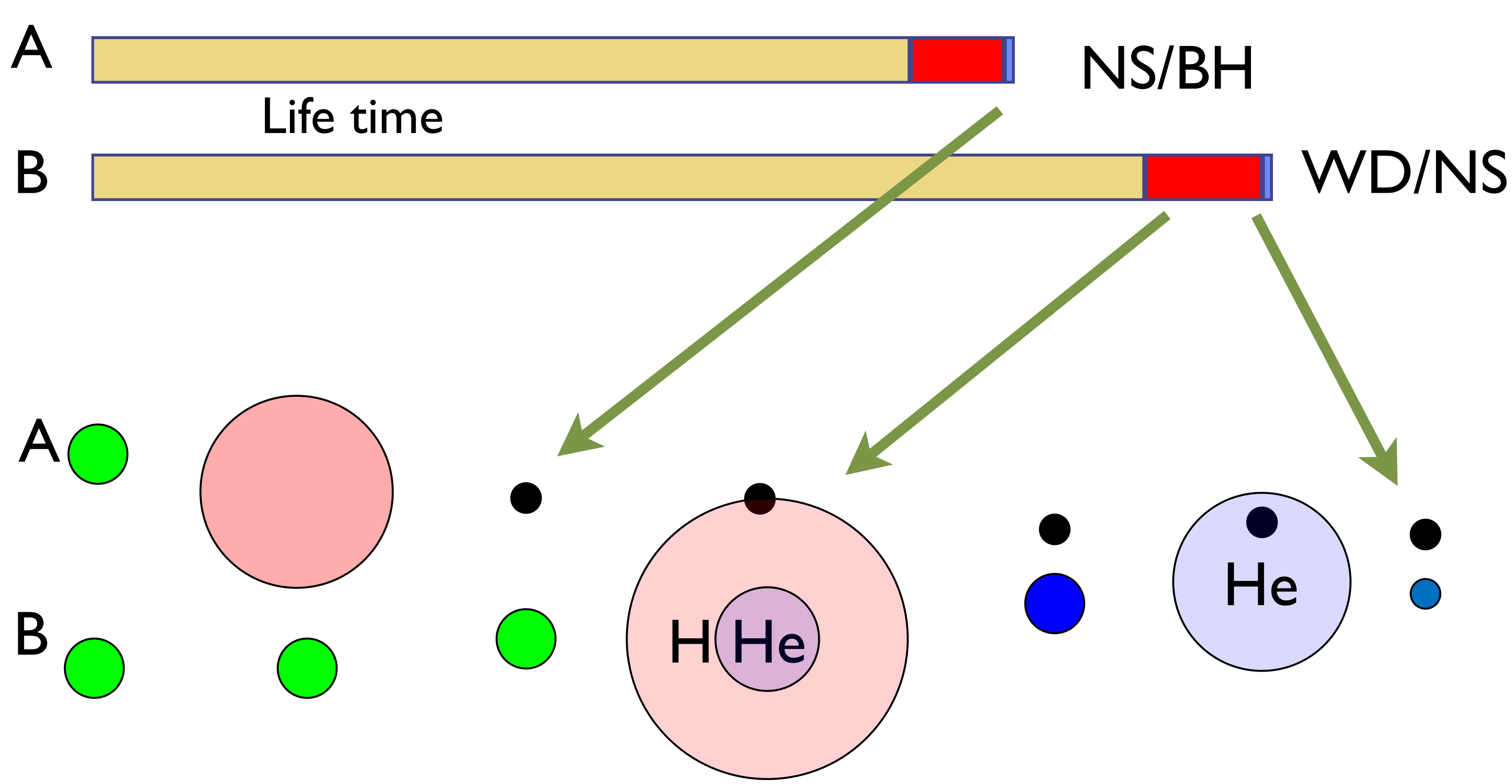
$$z = 0.09\ (1.3 \times 10^9 \text{ light year})$$

Formation of massive black holes
in low-metallicity galaxies
(from pop-III stars)

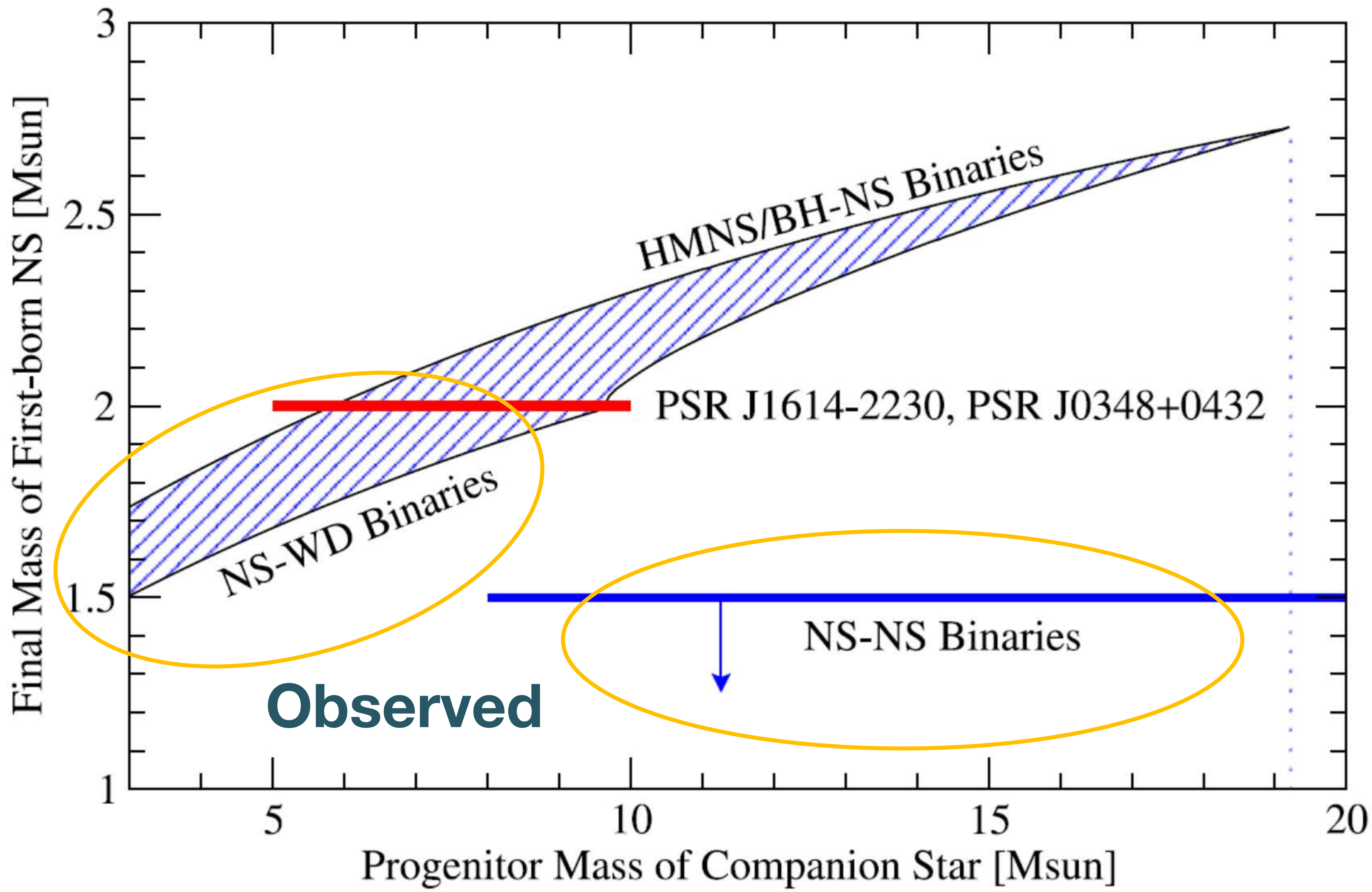
Binary Evolution

- black hole mass depends on the host galaxy
- black hole mass depends on the binary evolution

What about neutron stars ?



Mass Accretion after NS formation



BH-NS binaries

$$\tau_{\mathrm{recycled}} \gg \tau_{\mathrm{fresh}}$$

$$\tau_{\mathrm{pulsar}} \propto \frac{1}{B}$$

$$B_{\mathrm{fresh}} \sim 10^{12}\ \mathrm{G}$$

$$B_{\mathrm{recycled}} \sim 10^{8}\ \mathrm{G}$$

NS-NS

- if first-born NS is recycled by accretion
- longer pulsar life time, larger beaming angle

BH-NS

- no recycled pulsar
- smaller chances to be observed

Gravitational Wave Sources