제 1회 중력파 및 수치상대론 세미나

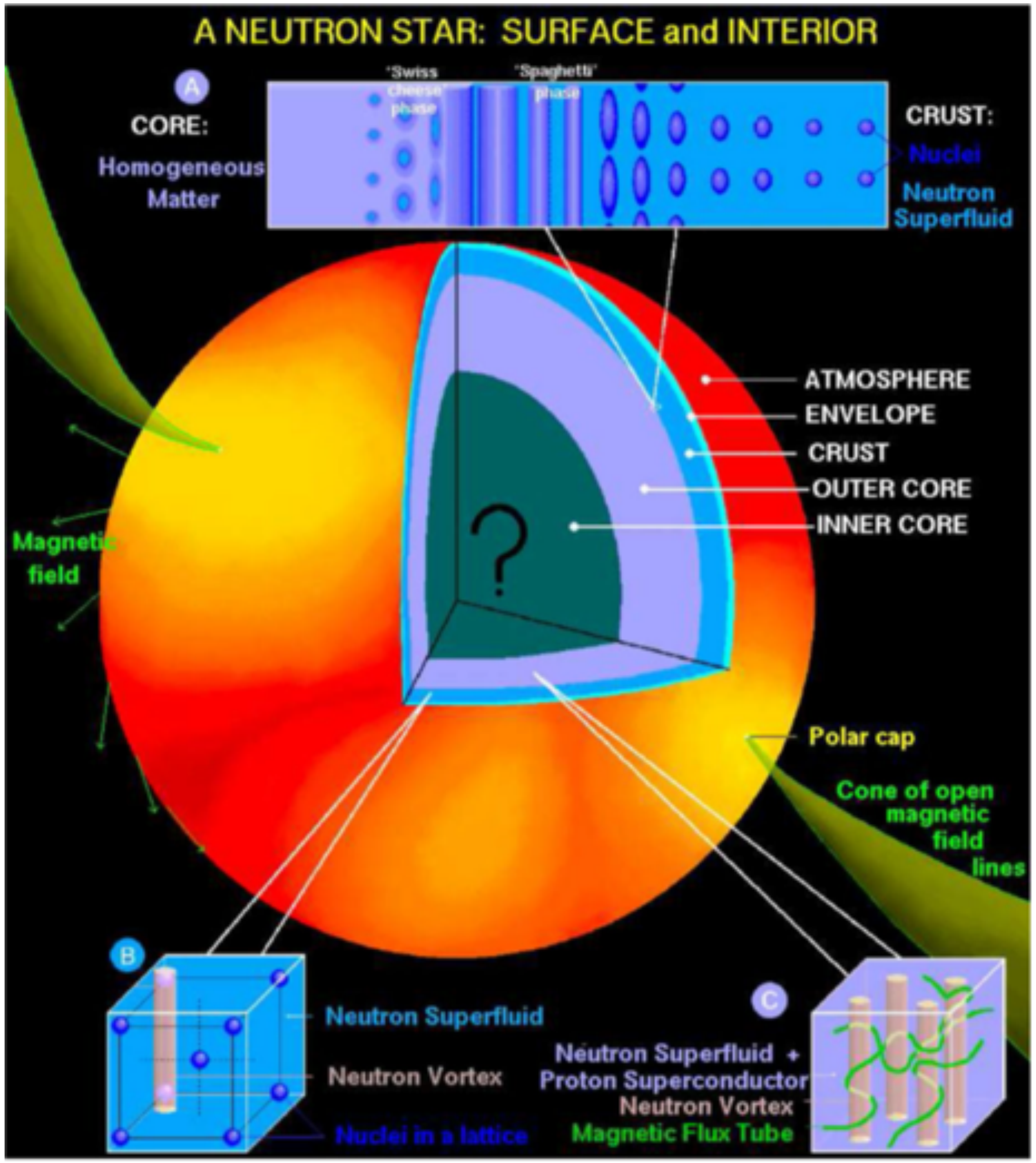
Tidal Deformability of a Neutron Star

Young-Min Kim (UNIST)

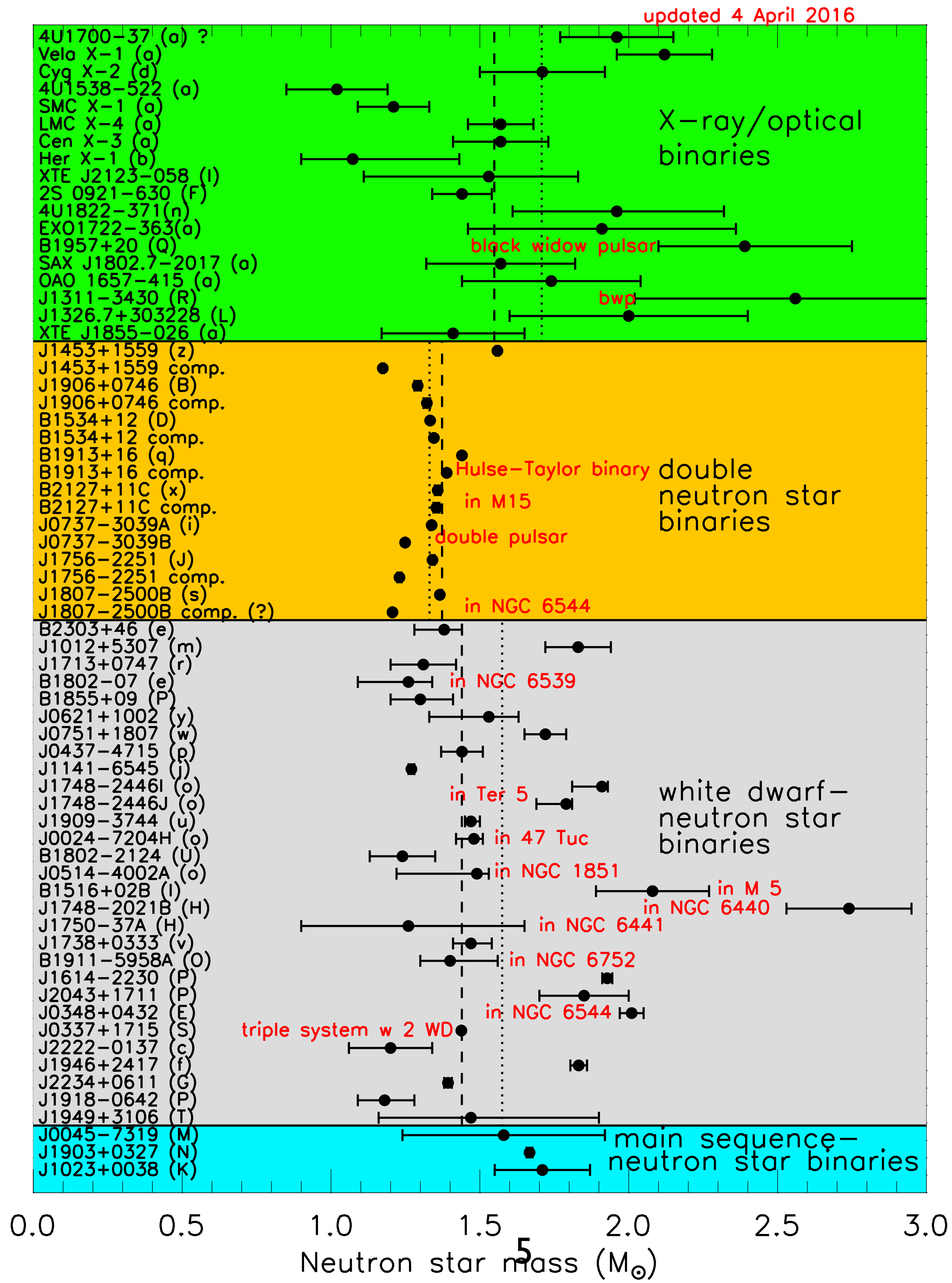
Contents

Introduction

Neutron Star

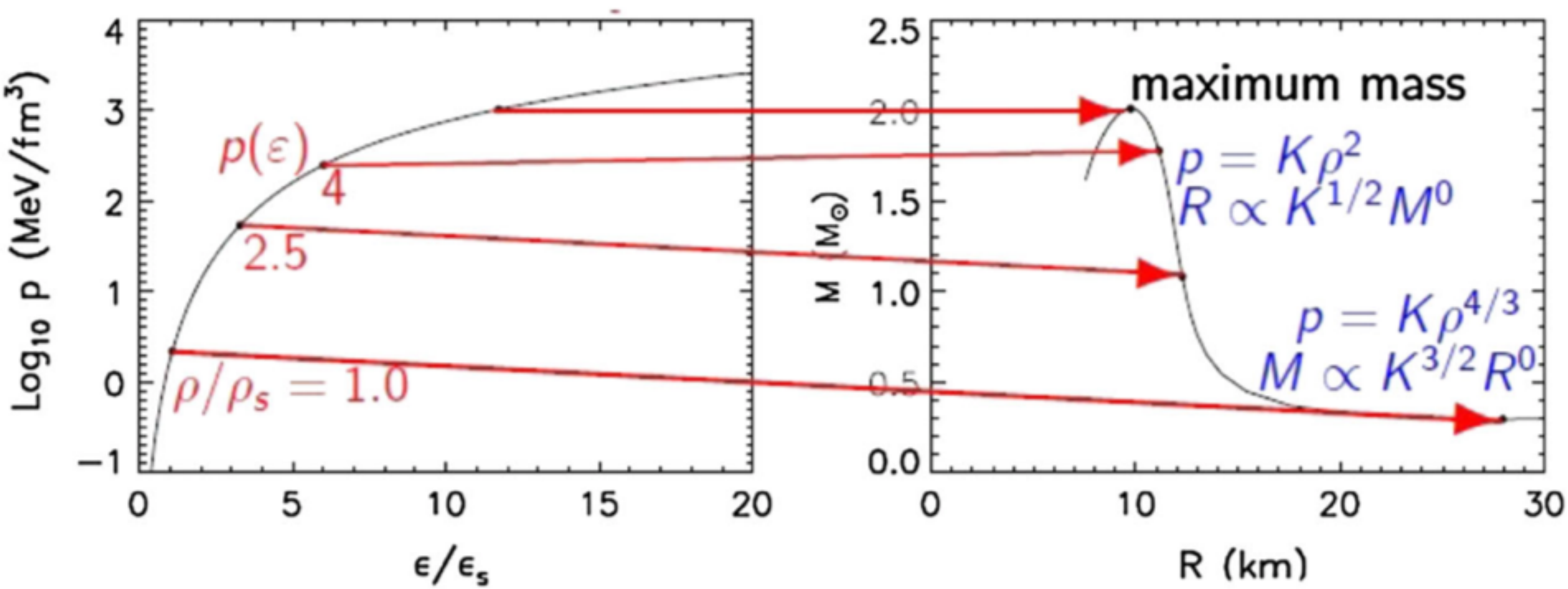


Neutron Star of Known Mass

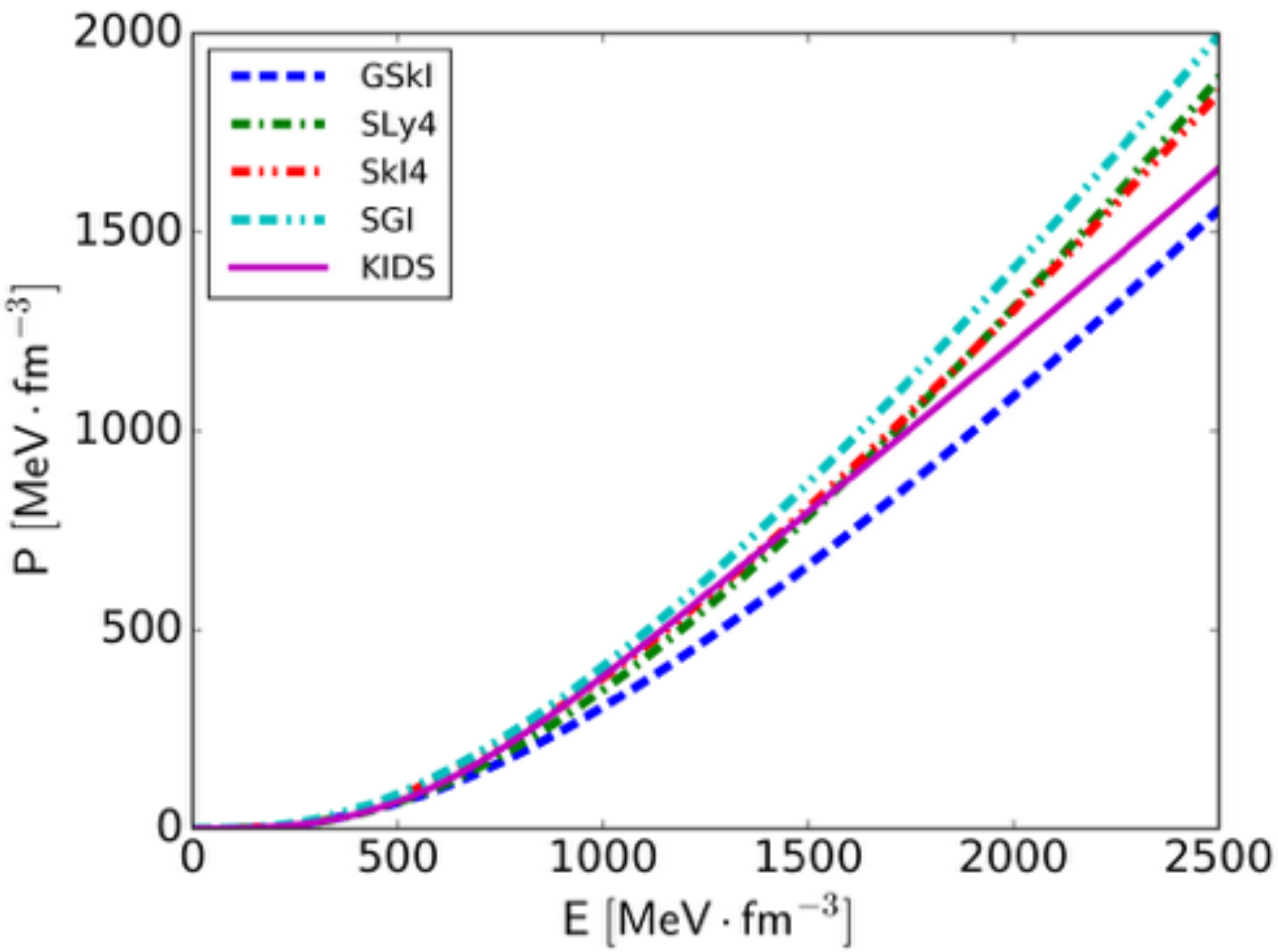


J. Lattimer, Annu.Rev.Nucl.Part.Sci. 62,485(2012) and https://stellarcollapse.org by C. Ott

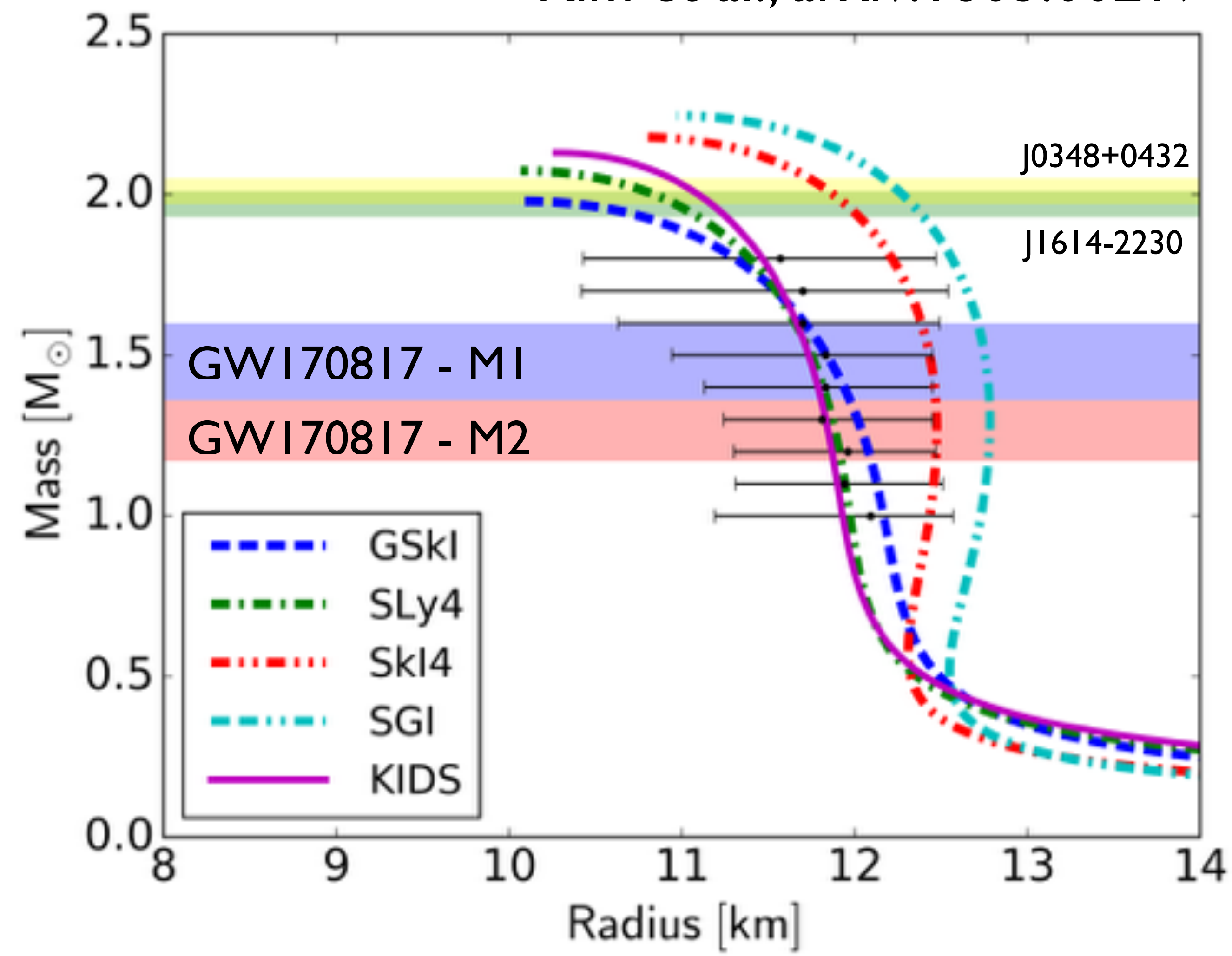
Maximum mass of Neutron Stars



Mass and Radius of NSs



Kim et al., arxiv:1805.00219



LMXB

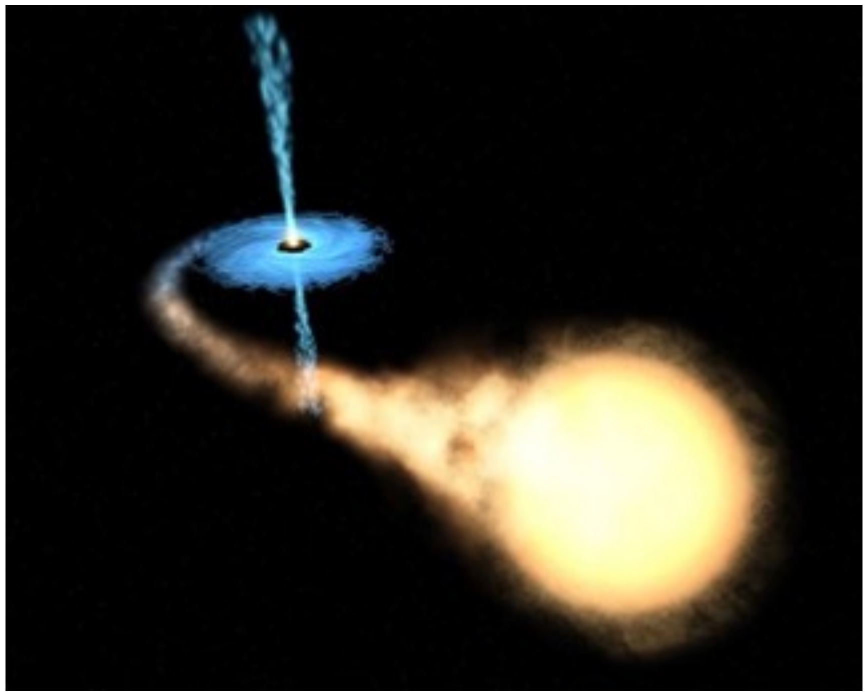
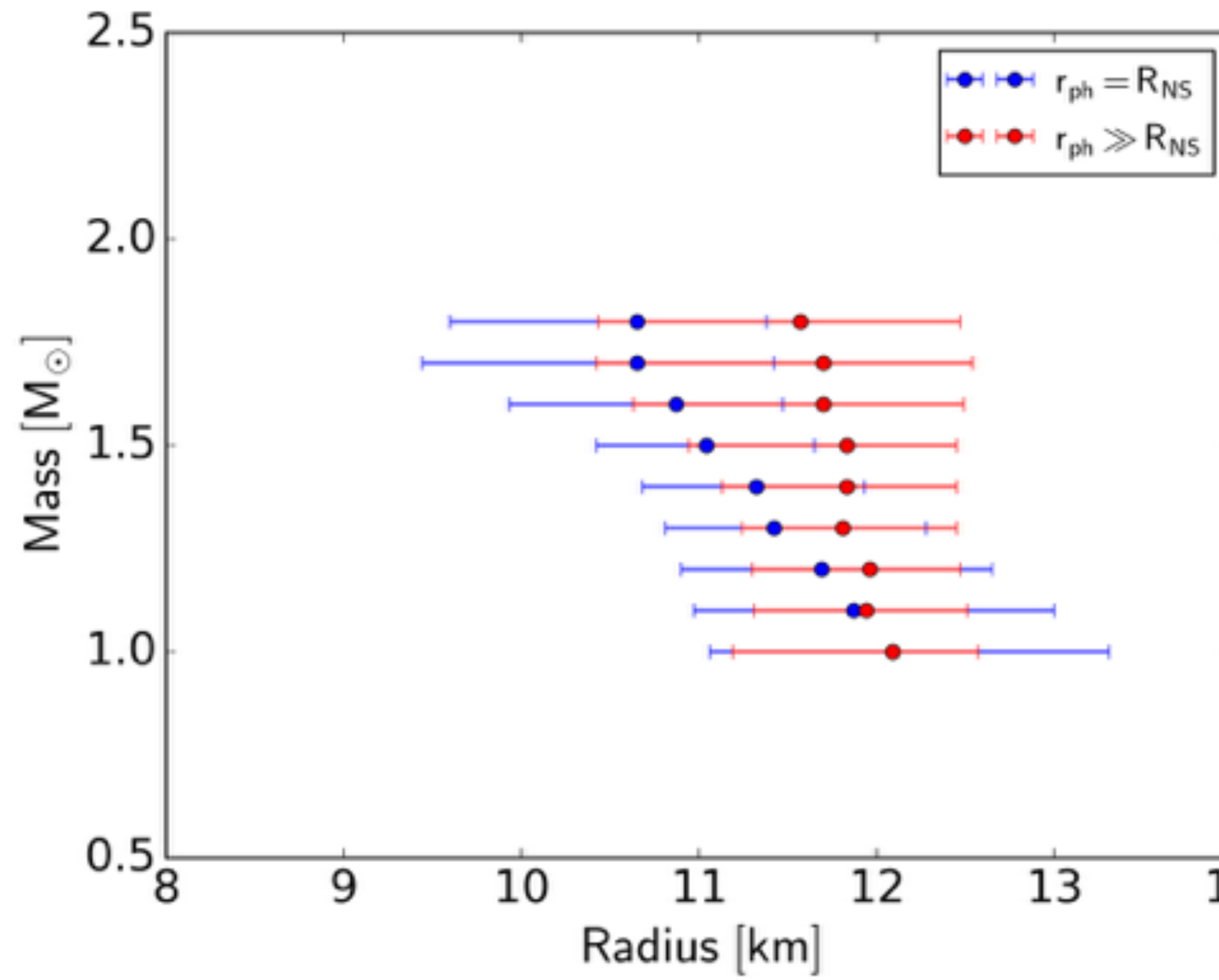


Table 9
Most Probable Values for Masses and Radii for Neutron Stars Constrained to Lie on One Mass Versus Radius Curve

Object	$M\ (M_\odot)$	R (km)	$M\ (M_\odot)$	R (km)
	$r_{ph} = R$		$r_{ph} \gg R$	
4U 1608–522	$1.52^{+0.22}_{-0.18}$	$11.04^{+0.53}_{-1.50}$	$1.64^{+0.34}_{-0.41}$	$11.82^{+0.42}_{-0.89}$
EXO 1745–248	$1.55^{+0.12}_{-0.36}$	$10.91^{+0.86}_{-0.65}$	$1.34^{+0.450}_{-0.28}$	$11.82^{+0.47}_{-0.72}$
4U 1820–30	$1.57^{+0.13}_{-0.15}$	$10.91^{+0.39}_{-0.92}$	$1.57^{+0.37}_{-0.31}$	$11.82^{+0.42}_{-0.82}$
M13	$1.48^{+0.21}_{-0.64}$	$11.04^{+1.00}_{-1.28}$	$0.901^{+0.28}_{-0.12}$	$12.21^{+0.18}_{-0.62}$
ω Cen	$1.43^{+0.26}_{-0.61}$	$11.18^{+1.14}_{-1.27}$	$0.994^{+0.51}_{-0.21}$	$12.09^{+0.27}_{-0.66}$
X7	$0.832^{+1.19}_{-0.051}$	$13.25^{+1.37}_{-3.50}$	$1.98^{+0.10}_{-0.36}$	$11.3^{+0.95}_{-1.03}$

95% confidence limits by using MC sampling



Steiner, Lattimer, Brown, ApJ 2010

Nuclear Equation of States

$$x = \frac{\rho_p}{\rho_p + \rho_n}$$

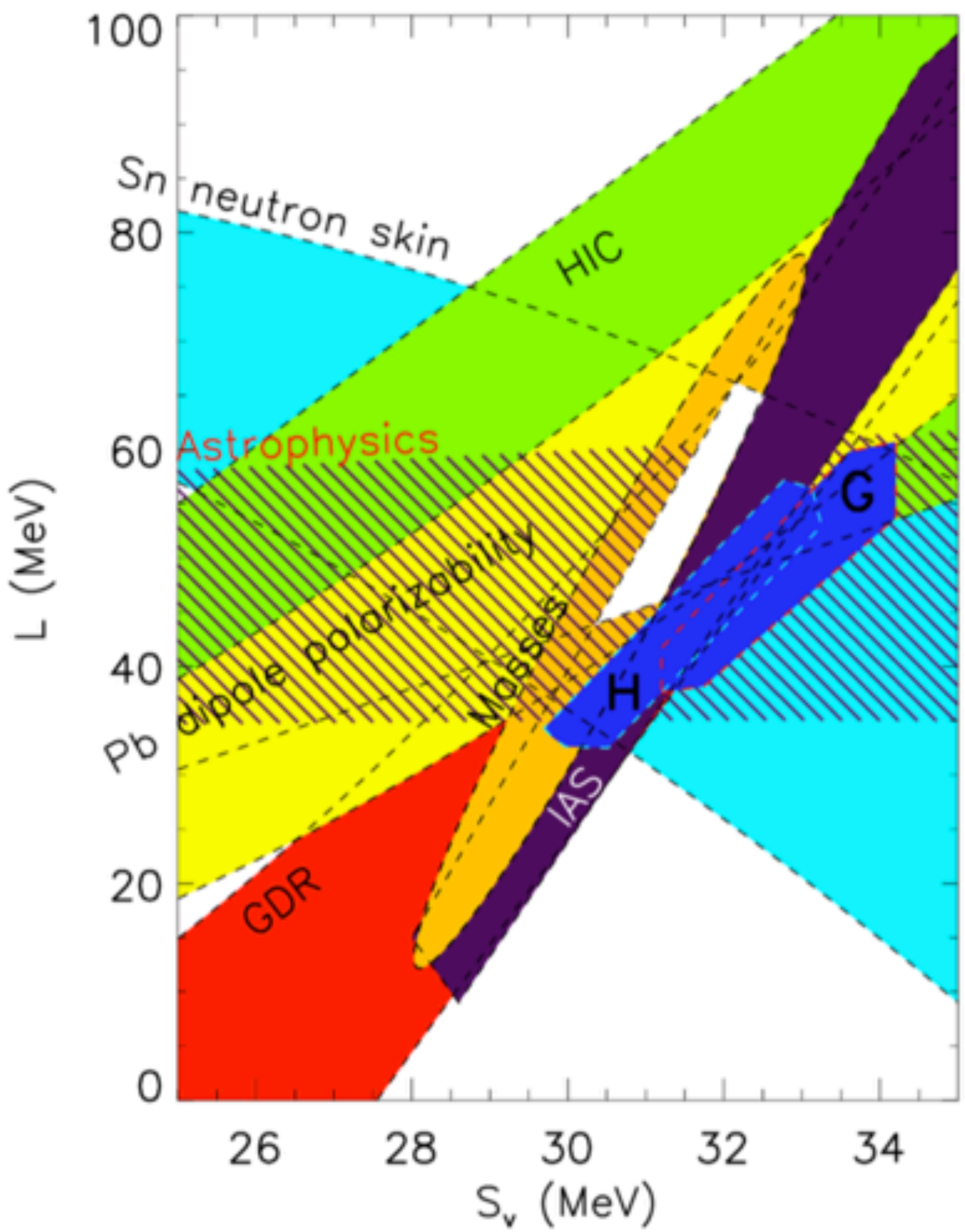
$$E(\rho, x) = -B + \frac{K_0}{18}\left(\frac{\rho}{\rho_0} - 1\right)^2 + \frac{K_0'}{162}\left(\frac{\rho}{\rho_0} - 1\right)^3 + E_{\rm sym}(\rho)(1-2x)^2 + \cdots$$

Incompressibility	$K_0 \simeq 230\ \mathrm{MeV}$
Skewness	$K_0' \sim -2000\ \mathrm{MeV}$

$S_0 \equiv E_{\rm sym}(\rho_0)$ **Symmetry Energy**

$$L \equiv 3\rho \left.\frac{\partial E_{\rm sym}}{\partial \rho}\right|_{\rho_0}$$

Symmetry Energy



RAON

$$S_0 \equiv E_{\rm sym}(\rho_0)$$

$$L \equiv 3\rho \left.\frac{\partial E_{\rm sym}}{\partial \rho}\right|_{\rho_0}$$

J.M. Latter, Y. Lim (2016)

First detection of GW from a BNS

GW170817

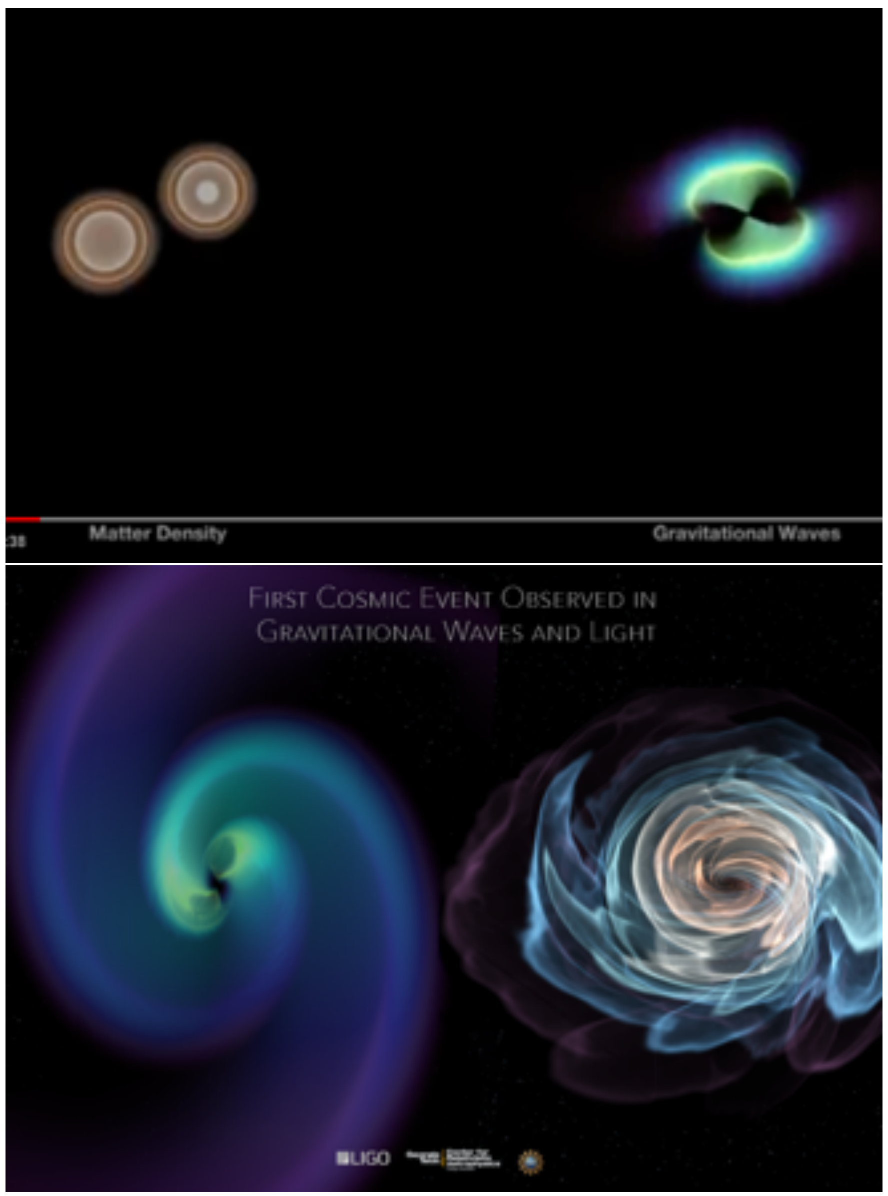
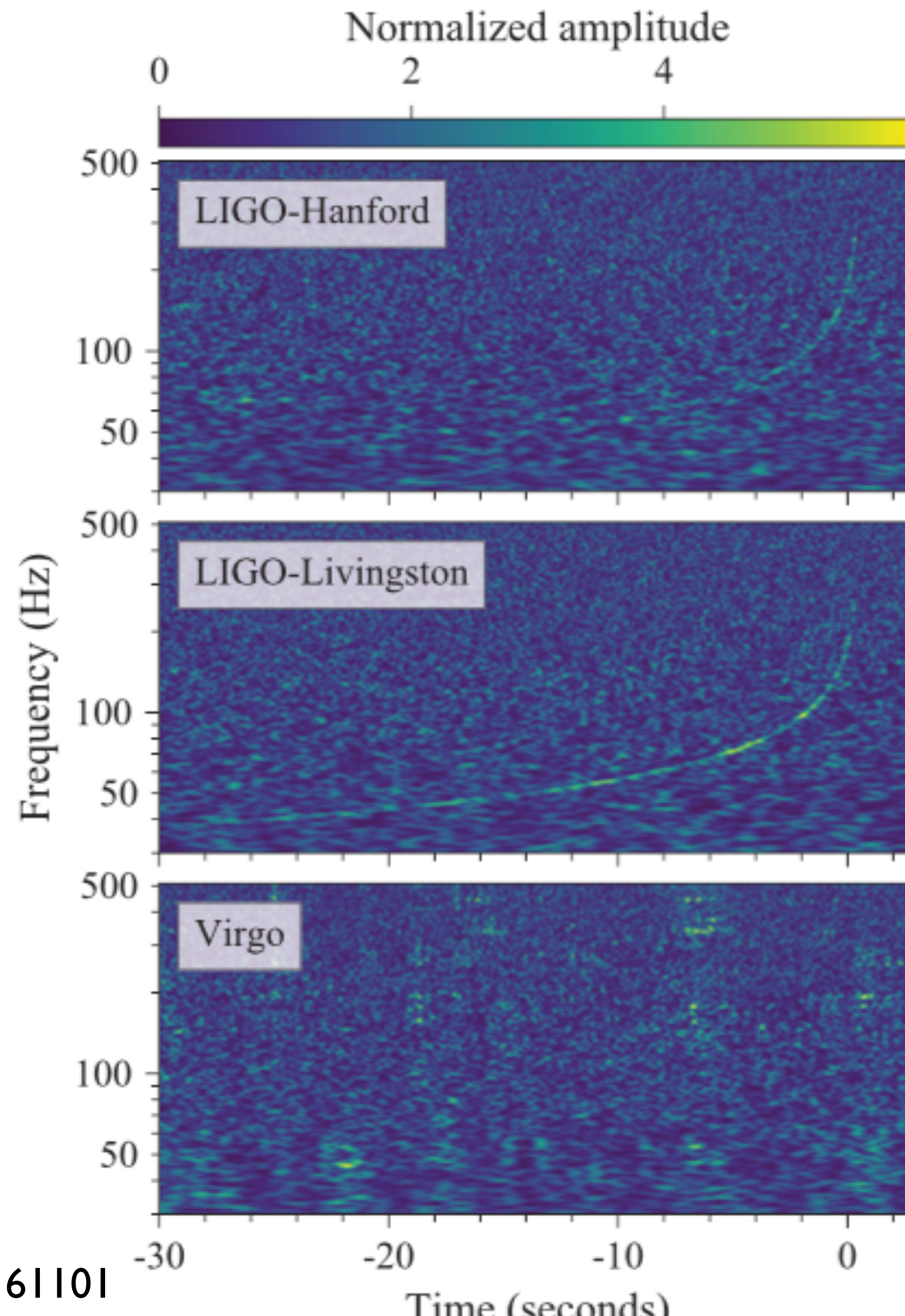
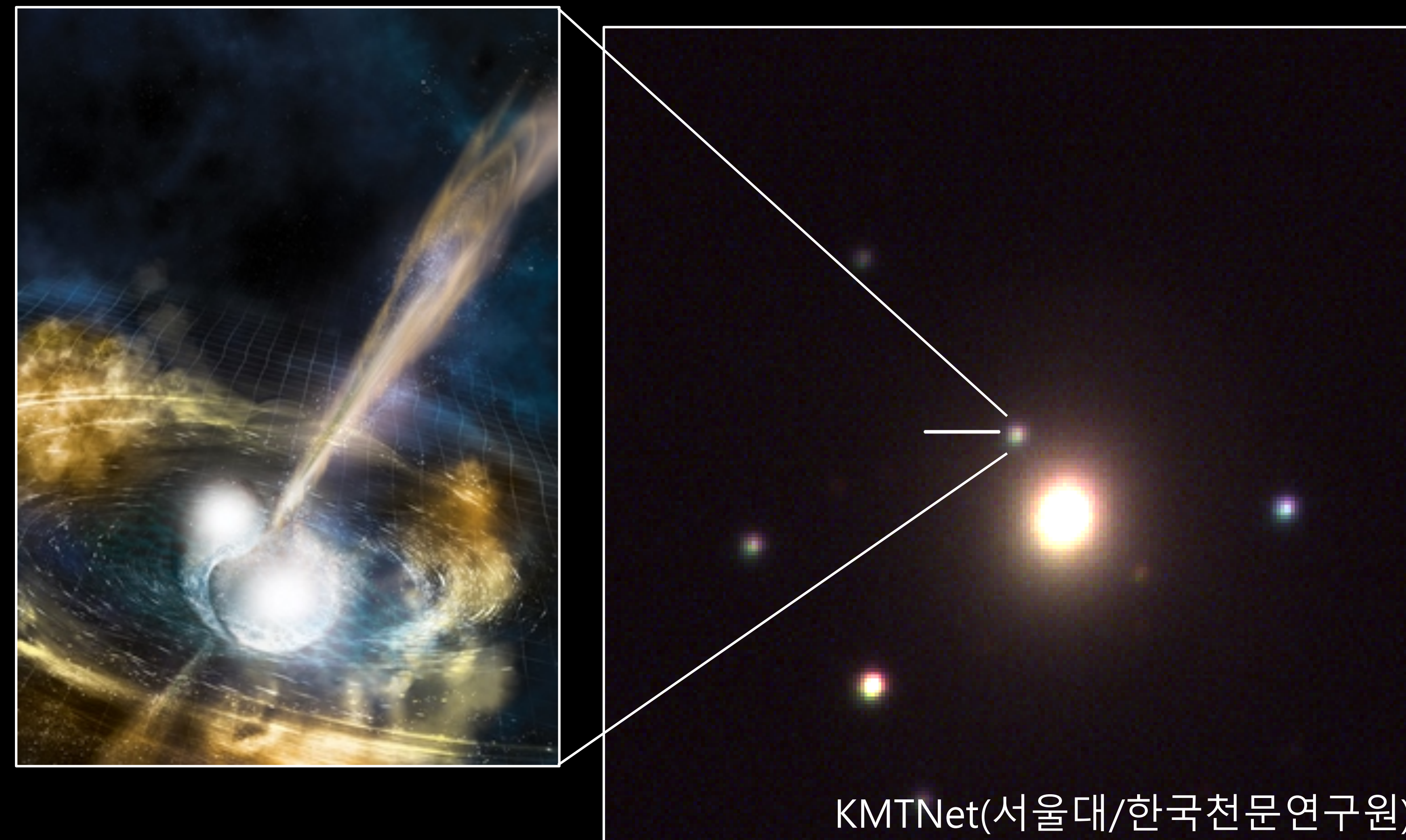


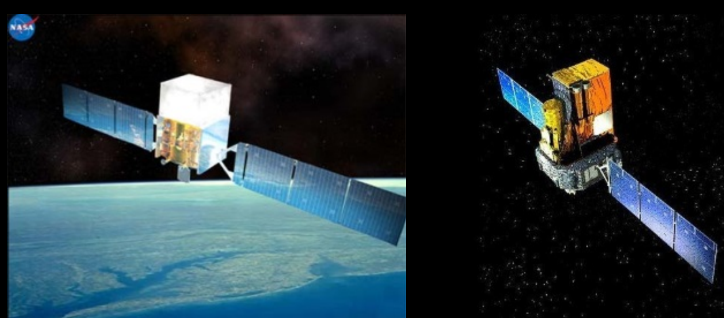
Image credit: Karan Jani/Georgia Tech.



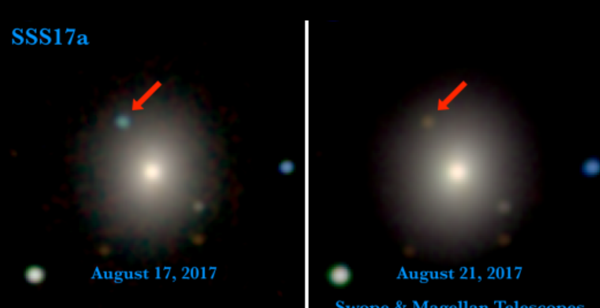
PhysRevLett.119.161101

2017-08-17 12:41:04 UTC
라이고 및 비르고 중력파 신호 포착

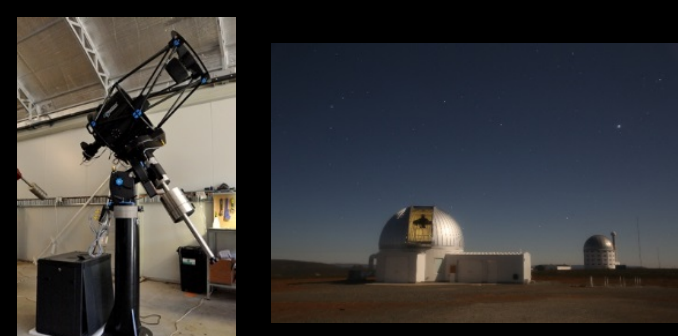




+2초 후
페르미 및 인티그랄 감마선 신호 포착



+약 11시간 후
칠레 천문대 망원경들이
가시광선 신호 포착



+약 21시간 후
국내연구진 호주 이상각망원경으로
추적관측시작. 이후 약 4주간 추적관측
(KMTNet, BOOTES-5망원경 등)

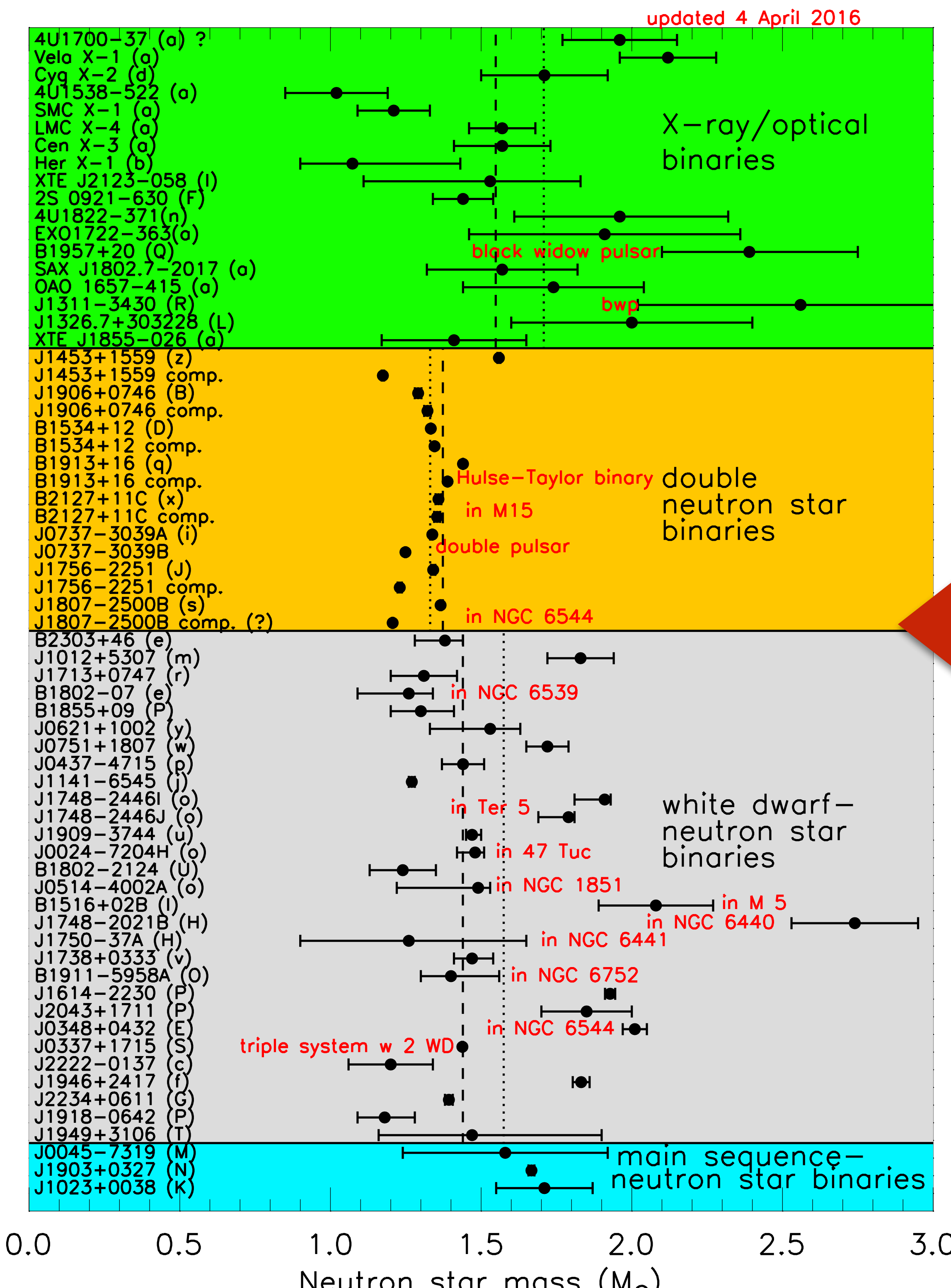
+약 9일 후
찬드라 우주망원경
X-선 신호 포착

+약 16일 후
지상 전파망원경
전파 신호 포착

그림출처: LIGO, NASA, 1M2H/UC Santa Cruz and Carnegie Observatories/Ryan Foley,
서울대/한국천문연구원/임명신, NSF/Sonoma 주립대/A. Simonnet, NRAO

서울대학교 초기우주천체연구단
(임명신)

Neutron Star of Known Mass



GW170817: BNS

M1: 1.36~1.60 $M_\odot$

(1.36~2.26)

M2: 1.17~1.36 $M_\odot$

(0.86~1.36)

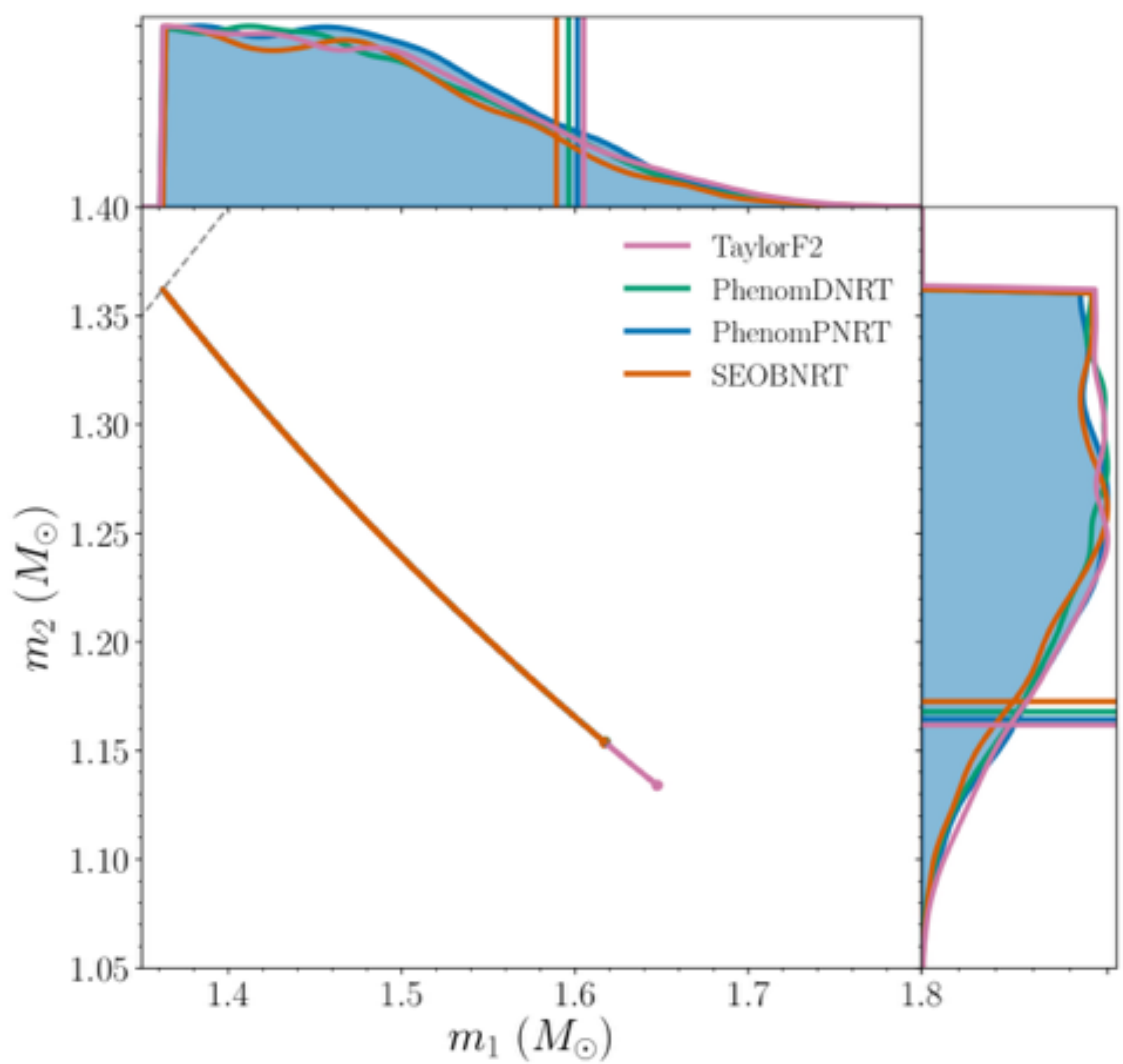
J. Lattimer, Annu.Rev.Nucl.Part.Sci.62,485(2012)
and https://stellarcollapse.org by C. Ott

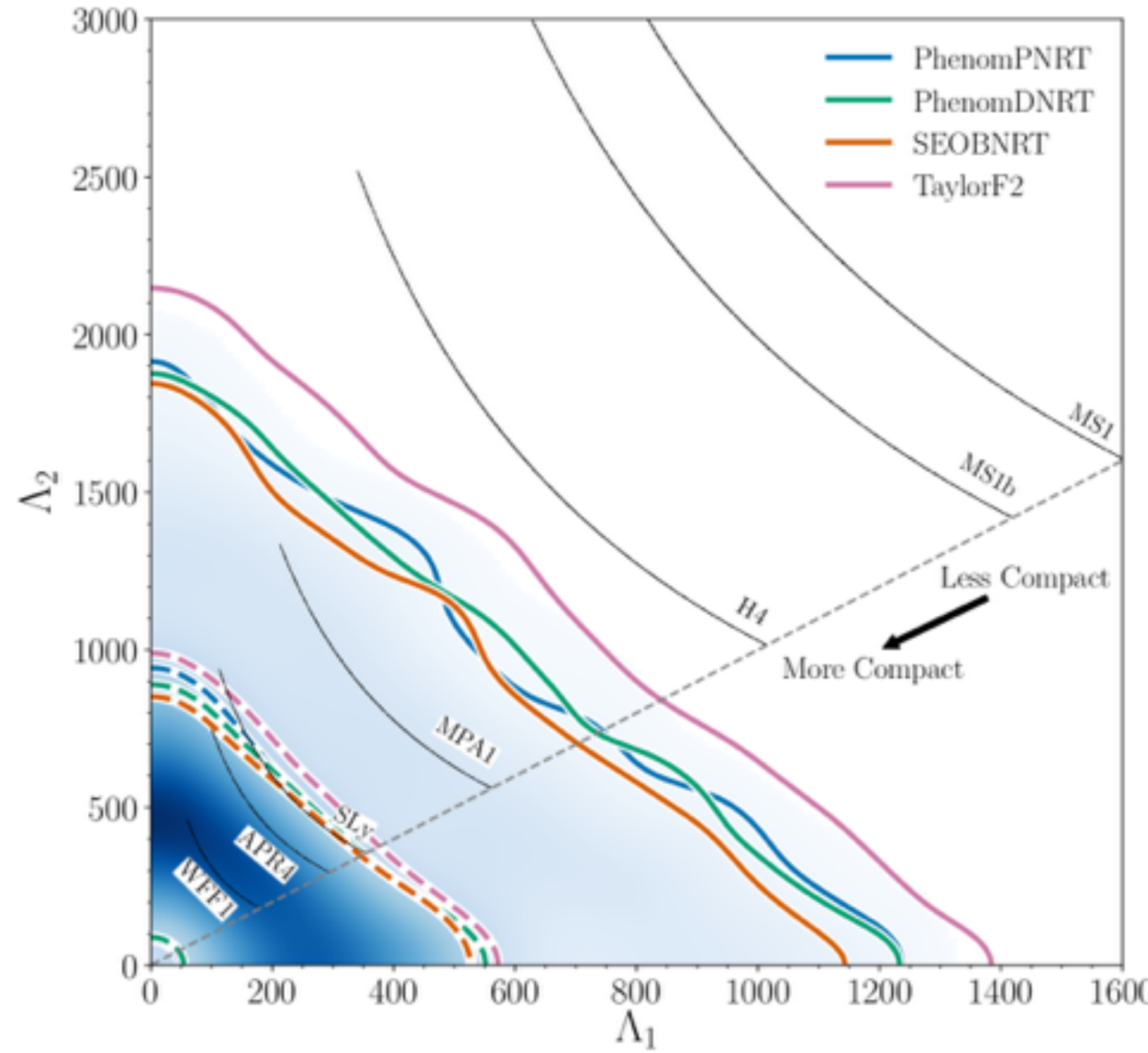
Properties of GW170817

Abbott et al. (LSC and Virgo), arxiv:1805.11579

	Low-spin prior ($\chi \leq 0.05$)	High-spin prior ($\chi \leq 0.89$)
Binary inclination θ_{JN}	146^{+25}_{-27} deg	152^{+21}_{-27} deg
Binary inclination θ_{JN} using EM distance constraint [104]	151^{+15}_{-11} deg	153^{+15}_{-11} deg
Detector frame chirp mass $\mathcal{M}^{\mathrm{det}}$	$1.1975^{+0.0001}_{-0.0001}\mathrm{M}_\odot$	$1.1976^{+0.0004}_{-0.0002}\mathrm{M}_\odot$
Chirp mass $\mathcal{M}$	$1.186^{+0.001}_{-0.001}\mathrm{M}_\odot$	$1.186^{+0.001}_{-0.001}\mathrm{M}_\odot$
Primary mass m_1	$(1.36, 1.60)\ \mathrm{M}_\odot$	$(1.36, 1.89)\ \mathrm{M}_\odot$
Secondary mass m_2	$(1.16, 1.36)\ \mathrm{M}_\odot$	$(1.00, 1.36)\ \mathrm{M}_\odot$
Total mass m	$2.73^{+0.04}_{-0.01}\mathrm{M}_\odot$	$2.77^{+0.22}_{-0.05}\mathrm{M}_\odot$
Mass ratio q	(0.73, 1.00)	(0.53, 1.00)
Effective spin χ_{eff}	$0.00^{+0.02}_{-0.01}$	$0.02^{+0.08}_{-0.02}$
Primary dimensionless spin χ_1	(0.00, 0.04)	(0.00, 0.50)
Secondary dimensionless spin χ_2	(0.00, 0.04)	(0.00, 0.61)
Tidal deformability $\tilde{\Lambda}$ with flat prior	300^{+500}_{-190}(symmetric)/ 300^{+420}_{-230}(HPD)	$(0, 630)$

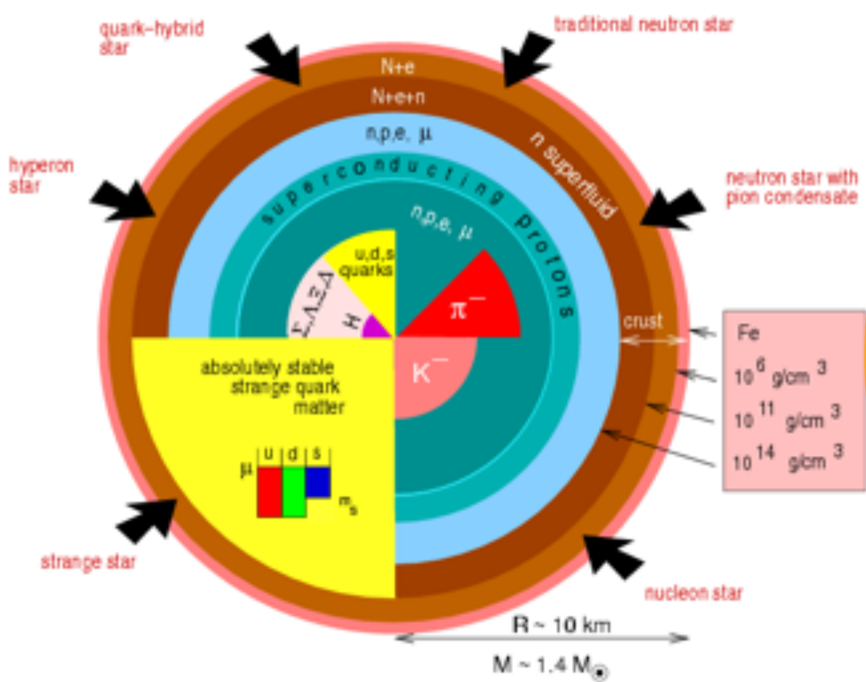
A new constraint by GW Observation

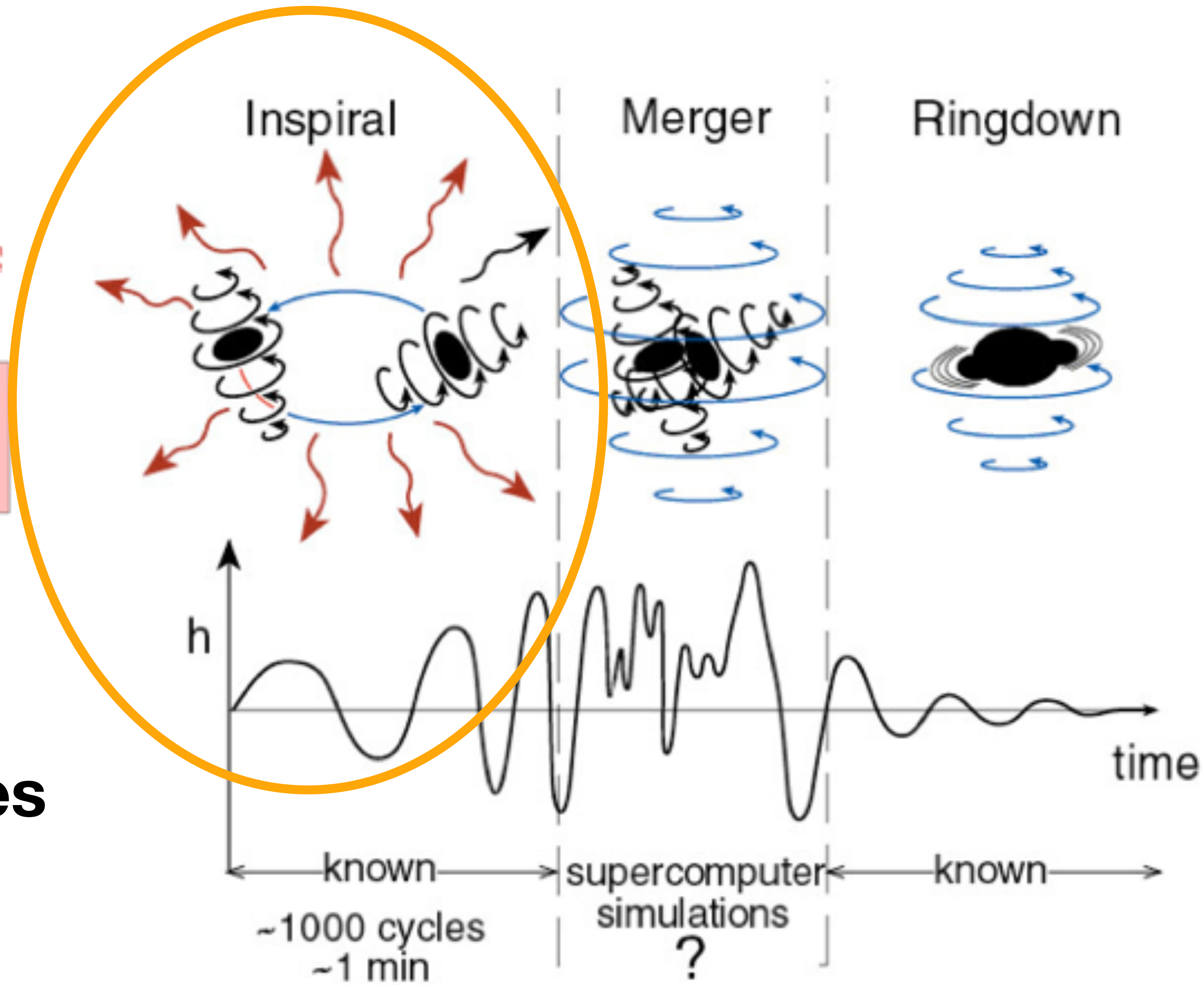




PhysRevLett.119.161101

Response of NS to GW during inspiral





perturbative approaches

$$Q_{ij} = -\lambda \mathcal{E}_{ij}$$

$$\lambda = \frac{2}{3}\frac{R^5}{G}k_2$$

λ : Tidal deformability
k2 : Tidal Love number

Measurability of tidal deformability

Tidal term in GW waveform

$$\tilde{h}_T(f) = \mathcal{A} f^{-7/6} e^{i\Psi_T(f)}$$

M. Favata, PRL.112.101101 (2014)

$$\Psi_{\rm T}(f) = \varphi_c + 2\pi f t_c + \frac{3}{128\eta v^5}(\Delta\Psi^{\rm pp}_{\rm 3.5PN} + \Delta\Psi^{\rm spin}_{\rm 3PN} + \Delta\Psi^{\rm ecc.}_{\rm 2PN} + \Delta\Psi^{\rm tidal}_{\rm 6PN} + \Delta\Psi^{\rm tm}_{\rm 6PN}), \qquad (1)$$

$$\Delta\Psi^{\rm tidal}_{\rm 6PN} = -\frac{39}{2}\tilde{\Lambda}v^{10} + v^{12}\left(\frac{6595}{364}\delta\tilde{\Lambda} - \frac{3115}{64}\tilde{\Lambda}\right), \qquad (4)$$

$$v = (\pi f M)^{1/3}$$

$$\tilde{\Lambda} = \frac{16}{13}\frac{(m_1 + 12m_2)m_1^4\Lambda_1 + (m_2 + 12m_1)m_2^4\Lambda_2}{(m_1+m_2)^5}$$

$$\Lambda = \lambda/M^5 \to G\left(\frac{c^2}{GM}\right)^5 \lambda = \frac{2}{3}\left(\frac{Rc^2}{GM}\right)^5 k_2$$

Accumulated GW phase (1)

the number of wave cycles in frequency domain

$$\Delta N_{\mathrm{cyc},\Psi} = \frac{1}{2\pi}\left[\Psi(f_2) - \Psi(f_1) + (f_1 - f_2)\frac{d\Psi}{df_1}\right], \qquad (7.8)$$

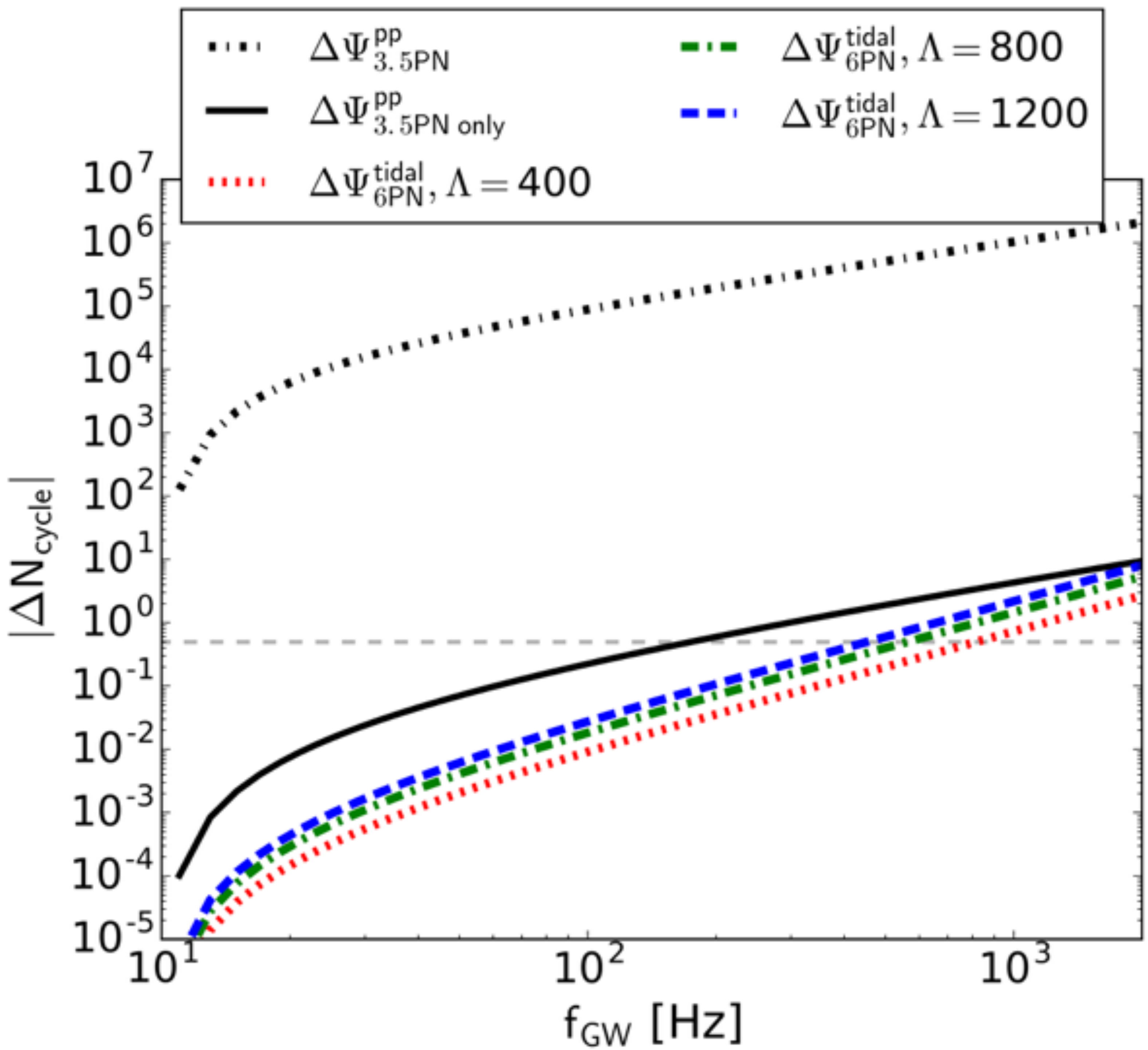
f_l = 10 Hz,
the low frequency cutoff for Advanced LIGO
due to seismic noises

Waveform models:
- TaylorT2 for ΔN_{cyc}
- TaylorF2(SPA) $\Delta N_{cyc,\Psi}$

Moore et al., PRD.93.124061(2016)

	$1.4M_\odot + 1.4M_\odot$, $f_2 = 1000$ Hz		
PN order	ΔN_{cyc}	$\Delta N_{\mathrm{cyc},\Psi}$	$\Delta N_{\mathrm{useful}}^{\mathrm{norm}}$
0PN(circ)	16 031	986 372	1821
0PN(ecc)	−463	−36 137	−6.37
1PN(circ)	439	21 743	125
1PN(ecc)	−15.8	−1193	−0.332
1.5PN(circ)	−208	−8520	−94.8
1.5PN(ecc)	1.67	103	0.113
2PN(circ)	9.54	294	6.70
2PN(ecc)	−0.215	−15.4	−0.008 17
2.5PN(circ)	−10.6	−218	−10.6
2.5PN(ecc)	0.0443	2.61	0.004 73
3PN(circ)	2.02	18.2	2.80
3PN(ecc)	0.002 00	0.119	−0.000 238
3.5PN(circ)	−0.662	−4.39	−0.977
Total	15 785	962 445	1843

Accumulated GW phase (2)

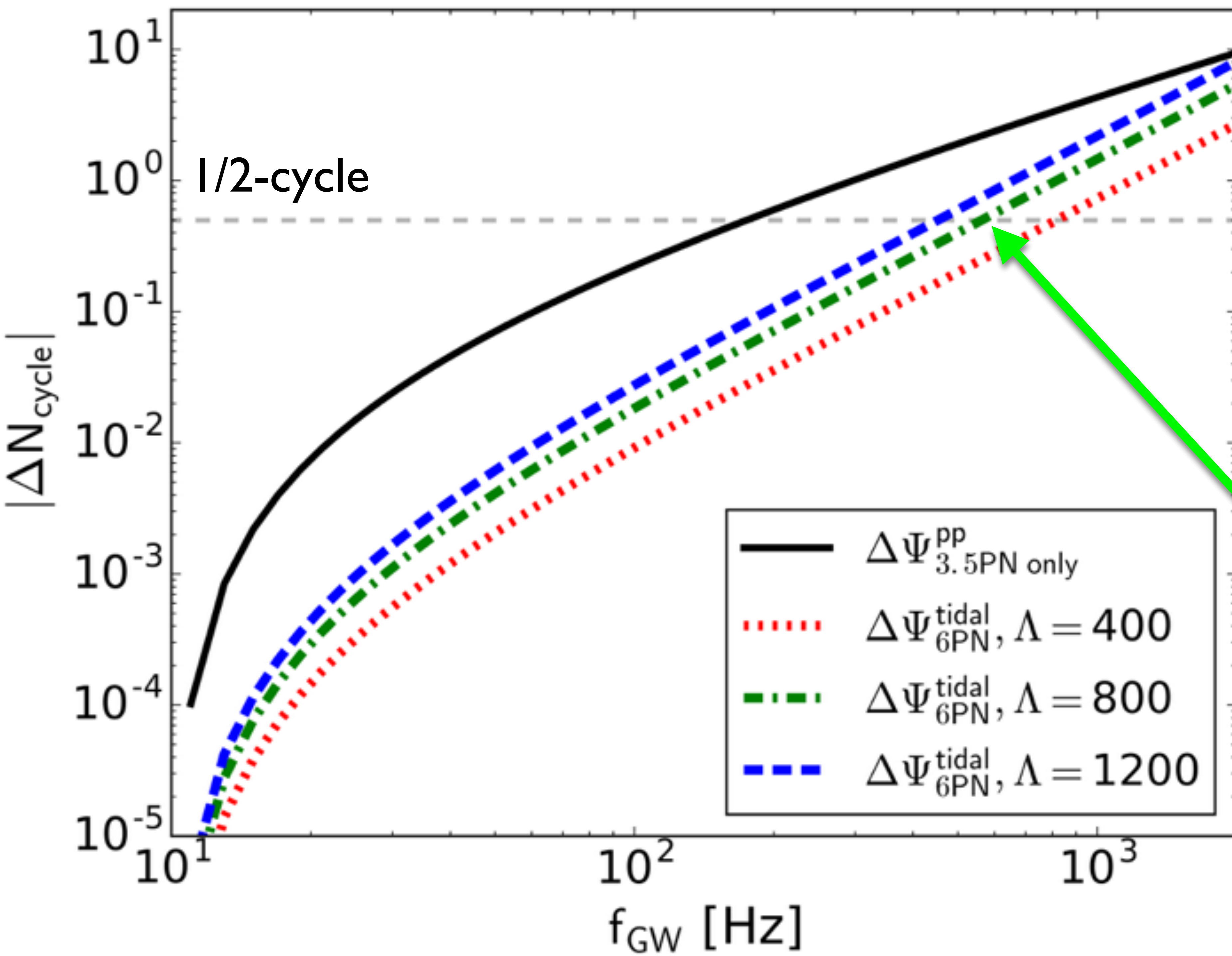


waveform model: TaylorF2(SPA)

Mch=1.188M$_\odot$

M1=M2=1.365M$_\odot$

Accumulated GW phase (2)



waveform model:
TaylorF2(SPA)

Mch=1.188M$_\odot$

M1=M2=1.365M$_\odot$

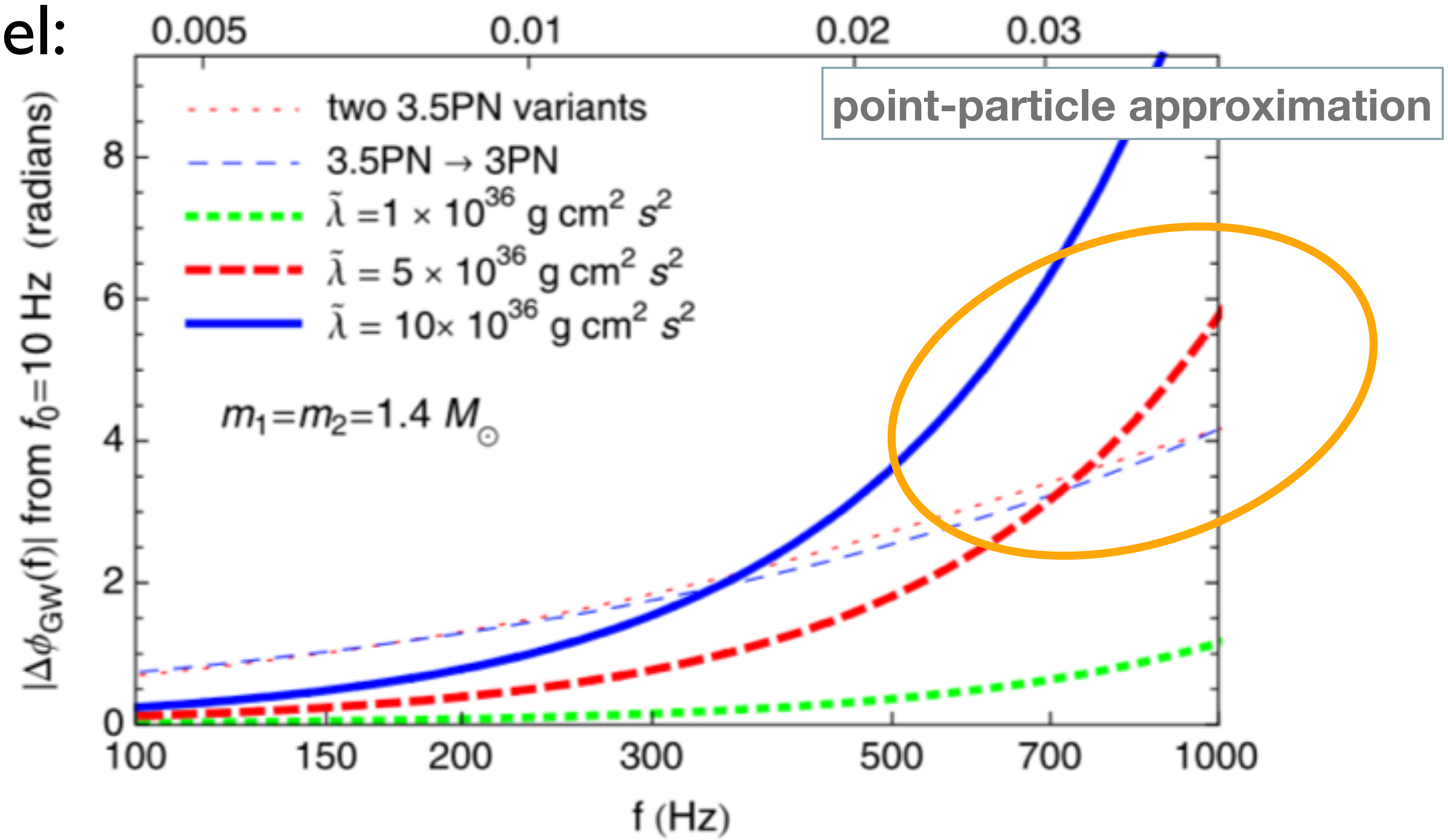
~600 Hz

Accumulated GW phase in time domain

$$|\Delta\phi_{\mathrm{GW}}(f)| = |\Phi_{3.5,\mathrm{pp}}(f_{\mathrm{GW}}) - \Phi_{3.5,\lambda}(f_{\mathrm{GW}})|$$

Hinderer et al., PHYSICAL REVIEW D 81, 123016 (2010)

waveform model:
TaylorT4



Measurement Errors via Fisher Matrix

Fisher matrix

$$\Gamma_{ij} = \mathrm{Re} < \frac{\partial h}{\partial \lambda_i} | \frac{\partial h}{\partial \lambda_j} >$$

$$\lambda_i = M_{\mathrm{chirp}}, \eta, \phi_c, t_c, s_{1,2}, e_0, \tilde{\Lambda}, ...$$

covariance matrix

$$\Sigma_{ij} = \Gamma_{ij}^{-1}$$

$$\tilde{h}_T(f) = \mathcal{A} f^{-7/6} e^{i\Psi_T}(f)$$

$$\Psi_{\mathrm{T}}(f) = \varphi_c + 2\pi f t_c + \frac{3}{128\eta v^5}(\Delta\Psi^{\mathrm{pp}}_{3.5\mathrm{PN}} + \Delta\Psi^{\mathrm{spin}}_{3\mathrm{PN}} + \Delta\Psi^{\mathrm{ecc.}}_{2\mathrm{PN}} + \Delta\Psi^{\mathrm{tidal}}_{6\mathrm{PN}} + \Delta\Psi^{\mathrm{tm}}_{6\mathrm{PN}})$$

correlation matrix

$$c_{ij} = \Sigma_{ij} / \sqrt{\Sigma_{ii}\Sigma_{jj}}$$

Measurement Errors

$$\sigma_i = \sqrt{\Sigma_{ii}}$$

Measurement error vs. Source Distance

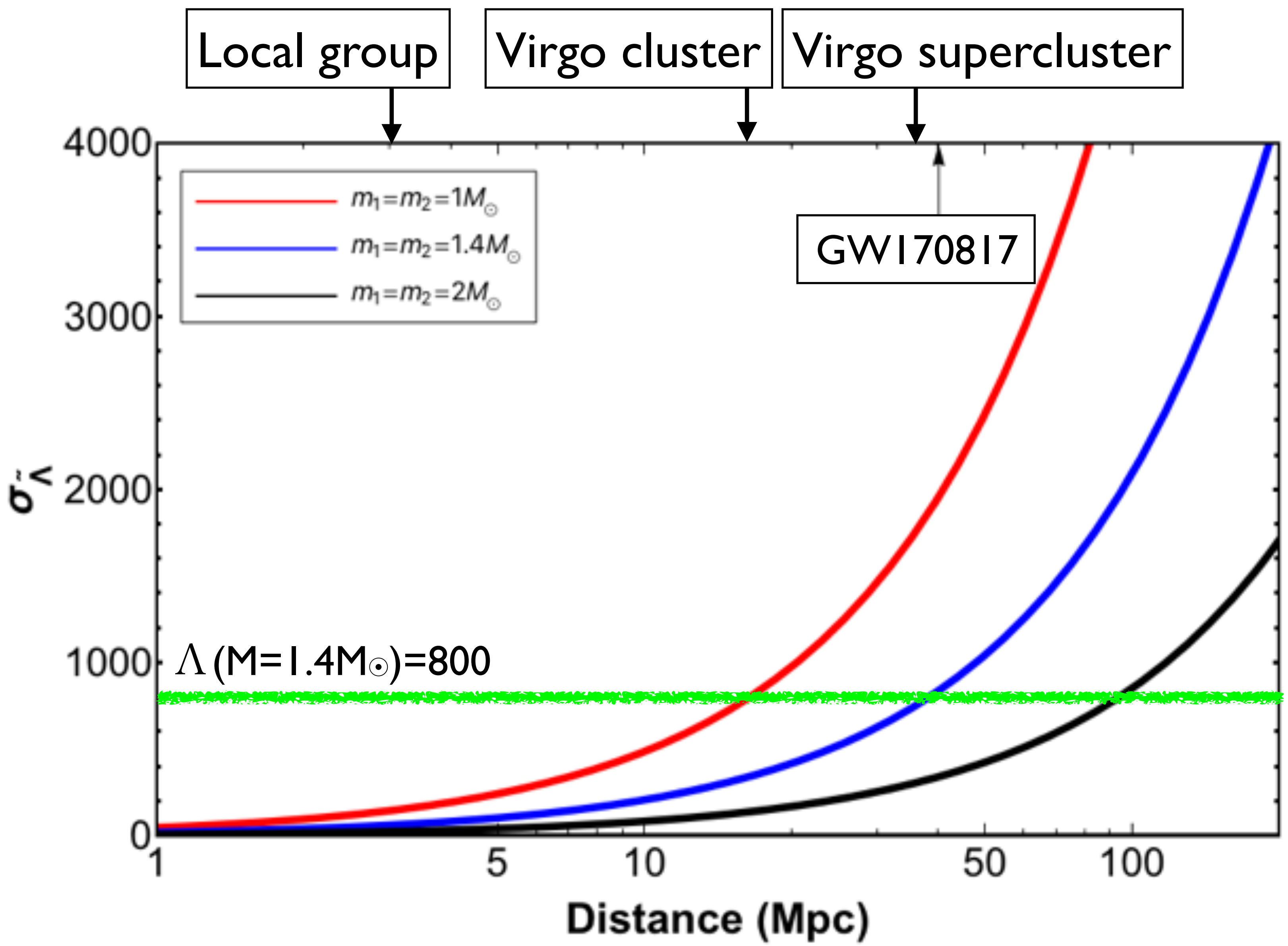


Fig. 1: Tidal deformability measurement error vs distance to the source.

Y.B. Choi, H. S. Cho, C.-H. Lee (PNU)

Derive 2nd order Diff. eq of H

TOV Eq. vs. Diff. Eq. for Tidal deformability

a spherical symmetric star in hydrostatic equilibrium

$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

$$\begin{aligned} ds_0^2 &= g^{(0)}_{\alpha\beta}\, dx^\alpha\, dx^\beta \\ &= -e^{\nu(r)}\, dt^2 + e^{\lambda(r)}\, dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right). \end{aligned}$$

$$T_{\alpha\beta} = (\rho + p)u_\alpha u_\beta + p g^{(0)}_{\alpha\beta},$$

TOV eq.

$$\boxed{\frac{dP}{dr} = -\frac{GM\rho}{r^2}\left(1 + \frac{P}{\rho c^2}\right)\left(1 + \frac{4\pi P r^3}{Mc^2}\right)\left(1 - \frac{2GM}{rc^2}\right)}$$

$$\frac{dM}{dr} = 4\pi r^2\left(\frac{\epsilon}{c^2}\right)$$

Mass & Radius

TOV Eq. vs. Diff. Eq. for Tidal deformability

static linearized perturbations due to an external tidal field

$$\delta G_{\mu\nu} = 8\pi \delta T_{\mu\nu}$$

$$g_{\alpha\beta} = g^{(0)}_{\alpha\beta} + h_{\alpha\beta},$$

T. Hinderer (2008), K. Thorne and A. Campolattaro (1967)

$$h_{\alpha\beta} =$$
$$\mathrm{diag}\left[-e^{\nu(r)}H_0(r), e^{\lambda(r)}H_2(r), r^2K(r), r^2\sin^2\theta K(r)\right]Y_{2m}(\theta,\varphi).$$

$$\delta T^0_0 = -\delta\rho = -(dp/d\rho)^{-1}\delta p \qquad \delta T^i_i = \delta p$$

$$\boxed{\begin{aligned} &H'' + H'\left\{\frac{2}{r} + e^{\lambda}\left[\frac{2m(r)}{r^2} + 4\pi r(p-\rho)\right]\right\} \\ &+ H\left[-\frac{6e^{\lambda}}{r^2} + 4\pi e^{\lambda}\left(5\rho + 9p + \frac{\rho+p}{dp/d\rho}\right) - \nu'^2\right] = 0, \end{aligned}}$$

k2 or λ

Selected references

- **A.E.H. Love** (1909) - The Yielding of the Earth to Disturbing Forces
- **K.S. Thorne** & A. Campolattaro (1967) - non-radial pulsation of NS
- J.B. Hartle & **K.S. Thorne** (1969) - stability of rotating NS
- … …
- **K.S. Thorne** (1998) - Tidal stabilization of rigid rotating, fully relativistic neutron star
- … …
- **T. Hinderer** (2008) - Tidal Love numbers of neutron stars
- **T. Damour and A. Nagar** (2009) - Relativistic tidal properties of neutron stars

Love number

The Yielding of the Earth to Disturbing Forces.

By A. E. H. LOVE, F.R.S., Sedleian Professor of Natural Philosophy in the University of Oxford.

(Received November 28, 1908,—Read January 14, 1909.)

1. Any estimate of the rigidity of the Earth must be based partly on some observations from which a deformation of the Earth's surface can be inferred, and partly on some hypothesis as to the internal constitution of the Earth. The observations may be concerned with tides of long period, variations of the vertical, variations of latitude, and so on. The hypothesis must relate to the arrangement of the matter as regards density in different parts, and to the state of the parts in respect of solidity, compressibility, and so on.

Love number

terms which are the products of the spherical solid harmonic W_n and functions of r. In the most important cases $n = 2$, and we may write

$$U = H(r)\frac{W_2}{g}, \qquad \Delta = f(r)\frac{W_2}{g}, \qquad V = V_0 + K(r)W_2, \tag{5}$$

where $H(r)$, $f(r)$, $K(r)$ are functions of r, and the constant $1/g$ has been inserted in the expression for U for the sake of convenience. If, in fact, W_2 is a periodic term of the tide-generating potential, W_2/g is the "true equilibrium height" of the corresponding tide, that is to say, it is the height of the harmonic inequality which the forces answering to W_2 would produce in an ocean covering a rigid spherical nucleus, of the same size and mass as the Earth, if the depth and density of the ocean were negligible. From the definition of $K(r)$ we easily find the formula

$$K(a) = \frac{3}{5\rho a^6}\int_0^a \rho_0 \left[\frac{d}{dr}\{r^6 H(r)\} - r^6 f(r)\right] dr. \tag{6}$$

In what follows we shall write h for $H(a)$ and k for $K(a)$, so that the equation of the disturbed surface is

$$r = a + hW_2/g,$$

Tidal Love number and Tidal deformability

T. Hinderer, ApJ, 677, 1216 (2008)

$$\frac{(1-g_{tt})}{2} = -\frac{M}{r} - \frac{3Q_{ij}}{2r^3}\left(n^i n^j - \frac{1}{3}\delta^{ij}\right) + O\left(\frac{1}{r^3}\right) + \frac{1}{2}\mathcal{E}_{ij}x^i x^j + O(r^3), \qquad (1)$$

$$Q_{ij} = -\lambda \mathcal{E}_{ij}$$

$$\lambda = \frac{2}{3}\frac{R^5}{G}k_2$$

λ : Tidal deformability

Q_{ij} : Quadrupole moment of NS

$\mathcal{E}_{ij}$: External quadrupole tidal field

k_2 : $l = 2$ Tidal Love number

$$\Lambda = \lambda / M^5 \rightarrow G\left(\frac{c^2}{GM}\right)^5 \lambda = \frac{2}{3}\left(\frac{Rc^2}{GM}\right)^5 k_2$$

Tidal coefficients (deformability)

Multipole moments induced by a external tidal field

electric-type tidal coefficient (even parity)

Mass moments $M_L^A = \mu_\ell^A G_L^A,$ electric-type tidal field

Spin moments $S_L^A = \sigma_\ell^A H_L^A.$ magnetic-type tidal field

magnetic-type tidal coefficient (odd parity)

Metric in linearized perturbation

even parity

$$h_{\mu\nu} = \begin{vmatrix} (1-2m^*/r)H_0(T,r)Y_L{}^M & H_1(T,r)Y_L{}^M & h_0(T,r)(\partial/\partial\theta)Y_L{}^M & h_0(T,r)(\partial/\partial\varphi)Y_L{}^M \\ H_1(T,r)Y_L{}^M & (1-2m^*/r)^{-1}H_2(T,r)Y_L{}^M & h_1(T,r)(\partial/\partial\theta)Y_L{}^M & h_1(T,r)(\partial/\partial\varphi)Y_L{}^M \\ \mathrm{Sym} & \mathrm{Sym} & \begin{matrix} r^2[K(T,r) \\ +G(T,r)(\partial^2/\partial\theta^2)]Y_L{}^M \end{matrix} & \mathrm{Sym} \\ \mathrm{Sym} & \mathrm{Sym} & \begin{matrix} r^2G(T,r)(\partial^2/\partial\theta\partial\varphi \\ -\cos\theta\partial/\sin\theta\partial\varphi)Y_L{}^M \end{matrix} & \begin{matrix} r^2[K(T,r)\sin^2\theta \\ +G(T,r)(\partial^2/\partial\varphi\partial\varphi \\ +\sin\theta\cos\theta\partial/\partial\theta)]Y_L{}^M \end{matrix} \end{vmatrix}. \tag{13}$$

odd parity

$$h_{\mu\nu} = \begin{vmatrix} 0 & 0 & -h_0(T,r)(\partial/\sin\theta\partial\varphi)Y_L{}^M & h_0(T,r)(\sin\theta\partial/\partial\theta)Y_L{}^M \\ 0 & 0 & -h_1(T,r)(\partial/\sin\theta\partial\varphi)Y_L{}^M & h_1(T,r)(\sin\theta\partial/\partial\theta)Y_L{}^M \\ \mathrm{Sym} & \mathrm{Sym} & h_2(T,r)(\partial^2/\sin\theta\partial\theta\partial\varphi-\cos\theta\partial/\sin^2\theta\partial\varphi)Y_L{}^M & \mathrm{Sym} \\ \mathrm{Sym} & \mathrm{Sym} & \frac{1}{2}h_2(T,r)(\partial^2/\sin\theta\partial\varphi\partial\varphi+\cos\theta\partial/\partial\theta-\sin\theta\partial^2/\partial\theta\partial\theta)Y_L{}^M & -h_2(T,r)(\sin\theta\partial^2/\partial\theta\partial\varphi-\cos\theta\partial/\partial\varphi)Y_L{}^M \end{vmatrix}. \tag{12}$$

Regge-Wheeler gauge and M=0

even parity

$$h_{\mu\nu}=\exp(-ikT)P_L(\cos\theta)$$

$$\times\begin{vmatrix} H_0(1-2m^*/r) & H_1 & 0 & 0 \\ H_1 & H_2(1-2m^*/r)^{-1} & 0 & 0 \\ 0 & 0 & r^2K & 0 \\ 0 & 0 & 0 & r^2K\sin^2\theta \end{vmatrix} \tag{20}$$

odd parity

$$h_{\mu\nu}=\begin{vmatrix} 0 & 0 & 0 & h_0(r) \\ 0 & 0 & 0 & h_1(r) \\ 0 & 0 & 0 & 0 \\ \mathrm{Sym} & \mathrm{Sym} & 0 & 0 \end{vmatrix}$$

$$\times\exp(-ikT)(\sin\theta\partial/\partial\theta)P_L(\cos\theta). \tag{18}$$

Regge-Wheeler gauge and M=0

even parity

$$h_{\mu\nu}=\exp(-ikT)P_L(\cos\theta)$$

H=H0=H2,K

$$\times\begin{vmatrix} H_0(1-2m^*/r) & \cancelto{0}{H_1} & 0 & 0 \\ \cancelto{0}{H_1} & H_2(1-2m^*/r)^{-1} & 0 & 0 \\ 0 & 0 & r^2K & 0 \\ 0 & 0 & 0 & r^2K\sin^2\theta \end{vmatrix} \qquad (20)$$

k=0, stationary case

odd parity

$$h_{\mu\nu}=\begin{vmatrix} 0 & 0 & 0 & h_0(r) \\ 0 & 0 & 0 & \cancelto{0}{h_1(r)} \\ 0 & 0 & 0 & 0 \\ \mathrm{Sym} & \cancelto{0}{\mathrm{Sym}} & 0 & 0 \end{vmatrix}$$

$$\times\exp(-ikT)(\sin\theta\,\partial/\partial\theta)P_L(\cos\theta). \qquad (18)$$

Ψ=h0

Even parity (Electric-type) metric Fn. H

Regge & Wheeler (1957), Thorne & Campolattaro (1967)

$$\delta G_{\mu\nu} = 8\pi \delta T_{\mu\nu}$$

$$h_{\alpha\beta} = \mathrm{diag}\left[-e^{\nu(r)} H_0(r), e^{\lambda(r)} H_2(r), r^2 K(r), r^2 \sin^2\theta K(r)\right] Y_{2m}(\theta, \varphi).$$

$$\delta T^0_0 = -\delta\rho = -(dp/d\rho)^{-1}\delta p \qquad \delta T^i_i = \delta p$$

$$H'' + C_1 H' + C_0 H = 0.$$

$$C_1 = \frac{2}{r} + \frac{1}{2}(\nu' - \lambda') = \frac{2}{r} + e^{\lambda}\left[\frac{2m}{r^2} + 4\pi r(p - e)\right], \tag{28}$$

$$\begin{aligned} C_0 = e^{\lambda}&\left[-\frac{\ell(\ell+1)}{r^2} + 4\pi(e+p)\frac{de}{dp} + 4\pi(e+p)\right] \\ &+ \nu'' + (\nu')^2 + \frac{1}{2r}(2 - r\nu')(3\nu' + \lambda') \\ = e^{\lambda}&\left[-\frac{\ell(\ell+1)}{r^2} + 4\pi(e+p)\frac{de}{dp} + 4\pi(5e + 9p)\right] \\ &- (\nu')^2, \end{aligned} \tag{29}$$

Electric-type tidal coefficients (1)

General solution

$$H = a_P \hat{P}_{\ell 2}(x) + a_Q \hat{Q}_{\ell 2}(x),$$

$$x \equiv r/M - 1,$$

$$c \equiv M/R$$

$$y^{\text{ext}} \equiv rH'/H$$

$$a_\ell \equiv a_Q/a_P$$

$$y_\ell^{\text{ext}}(x) = (1+x)\frac{\hat{P}'_{\ell 2}(x) + a_\ell \hat{Q}'_{\ell 2}(x)}{\hat{P}_{\ell 2}(x) + a_\ell \hat{Q}_{\ell 2}(x)}.$$

$$a_\ell = -\frac{\hat{P}'_{\ell 2}(x) - c y_\ell \hat{P}_{\ell 2}(x)}{\hat{Q}'_{\ell 2}(x) - c y_\ell \hat{Q}_{\ell 2}(x)}\Bigg|_{x=1/c-1}.$$

Electric-type tidal coefficients (2)

Damour and Nagar , PRD 80, 084035 (2009)

$$-(\delta H_{00}e^{-\nu})^{\rm growing} = H^{\rm growing}(r) = a_P \hat{P}_{\ell 2}(x) Y_{\ell m} \simeq a_P \left(\frac{r}{M}\right)^{\ell} Y_{\ell m}, \quad (40)$$

$$-(\delta H_{00}e^{-\nu})^{\rm decreasing} = H^{\rm decreasing}(r) = a_Q \hat{Q}_{\ell 2}(x) Y_{\ell m} \simeq a_Q \left(\frac{r}{M}\right)^{-(\ell+1)} Y_{\ell m}, \quad (41)$$

$$\partial_L r^{-1} = (-)^{\ell}(2\ell-1)!!\hat{n}^L r^{-(\ell+1)},$$

$$M_L^A = \mu_\ell^A G_L^A,$$

$$(2\ell-1)!! G\mu_\ell = \frac{a_Q}{a_P}\left(\frac{GM}{c_0^2}\right)^{2\ell+1} = a_\ell \left(\frac{GM}{c_0^2}\right)^{2\ell+1}.$$

$$\bar{W} = \frac{1}{\ell!}\hat{X}^L G_L^A = \frac{1}{\ell!} r^\ell \hat{n}^L G_L^A,$$

$$W^+ = G\frac{(-)^\ell}{\ell!}\partial_L\left(\frac{M_L^A}{r}\right),$$

$$G_{00}^A(X) = -\exp(-2W^A/c^2),$$

$$G_{0a}^A(X) = -\frac{4}{c^3}W_a^A,$$

$$E_a^A(X) = \partial_a W^A + \frac{4}{c^2}\partial_T W_a^A,$$

$$G_L^A(T) \equiv [\partial_{\langle L-1}\bar{E}^A_{a_\ell\rangle}(T, \mathbf{X})]_{X^a\to 0}$$

Electric-type tidal coefficients (3)

Damour and Nagar , PRD 80, 084035 (2009)

$$(2\ell-1)!!G\mu_\ell = \frac{a_Q}{a_P}\left(\frac{GM}{c_0^2}\right)^{2\ell+1} = a_\ell\left(\frac{GM}{c_0^2}\right)^{2\ell+1}. \quad (45)$$

$$G\mu_\ell = \frac{a_\ell}{(2\ell-1)!!}\left(\frac{GM}{c_0^2}\right)^{2\ell+1} = \frac{2k_\ell}{(2\ell-1)!!}R^{2\ell+1}. \quad (48)$$

$$\begin{aligned} k_\ell &= \frac{1}{2}c^{2\ell+1}a_\ell \\ &= -\frac{1}{2}c^{2\ell+1}\frac{\hat{P}'_{\ell 2}(x) - cy_\ell\hat{P}_{\ell 2}(x)}{\hat{Q}'_{\ell 2}(x) - cy_\ell\hat{Q}_{\ell 2}(x)}\Bigg|_{x=1/c-1}. \end{aligned} \quad (49)$$

Electric-type tidal coefficients (4)

Damour and Nagar , PRD 80, 084035 (2009)

Electric-type tidal coefficients

$$G\mu_\ell = \frac{a_\ell}{(2\ell-1)!!}\left(\frac{GM}{c_0^2}\right)^{2\ell+1} = \frac{2k_\ell}{(2\ell-1)!!}R^{2\ell+1}. \quad (48)$$

$$H'' + C_1 H' + C_0 H = 0.$$

$$C_1 = \frac{2}{r} + \frac{1}{2}(\nu' - \lambda') = \frac{2}{r} + e^{\lambda}\left[\frac{2m}{r^2} + 4\pi r(p-e)\right], \quad (28)$$

$$\begin{aligned} C_0 &= e^{\lambda}\left[-\frac{\ell(\ell+1)}{r^2} + 4\pi(e+p)\frac{de}{dp} + 4\pi(e+p)\right] \\ &\quad + \nu'' + (\nu')^2 + \frac{1}{2r}(2 - r\nu')(3\nu' + \lambda') \\ &= e^{\lambda}\left[-\frac{\ell(\ell+1)}{r^2} + 4\pi(e+p)\frac{de}{dp} + 4\pi(5e+9p)\right] \\ &\quad - (\nu')^2, \end{aligned} \quad (29)$$

we calculated the case, l=2

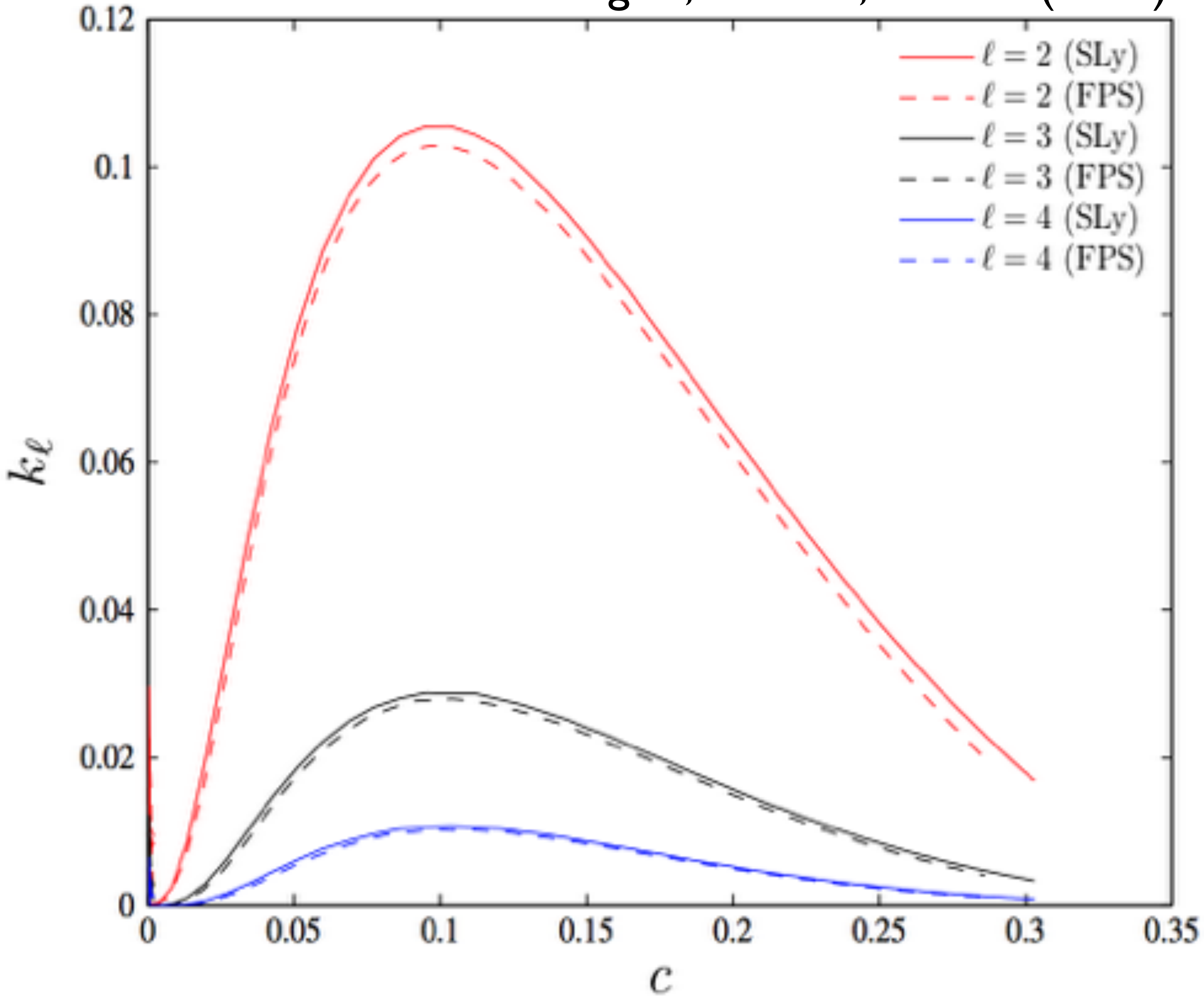
$$\boxed{\begin{aligned} k_2 &= \frac{8}{5}(1-2c)^2 c^5[2c(y-1) - y + 2]\Big\{2c(4(y+1)c^4 + (6y-4)c^3 + (26-22y)c^2 + 3(5y-8)c - 3y + 6) \\ &\quad - 3(1-2c)^2(2c(y-1) - y + 2)\log\left(\frac{1}{1-2c}\right)\Big\}^{-1}, \end{aligned} \quad (50)}$$

$$y_R = \frac{rH'(r)}{H(r)}\Big|_{r=R}$$

$$\begin{aligned} k_3 &= \frac{8}{7}(1-2c)^2 c^7[2(y-1)c^2 - 3(y-2)c + y - 3]\Big\{2c[4(y+1)c^5 + 2(9y-2)c^4 - 20(7y-9)c^3 + 5(37y-72)c^2 \\ &\quad - 45(2y-5)c + 15(y-3)] - 15(1-2c)^2(2(y-1)c^2 - 3(y-2)c + y - 3)\log\left(\frac{1}{1-2c}\right)\Big\}^{-1}, \end{aligned} \quad (51)$$

Higher Tidal Love Numbers

Damour and Nagar , PRD 80, 084035 (2009)



$$x \equiv (M\omega)^{2/3}$$

$$\Delta\Psi_2^{tidal} \sim \lambda_2 \; x^{5/2}$$

$$\Delta\Psi_3^{tidal} \sim \lambda_3 \; x^{9/2}$$

$$|\Delta\Psi_3^{tidal}/\Delta\Psi_2^{tidal}| \sim \mathcal{O}(10^{-3})$$

We hardly expect to observe higher terms of tidal deformability in the waveform

Magnetic-type tidal coefficients

Damour and Nagar , PRD 80, 084035 (2009)

Likewise,

Magnetic-type tidal coefficients

$$S_L^A = \sigma_\ell^A H_L^A.$$

$$\psi'' + \frac{e^\lambda}{r^2}[2m + 4\pi r^3(p-e)]\psi' - e^\lambda\left[\frac{\ell(\ell+1)}{r^2} - \frac{6m}{r^3} + 4\pi(e-p)\right]\psi = 0. \quad (31)$$

$$G\sigma_\ell = \frac{\ell-1}{4(\ell+2)}\frac{b_\ell}{(2\ell-1)!!}\left(\frac{GM}{c_0^2}\right)^{2\ell+1} = \frac{\ell-1}{4(\ell+2)}\frac{j_\ell}{(2\ell-1)!!}R^{2\ell+1},$$

$$j_\ell \equiv c^{2\ell+1}b_\ell = -c^{2\ell+1}\left.\frac{\psi'_P(\hat{r}) - cy_{\rm odd}\psi_P(\hat{r})}{\psi'_Q(\hat{r}) - cy_{\rm odd}\psi_Q(\hat{r})}\right|_{\hat{r}=1/c}.$$

$$G\sigma_2 = \frac{1}{48}j_2R^5 = \frac{1}{48}b_2\left(\frac{GM}{c_0^2}\right)^5,$$

$$j_2 = \frac{96c^5(2c-1)(y-3)}{5(2c(12(y+1)c^4 + 2(y-3)c^3 + 2(y-3)c^2 + 3(y-3)c - 3y + 9) + 3(2c-1)(y-3)\log(1-2c))}. \quad (73)$$

Magnetic-type tidal coefficients

Damour and Nagar , PRD 80, 084035 (2009)

Likewise,

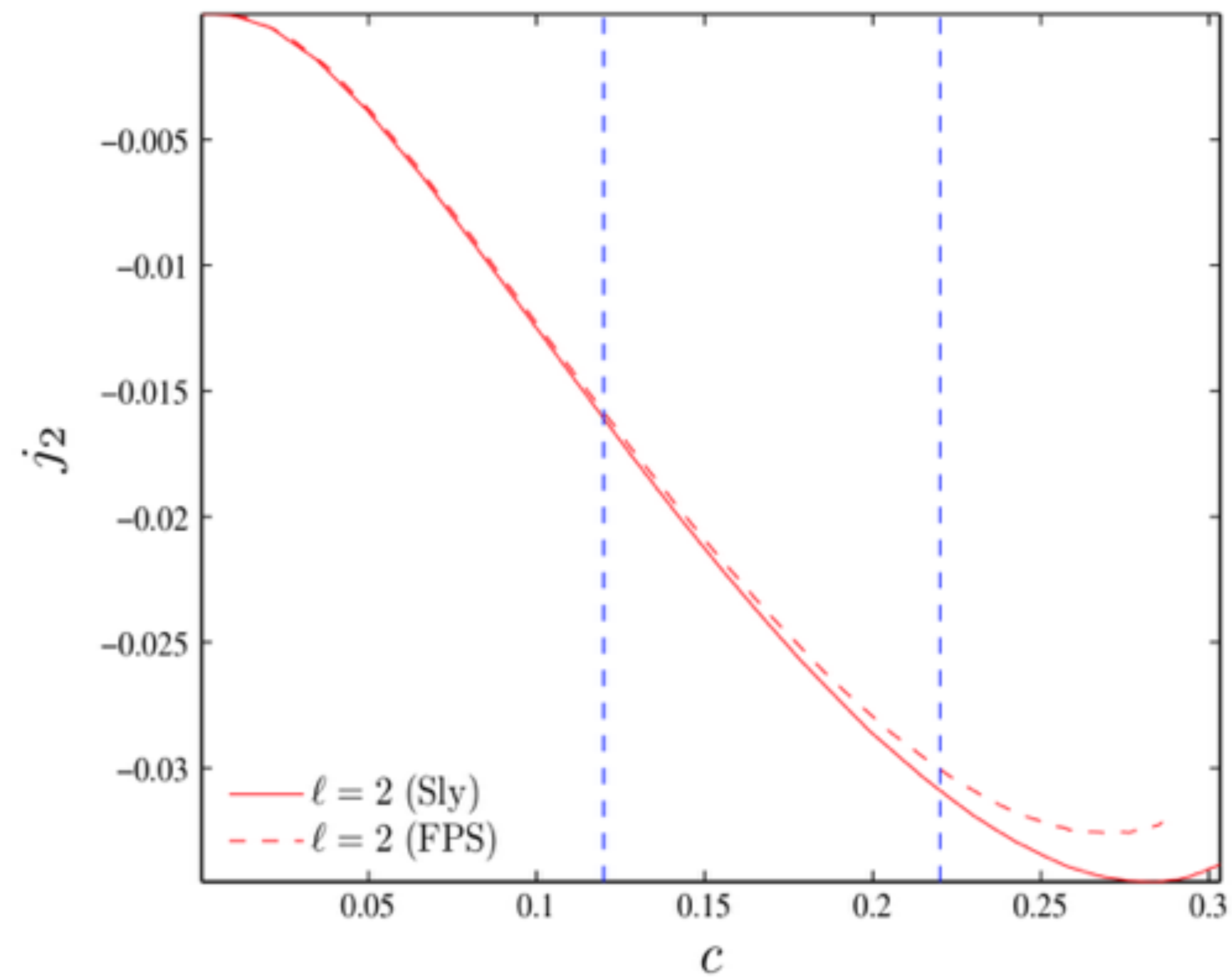
Magnetic-type tidal coefficients

$$S_L^A = \sigma_\ell^A H_L^A.$$

$$\psi'' +$$

$$G\sigma_\ell = \frac{\ell-1}{4(\ell+2)}\frac{b_\ell}{(2\ell-1)!!}\left(\frac{GM}{c_0^2}\right)^{2\ell+1}$$
$$= \frac{\ell-1}{4(\ell+2)}\frac{j_\ell}{(2\ell-1)!!}R^{2\ell+1},$$

$$j_\ell \equiv$$



$$G\sigma_2 = \frac{1}{48}j_2R^5 = \frac{1}{48}b_2\left(\frac{GM}{c_0^2}\right)^5,$$

$$j_2 = \frac{96c^5(2c-1)(y-3)}{5(2c(12(y+1)c^4 + 2(y-3)c^3 + 2(y-3)c^2 + 3(y-3)c - 3y + 9) + 3(2c-1)(y-3)\log(1-2c))}. \qquad (73)$$

Tidal Love number and Tidal deformability

$$\frac{(1-g_{tt})}{2} = -\frac{M}{r} - \frac{3Q_{ij}}{2r^3}\left(n^i n^j - \frac{1}{3}\delta^{ij}\right) + O\left(\frac{1}{r^3}\right) + \frac{1}{2}\mathcal{E}_{ij}x^i x^j + O(r^3), \qquad (1)$$

T. Hinderer, ApJ, 677, 1216 (2008)

$$Q_{ij} = -\lambda \mathcal{E}_{ij}$$

$$\lambda = \frac{2}{3}\frac{R^5}{G}k_2$$

λ : Tidal deformability

Q_{ij} : Quadrupole moment of NS

$\mathcal{E}_{ij}$: External quadrupole tidal field

k_2 : $\mathrm{l} = 2$ Tidal Love number

$$y = \frac{R\beta(R)}{H(R)} \qquad C = \frac{GM}{Rc^2}$$

$$\begin{aligned} k_2 =& \frac{8C^5}{5}(1-2C)^2[2+2C(y-1)-y] \\ &\times \Big\{2C[6-3y+3C(5y-8)] + 4C^3[13-11y+C(3y-2)+2C^2(1+y)] \\ &+ 3(1-2C)^2[2-y+2C(y-1)]\ln(1-2C)\Big\}^{-1} \end{aligned}$$

Tidal term in GW waveform

$$\tilde{h}_T(f) = \mathcal{A} f^{-7/6} e^{i\Psi_T}(f)$$

M. Favata, PRL.112.101101 (2014)

$$\Psi_{\rm T}(f) = \varphi_c + 2\pi f t_c + \frac{3}{128\eta v^5}(\Delta\Psi^{\rm pp}_{\rm 3.5PN} + \Delta\Psi^{\rm spin}_{\rm 3PN} + \Delta\Psi^{\rm ecc.}_{\rm 2PN} + \Delta\Psi^{\rm tidal}_{\rm 6PN} + \Delta\Psi^{\rm tm}_{\rm 6PN}), \qquad (1)$$

$$\Delta\Psi^{\rm tidal}_{\rm 6PN} = -\frac{39}{2}\tilde{\Lambda}v^{10} + v^{12}\left(\frac{6595}{364}\delta\tilde{\Lambda} - \frac{3115}{64}\tilde{\Lambda}\right), \qquad (4)$$

$$\tilde{\Lambda} = \frac{16}{13}\frac{(m_1+12m_2)m_1^4\Lambda_1 + (m_2+12m_1)m_2^4\Lambda_2}{(m_1+m_2)^5}$$

$$\Lambda = \lambda/M^5 \to G\left(\frac{c^2}{GM}\right)^5\lambda = \frac{2}{3}\left(\frac{Rc^2}{GM}\right)^5 k_2$$

Higher order term in GW phase (1)

Post-Newtonian spin-tidal couplings for compact binaries - arxiv:1805.01487

PN order	λ_2	σ_2	$\lambda_{23,32}$, $\sigma_{23,32}$	λ_3	σ_3
5	LO $\propto \Lambda$				
6	NLO $\propto \delta\Lambda$	LO $\propto \Sigma$			
6.5	**NNLO $\propto \tilde{\Lambda}$**	**NLO $\propto \tilde{\Sigma}$**	**LO $\propto \tilde{\Gamma}$**		
7	…	…	…	LO	
8	…	…	…	…	LO

$$\psi(x) = \frac{3}{128\nu x^{5/2}}\Bigg\{1 + \left(\frac{3715}{756} + \frac{55}{9}\nu\right)x + \left(\frac{113}{3} \times \right.$$
$$\left. \times (\eta_1\chi_1 + \eta_2\chi_2) - \frac{38}{3}\nu(\chi_1 + \chi_2)\right)x^{1.5} + O(x^2)$$
$$+ \Lambda x^5 + (\delta\Lambda + \Sigma)x^6 + (\tilde{\Lambda} + \tilde{\Sigma} + \tilde{\Gamma})x^{6.5} + O(x^7)\Bigg\}, \tag{15}$$

$$\Lambda = \left(264 - \frac{288}{\eta_1}\right)\frac{c^{10}\lambda_2^{(1)}}{M^5} + (1 \leftrightarrow 2), \tag{16}$$

$$\delta\Lambda = \left(\frac{4595}{28} - \frac{15895}{28\eta_1} + \frac{5715\eta_1}{14} - \frac{325\eta_1^2}{7}\right)\frac{c^{10}\lambda_2^{(1)}}{M^5} + (1 \leftrightarrow 2), \tag{17}$$

$$\Sigma = \left(\frac{6920}{7} - \frac{20740}{21\eta_1}\right)\frac{c^8\sigma_2^{(1)}}{M^5} + (1 \leftrightarrow 2).$$

$$\tilde{\Lambda} = \left[\left(\frac{593}{4} - \frac{1105}{8\eta_1} + \frac{567\eta_1}{8} - 81\eta_1^2\right)\chi_2 + \left(-\frac{6607}{8} + \frac{6639\eta_1}{8} - 81\eta_1^2\right)\chi_1\right]\frac{c^{10}\lambda_2^{(1)}}{M^5} + (1 \leftrightarrow 2), \tag{19}$$

$$\tilde{\Sigma} = \left[\left(-\frac{9865}{3} + \frac{4933}{3\eta_1} + 1644\eta_1\right)\chi_2 - \chi_1\right]\frac{c^8\sigma_2^{(1)}}{M^5} + (1 \leftrightarrow 2), \tag{20}$$

$$\tilde{\Gamma} = \frac{c^{10}\chi_1}{M^4}\left[\left(856\eta_1 - 816\eta_1^2\right)\lambda_{23}^{(1)} - \left(\frac{4993\eta_1}{18} - \frac{2497\eta_1^2}{9}\right)\sigma_{23}^{(1)} - \nu\left(272\lambda_{32}^{(1)} - 204\sigma_{32}^{(1)}\right)\right] + (1 \leftrightarrow 2). \tag{21}$$

Higher order term in GW phase (2)

Rotational-tidal phasing of the binary neutron star waveform

- arxiv:1805.01882

$$\Psi = \frac{3M}{128\mu} x^{-2.5} \Big[1 - \frac{39}{2}\tilde{\Lambda}\, x^5 + \tilde{\Sigma}\, x^6 - \tilde{X}\, x^{6.5} - \tilde{\Lambda}_3\, x^7 + \tilde{\Sigma}_3\, x^8 \Big], \tag{8}$$

$$\tilde{X} = \frac{1}{21M^6} c^{12} \Big\{ \chi^{(1)} \left[36(35+614q)\hat{\lambda}_2^{(1)} - (7-4751q)\hat{\sigma}_2^{(1)} - 2316q\hat{\lambda}_3^{(1)} - 3474q\hat{\sigma}_3^{(1)} \right] + \chi^{(2)} \left[36(35+614/q)\hat{\lambda}_2^{(2)} - (7-4751/q)\hat{\sigma}_2^{(2)} - 2316\hat{\lambda}_3^{(2)}/q - 3474\hat{\sigma}_3^{(2)}/q \right] \Big\},$$

$$\tilde{\Lambda}_3 = \frac{4000}{9M^7} c^{14} (q\lambda_3^{(1)} + \lambda_3^{(2)}/q),$$

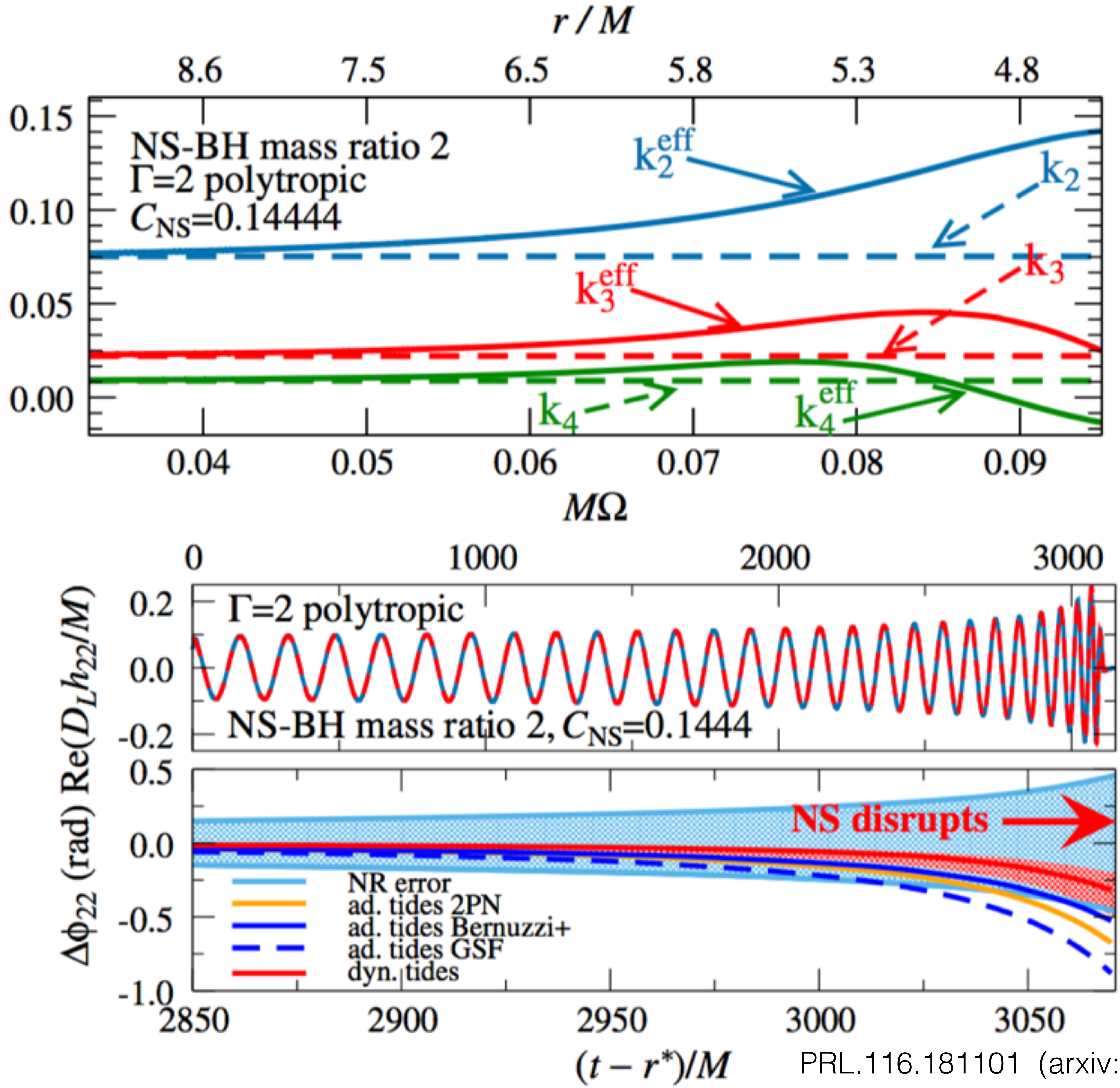
$$\tilde{\Sigma}_3 = \frac{29925}{11M^7} c^{14} (q\sigma_3^{(1)} + \sigma_3^{(2)}/q).$$

Dynamic tide

$$\boxed{k_\ell^{\mathrm{eff}} = k_\ell\left[a_\ell + \frac{b_\ell}{2}\left(\frac{Q_{m=\ell}^{\mathrm{DT}}}{Q_{m=\ell}^{\mathrm{AT}}} + \frac{Q_{m=-\ell}^{\mathrm{DT}}}{Q_{m=-\ell}^{\mathrm{AT}}}\right)\right],}$$

$$\frac{Q_m^{\mathrm{DT}}}{Q_m^{\mathrm{AT}}} \approx \frac{\omega_f^2}{\omega_f^2 - (m\Omega)^2} + \frac{\omega_f^2}{2(m\Omega)^2\epsilon_f\Omega_f'(\phi-\phi_f)} \pm \frac{i\omega_f^2}{(m\Omega)^2\sqrt{\epsilon_f}} e^{\pm i\Omega_f'\epsilon_f(\phi-\phi_f)^2}\int_{-\infty}^{\sqrt{\epsilon_f}(\phi-\phi_f)} e^{\mp i\Omega_f' s^2} ds, \tag{2}$$

PRL.116.181101 (arxiv:1602.00599)



PRL.116.181101 (arxiv:1602.00599)

Calculation of Tidal deformability

To obtain Tidal Love number, k2 (1)

T. Hinderer, ApJ, 677, 1216 (2008)

$$
\begin{aligned}
k_2 =& \frac{8C^5}{5}(1-2C)^2[2+2C(y-1)-y] \\
&\times \Big\{ 2C[6-3y+3C(5y-8)] + 4C^3[13-11y+C(3y-2)+2C^2(1+y)] \\
&+ 3(1-2C)^2[2-y+2C(y-1)]\ln(1-2C) \Big\}^{-1}
\end{aligned}
$$

$$
y = \frac{R\beta(R)}{H(R)} \quad C = \frac{GM}{Rc^2}
$$

$$
\frac{dH}{dr} = \beta
$$

$$
\begin{aligned}
\frac{d\beta}{dr} =& 2\left(1-2\frac{M}{r}\right)^{-1} \times H \Big\{ -2\pi\left[5\epsilon + 9P + (d\epsilon/dP)(\epsilon+P)\right] \\
&\quad + \frac{3}{r^2} + 2\left(1-2\frac{M}{r}\right)^{-1}\left(\frac{M}{r^2} + 4\pi r P\right)^2 \Big\} \\
&+ \frac{2\beta}{r}\left(1-2\frac{M}{r}\right)^{-1}\left\{ -1 + \frac{M}{r} + 2\pi r^2(\epsilon - P) \right\}
\end{aligned}
$$

To obtain Tidal Love number, k2 (2)

$$\frac{dH}{dr} = \beta \qquad \frac{d\beta}{dr} = 2\left(1-2\frac{M}{r}\right)^{-1} \times H\left\{-2\pi\left[5\epsilon+9P+(d\epsilon/dP)(\epsilon+P)\right] + \frac{3}{r^2} + 2\left(1-2\frac{M}{r}\right)^{-1}\left(\frac{M}{r^2}+4\pi r P\right)^2\right\} + \frac{2\beta}{r}\left(1-2\frac{M}{r}\right)^{-1}\left\{-1+\frac{M}{r}+2\pi r^2(\epsilon-P)\right\}$$

TOV

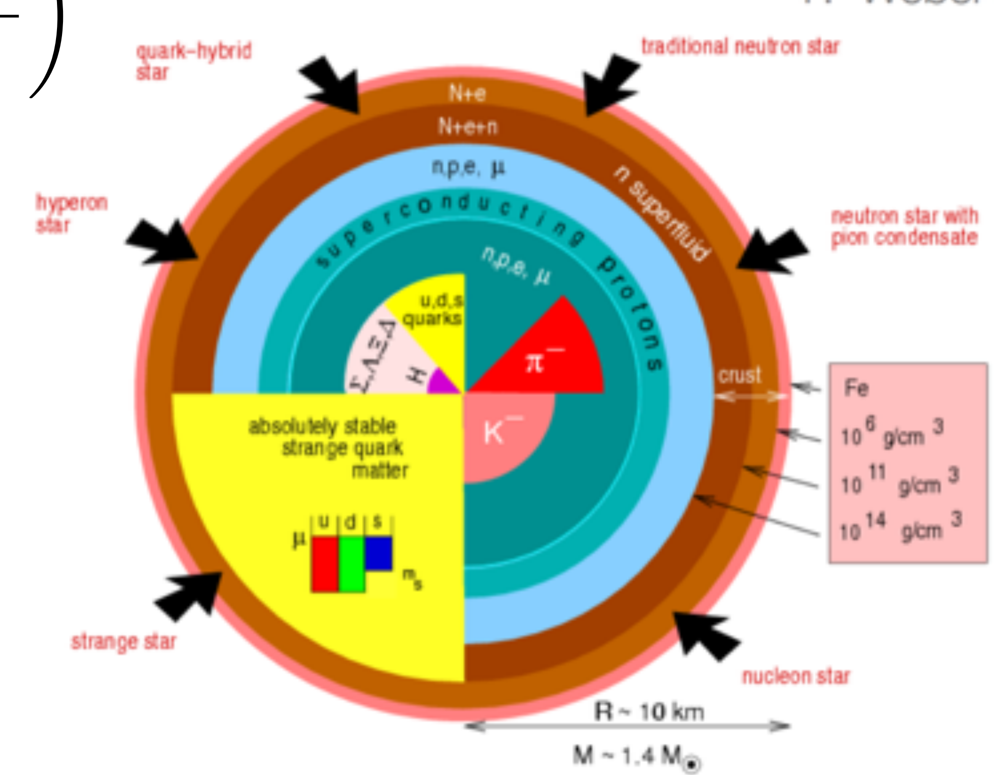
$$\frac{dP}{dr} = -\frac{GM\rho}{r^2}\left(1+\frac{P}{\rho c^2}\right)\left(1+\frac{4\pi P r^3}{Mc^2}\right)\left(1-\frac{2GM}{rc^2}\right)$$

$$\frac{dM}{dr} = 4\pi r^2\left(\frac{\epsilon}{c^2}\right)$$

k2 (λ,Λ) depends on NS EoS !!

Compositions of a NS

F. Weber 2005



Various Nuclear EoSs

For plain $npe\mu$ nuclear matter, we include:

(i) two potential-method EOSs (PAL6 [15] and SLy [16]);

(ii) eight variational-method EOSs (APR1-4 [17], FPS [18], and WFF1-3 [19]);

(iii) one nonrelativistic (BBB2 [20]) and three relativistic (BPAL12 [21], ENG [1] and MPA1 [22]) Brueckner-Hartree-Fock EOSs; and

(iv) three relativistic mean-field theory EOSs (MS1-2 and one we call MS1b, which is identical to MS1 except with a low symmetry energy of 25 MeV [23]).

EOSs as $K/\pi/H/q$ models:

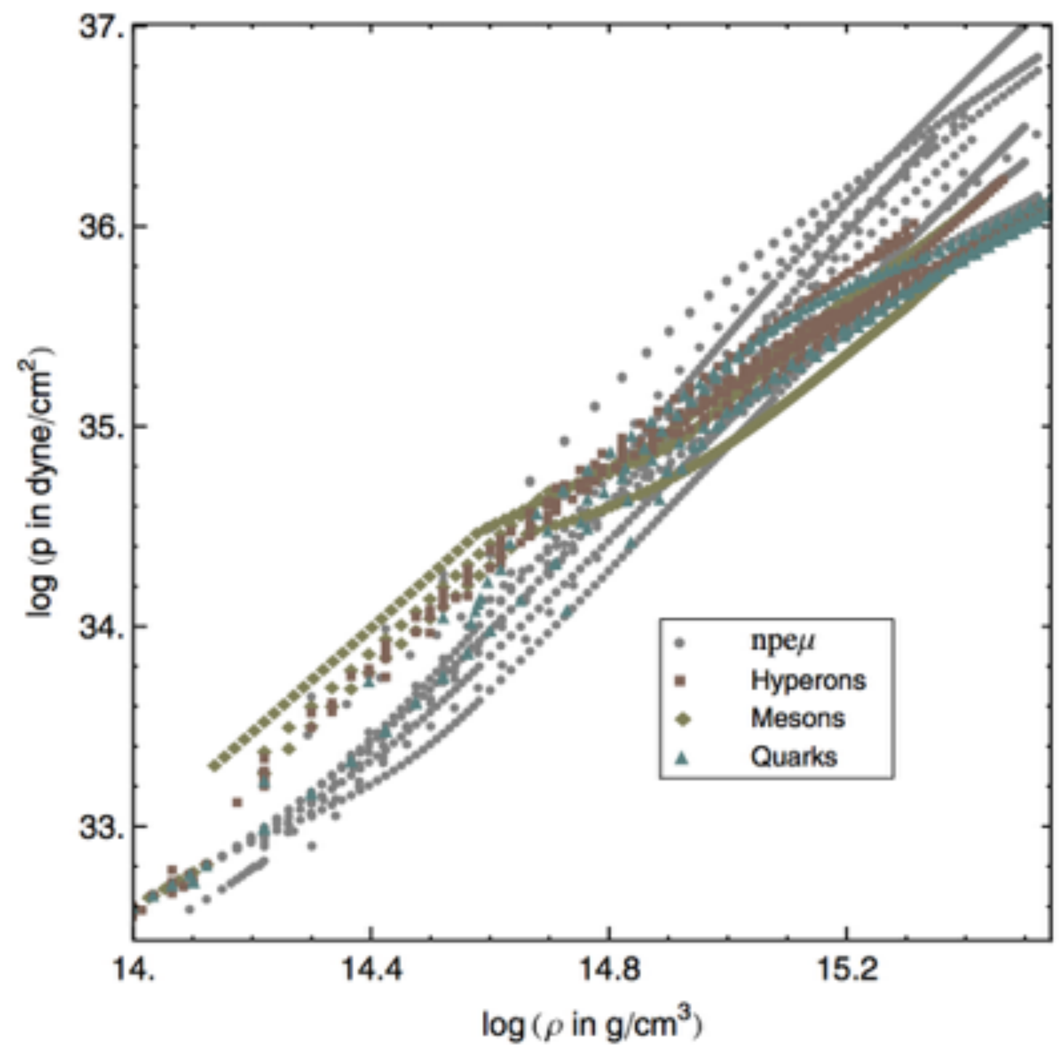
(i) one neutron-only EOS with pion condensates (PS [24]);

(ii) two relativistic mean-field theory EOSs with kaons (GS1-2 [25]);

(iii) one effective-potential EOS including hyperons (BGN1H1 [26]); eight relativistic mean-field theory EOSs with hyperons (GNH3 [27], and seven variants H1-7 [12]; one relativistic mean-field theory EOS with hyperons and quarks (PCL2 [28]); and

(iv) four hybrid EOSs with mixed APR nuclear matter and color-flavor-locked quark matter (ALF1-4 with transition density ρ_c and interaction parameter c given by $\rho_c = 2n_0$, $c = 0$; $\rho_c = 3n_0$, $c = 0.3$; $\rho_c = 3n_0$, $c = 0.3$; and $\rho_c = 4.5n_0$, $c = 0.3$, respectively [29]).

Read et al. , PRD 79, 124032 (2009)



Constraints on Nuclear EoS

- Nuclear data: hundreds of models (Skyrme force, RMF, …)
- Neutron star maximum mass

  1.97 ± 0.04 M⊙ [Nature 467, 1081 (2010)]

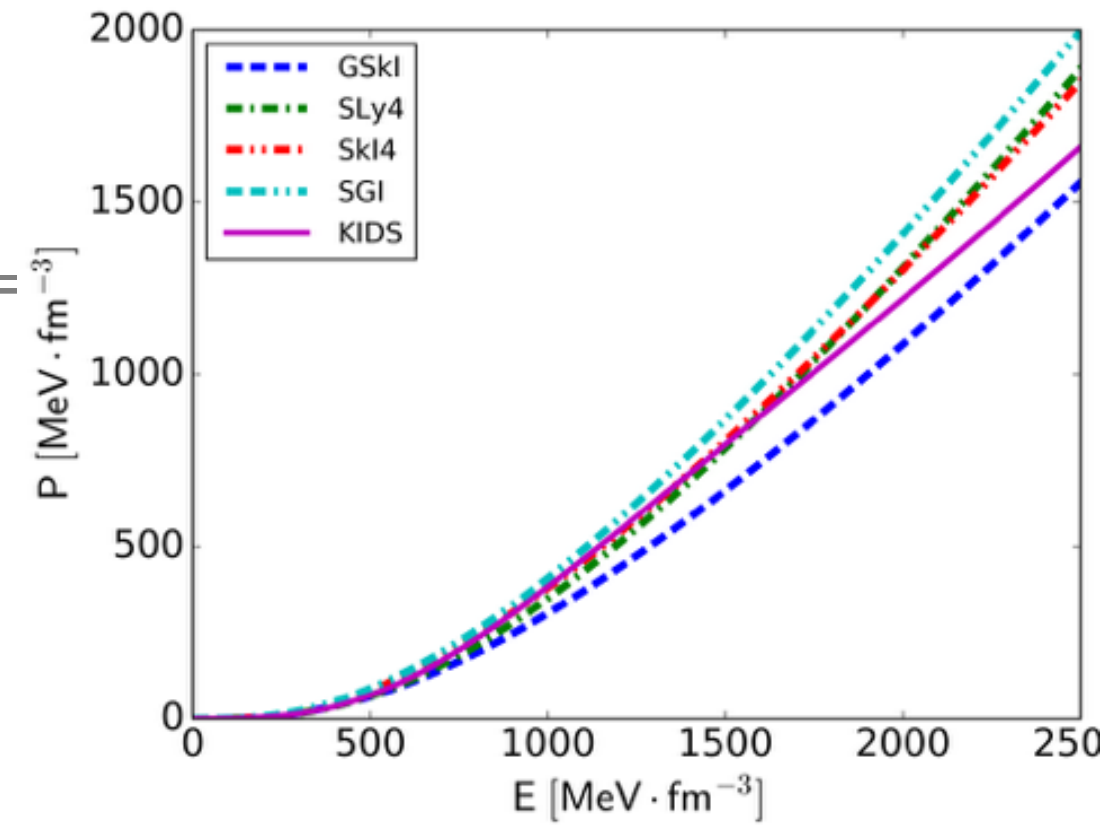
  2.01 ± 0.04 M⊙ [Science 340, 448 (2013)]

- 11 experimental/empirical data for nuclear matter around saturation density [Phys.Rev. C 85, 035201 (2012)]

Constraint	Quantity	Eq.	Density Region	Range of constraint exp/emp	Range of constraint from CSkP	Ref.
SM1	K_o	(7),(15)	ρ_o (fm^{-3})	200 – 260 MeV	202.0 – 240.3 MeV	[64]
SM2	$K' = -Q_o$	(8),(16)	ρ_o (fm^{-3})	200 – 1200 MeV	362.5 – 425.6 MeV	[65]
SM3	$P(\rho)$	(6)	$2 < \frac{\rho}{\rho_o} < 3$	Band Region	see Fig. 1	[78]
SM4	$P(\rho)$	(6)	$1.2 < \frac{\rho}{\rho_o} < 2.2$	Band Region	see Fig. 2	[80]
PNM1	$\frac{E_{PNM}}{E^o_{PNM}}$	(31)	$0.014 < \frac{\rho}{\rho_o} < 0.106$	Band Region	see Fig. 3	[39, 40]
PNM2	$P(\rho)$	(6)	$2 < \frac{\rho}{\rho_o} < 3$	Band Region	see Fig. 5	[78]
MIX1	J	(9)	ρ_o (fm^{-3})	30 – 35 MeV	30.0 – 35.5 MeV	[44]
MIX2	L	(10)	ρ_o (fm^{-3})	40 – 76 MeV	48.6 – 67.1 MeV	[101]
MIX3	$K_{\tau,v}$	(21)	ρ_o (fm^{-3})	-760 – -372 MeV	-407.1 – -360.1 MeV	[107]
MIX4	$\frac{S(\rho_o/2)}{J}$	-	ρ_o (fm^{-3})	0.57 – 0.86	0.61 – 0.67	[110]
MIX5	$\frac{3P_{PNM}}{L\rho_o}$	(41)	ρ_o (fm^{-3})	0.90 – 1.10	1.02 – 1.10	[112]

Selected EoSs

- Skyrme force models
- Basically fitted to properties of well-known nuclei
- Good saturation properties
- Mmax more than 2Msun



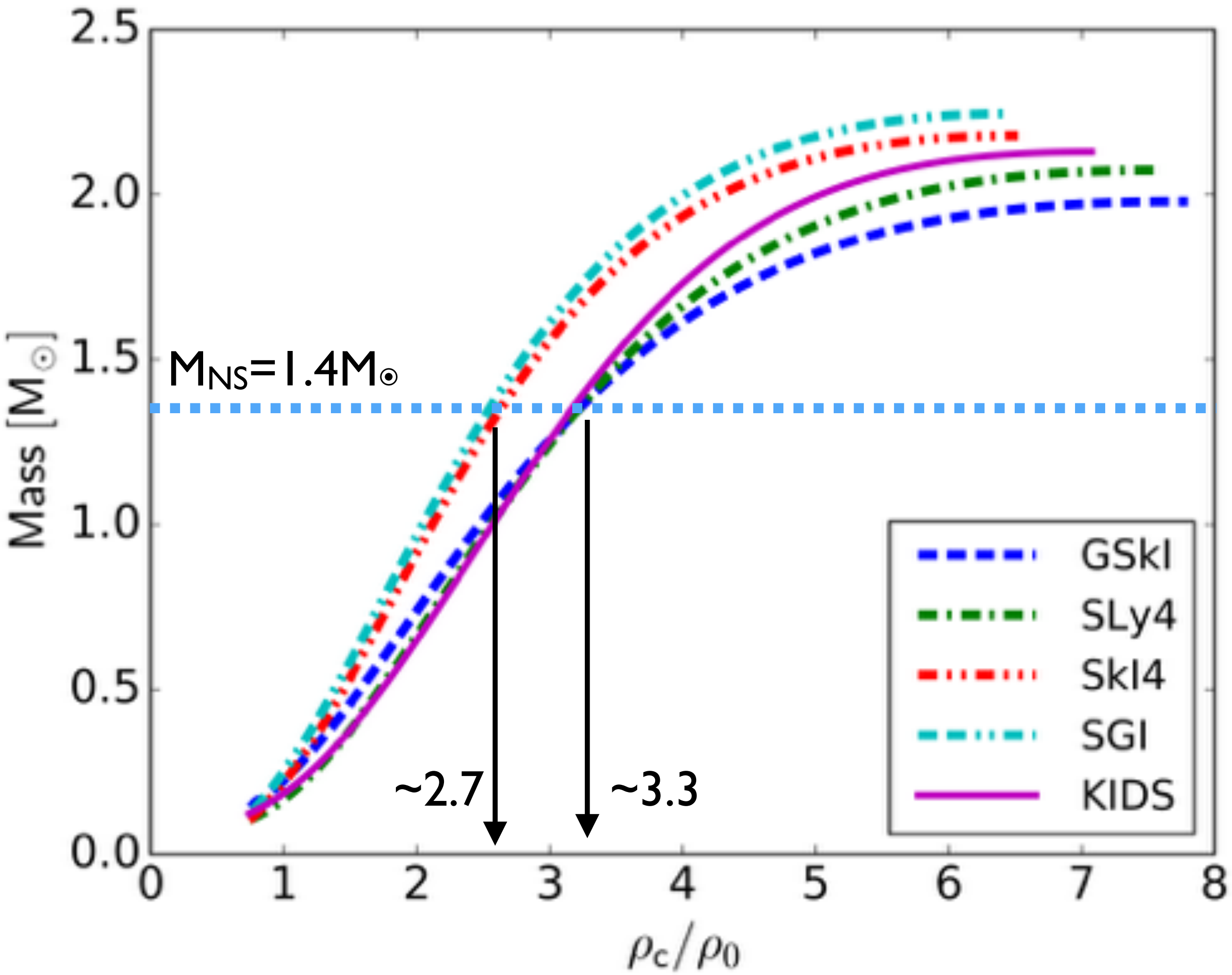
Model	ρ_0	E_0	K_0	$-Q_0$	J	L	$-K_\tau$	$M_{\max}$
Exp/Emp	$\simeq 0.16$	$\simeq 16.0$	$200 \sim 260$	$200 \sim 1200$	$30 \sim 35$	$40 \sim 76$	$372 \sim 760$	$\geq 1.93 \sim 2.05$
CSkP	-	-	$202.0 \sim 240.3$	$362.5 \sim 425.6$	$30.0 \sim 35.5$	$48.6 \sim 67.1$	$360.1 \sim 407.1$	-
GSkI	0.159	16.02	230.2	405.6	32.0	63.5	364.2	1.98
SLy4	0.160	15.97	229.9	363.1	32.0	45.9	322.8	2.07
SkI4	0.160	15.95	248.0	331.2	29.5	60.4	322.2	2.19
SGI	0.154	15.89	261.8	297.9	28.3	63.9	362.5	2.25
KIDS	0.160	16.00	240.0	372.7	32.8	49.1	375.1	2.14

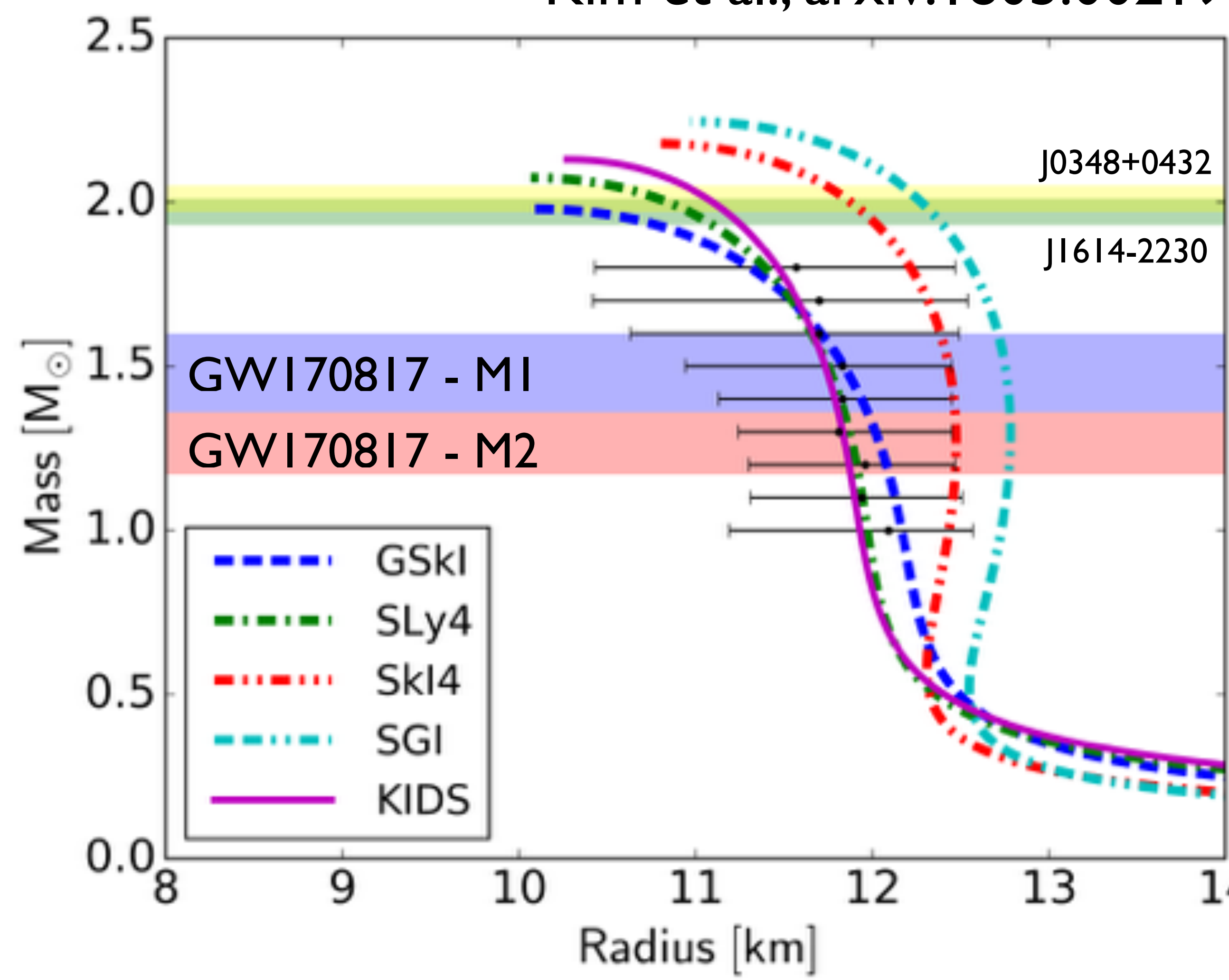
Kim et al., arxiv:1805.00219

KIDS (Korea: IBS-Daegu-Sungkyunkwan): A new systematic expansion scheme for nuclear EDF [Phys. Rev. C 97, 014312 (2018)]

Mass-Radius relations

Kim et al., arxiv:1805.00219

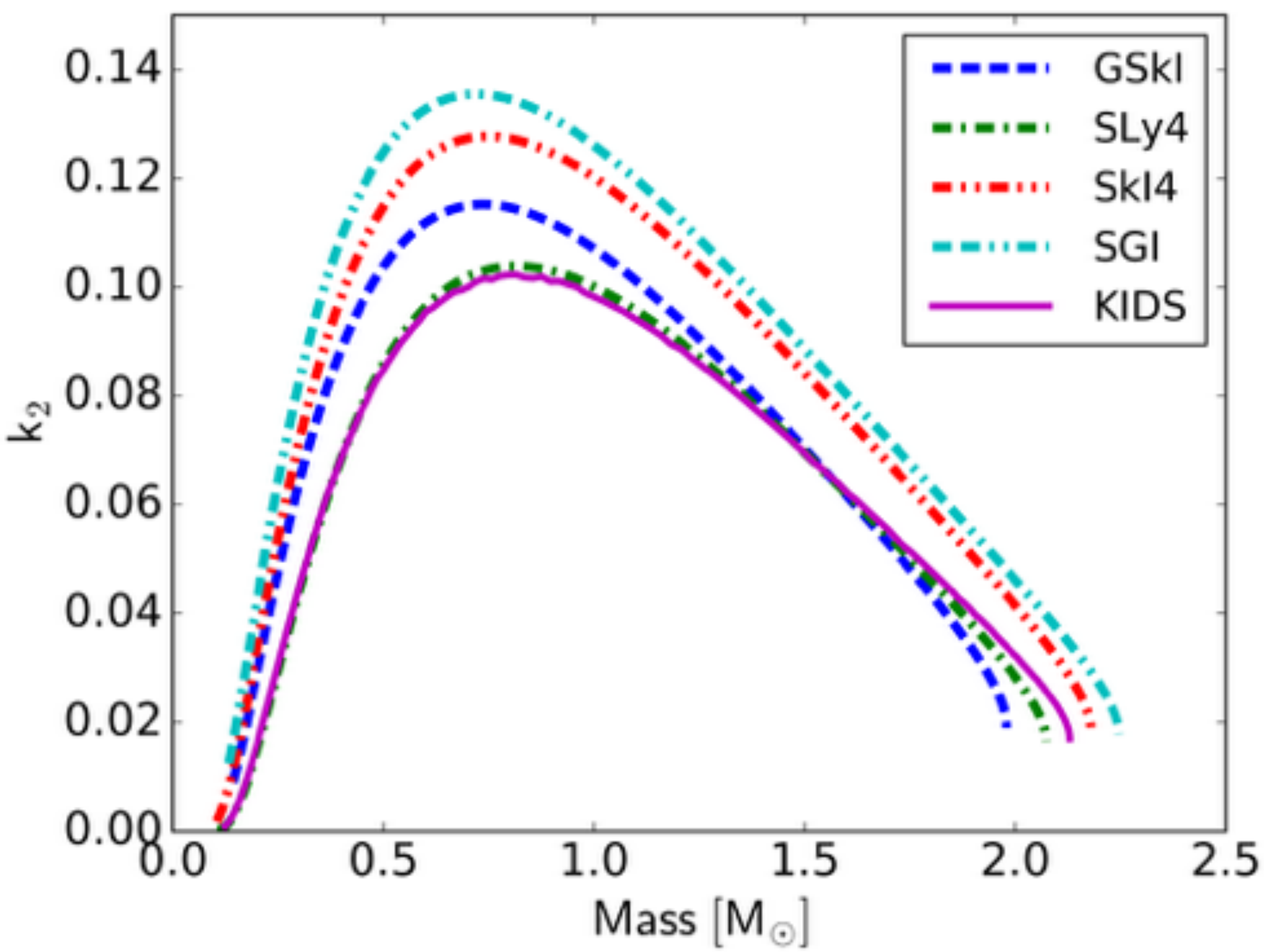


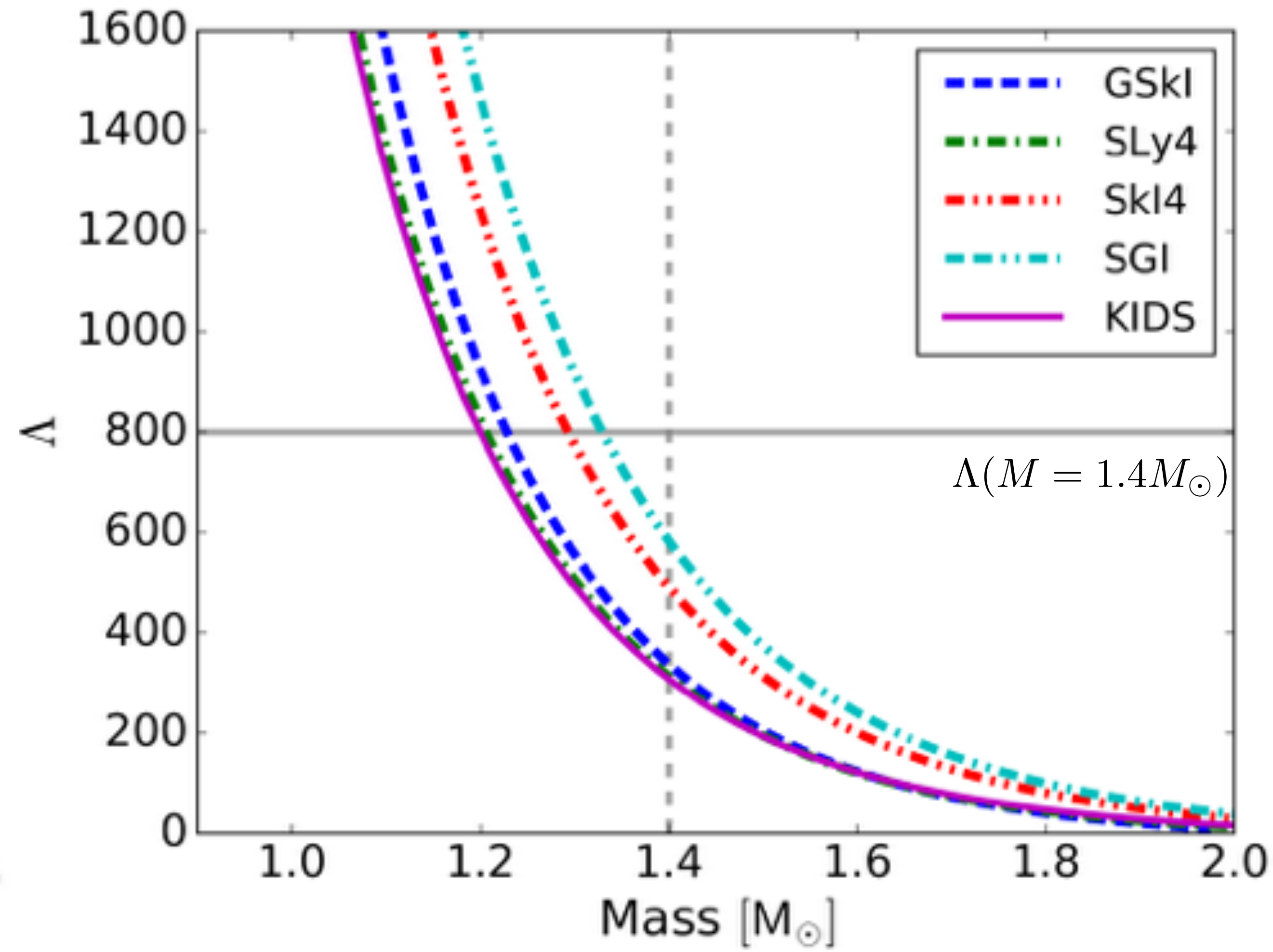


GW170817
- Mchirp = 1.188 M⊙
- low spin prior : M1 = 1.36 ~ 1.60 M⊙ , M2 = 1.17 ~ 1.36 M⊙
- high spin prior : M1 = 1.36 ~ 2.26 M⊙ , M2 = 0.86 ~ 1.36 M⊙

Tidal deformability of a NS

Kim et al., arxiv:1805.00219

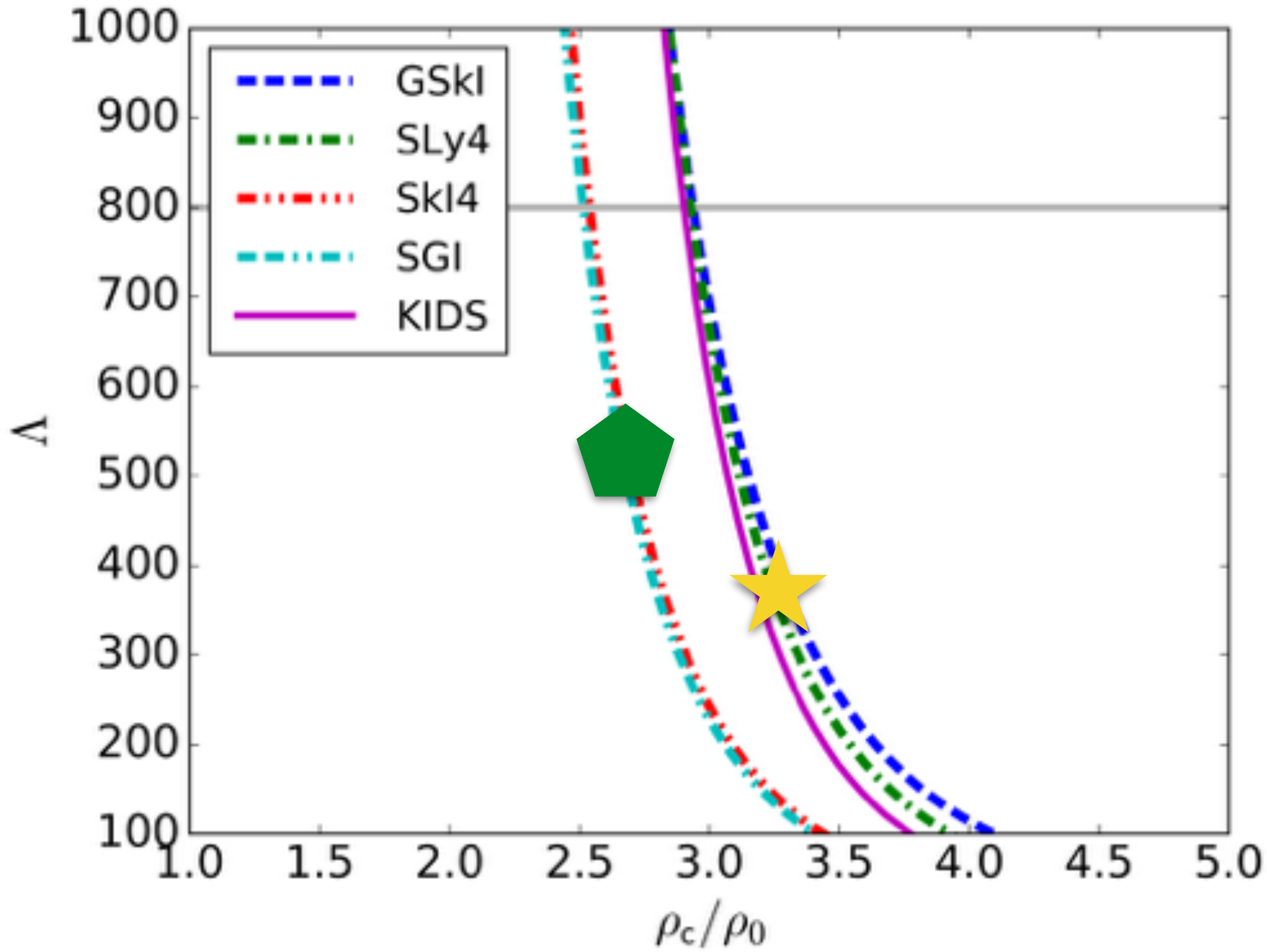


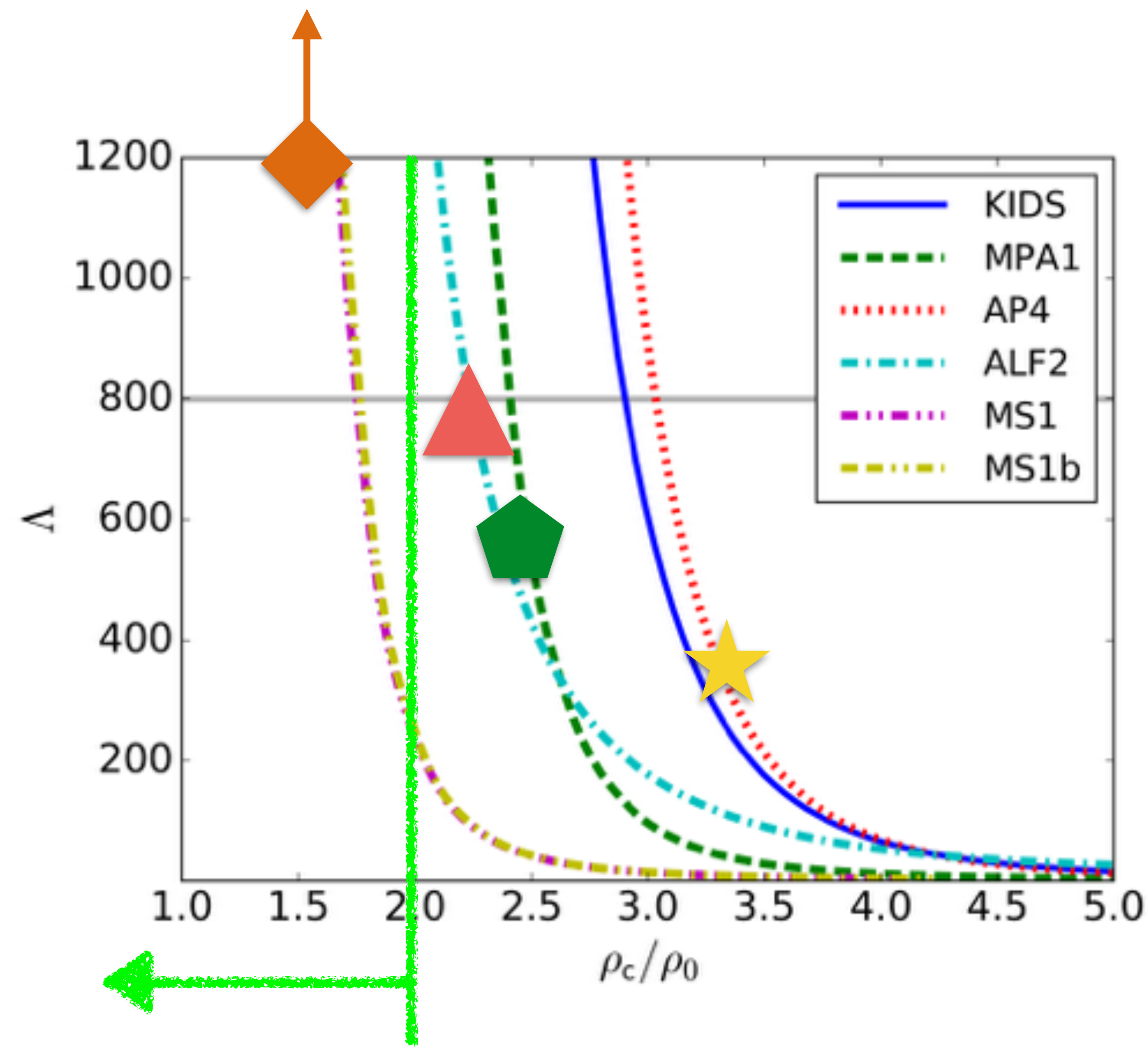


GW170817, Mchirp = 1.188 M☉
- low spin prior : Lambda(1.4 M☉) < 800
- high spin prior : Lambda(1.4 M☉) < 1400

Central Density at M_{NS}=1.4 $M_{\odot}$

well-selected EoSs

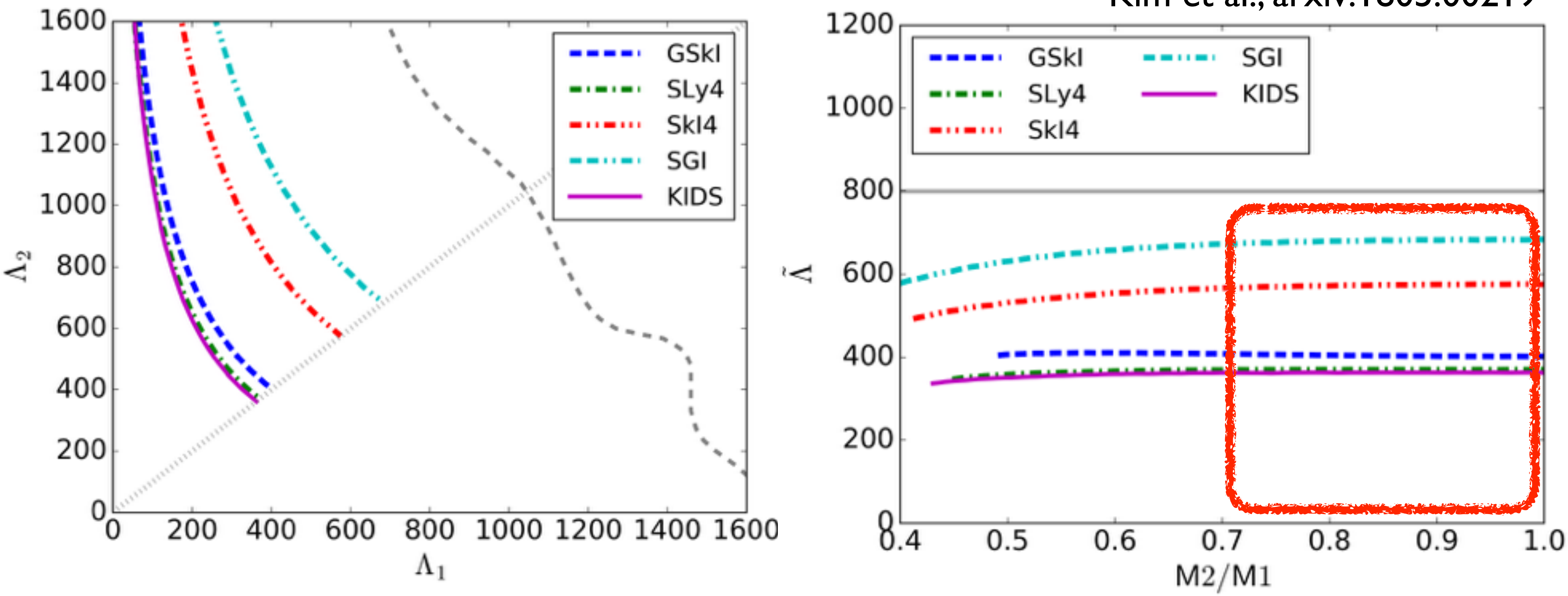




ruled out ?

Tidal deformability in BNS

Kim et al., arxiv:1805.00219

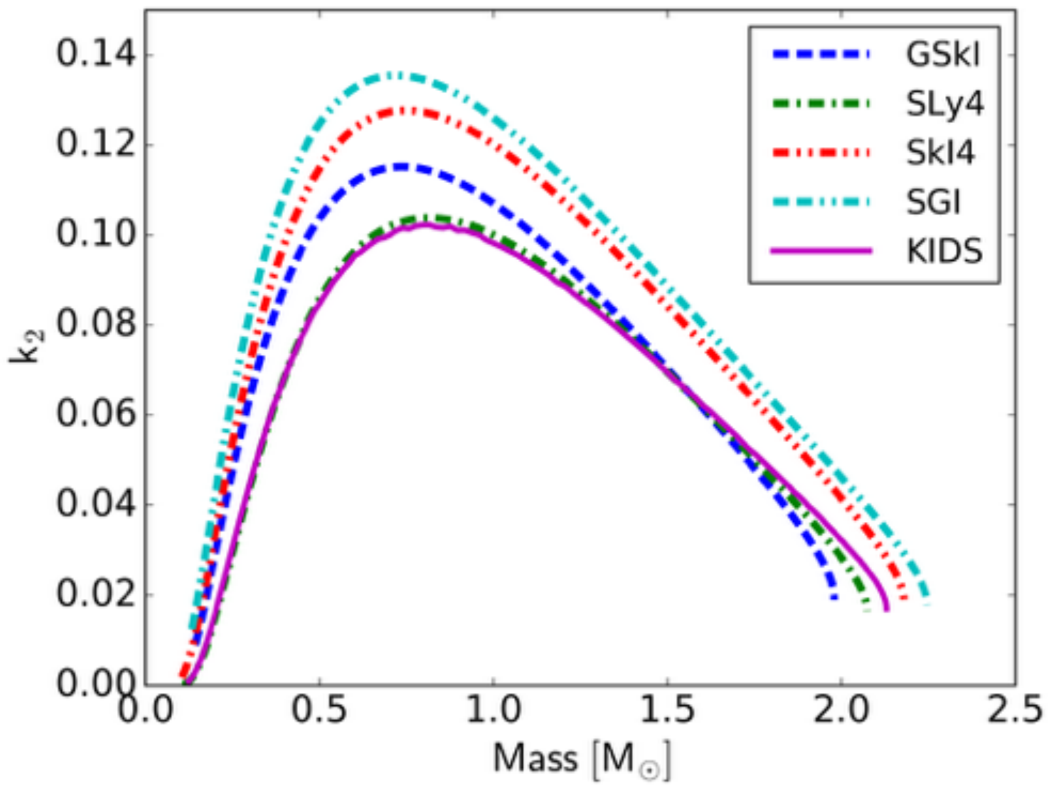


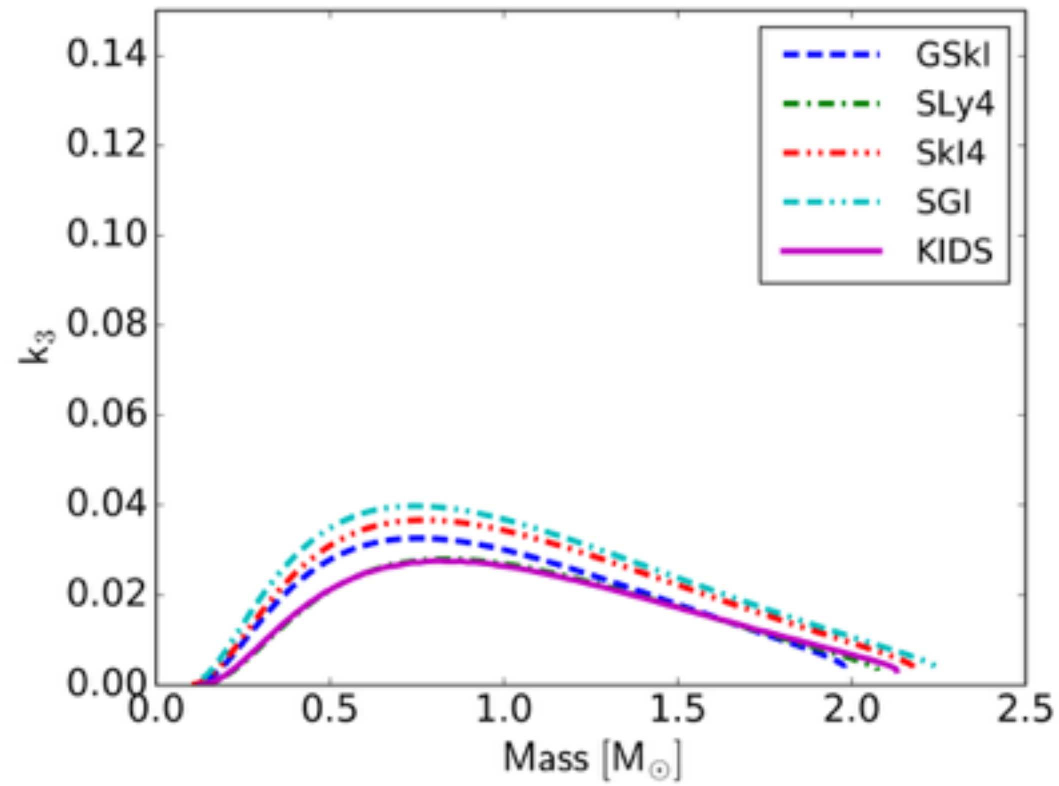
$$\tilde{\Lambda} = \frac{16}{13}\frac{(m_1 + 12m_2)m_1^4\Lambda_1 + (m_2 + 12m_1)m_2^4\Lambda_2}{(m_1+m_2)^5}$$

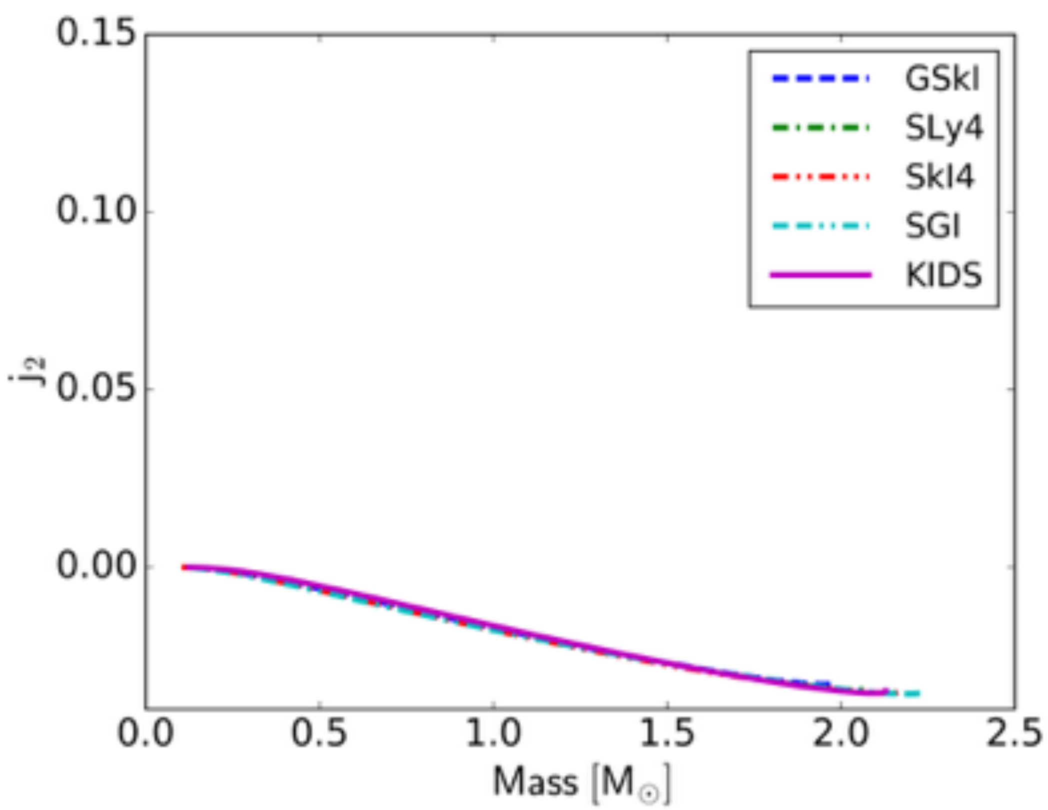
GW170817, Mchirp = 1.188 M☉

- low spin prior : reduced Lambda < 800
- high spin prior : reduced Lambda < 700

Beyond k2







possible?

$$\psi(x) = \frac{3}{128\nu x^{5/2}}\Bigg\{1 + \left(\frac{3715}{756} + \frac{55}{9}\nu\right)x + \Bigg(\frac{113}{3} \times \times (\eta_1\chi_1 + \eta_2\chi_2) - \frac{38}{3}\nu(\chi_1+\chi_2)\Bigg)x^{1.5} + O(x^2) + \Lambda x^5 + (\delta\Lambda + \Sigma)x^6 + (\tilde{\Lambda} + \tilde{\Sigma} + \tilde{\Gamma})x^{6.5} + O(x^7)\Bigg\}, \tag{15}$$

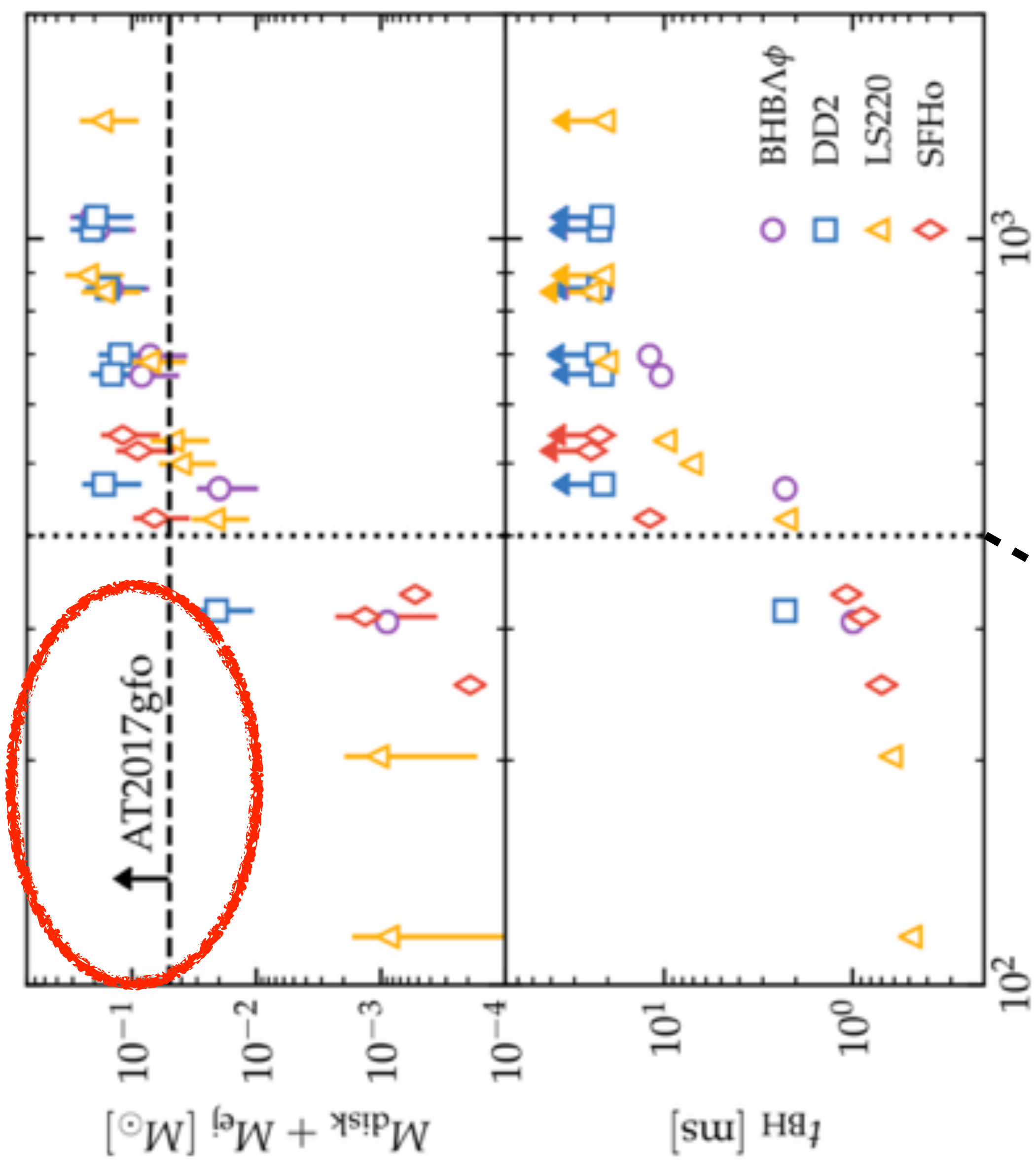
Recent Recearches

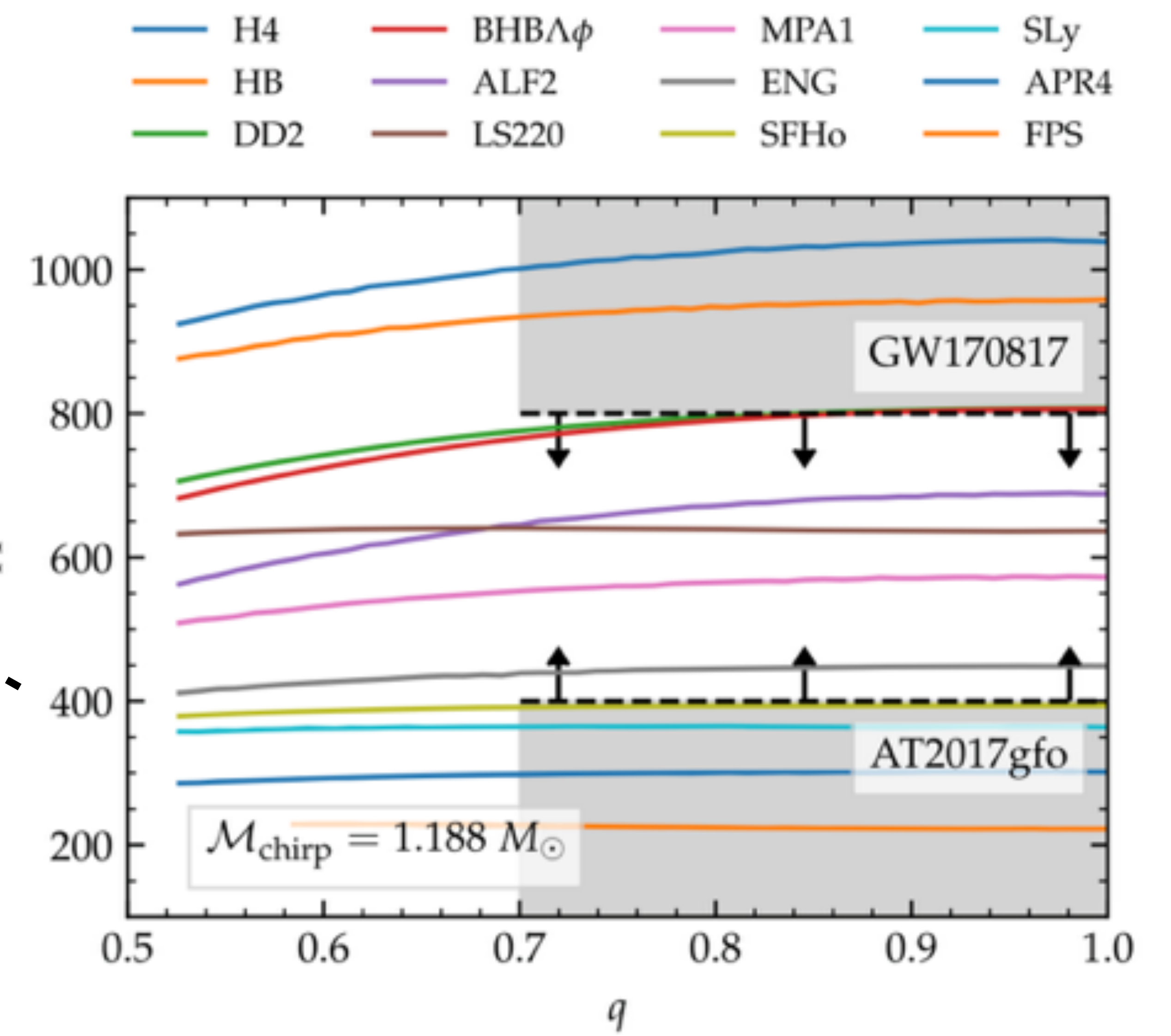
a lower bound of Λ

AT2017gfo
$M_{disk} + M_{ej} > 0.05\ M_\odot$

GR Hydrodynamics

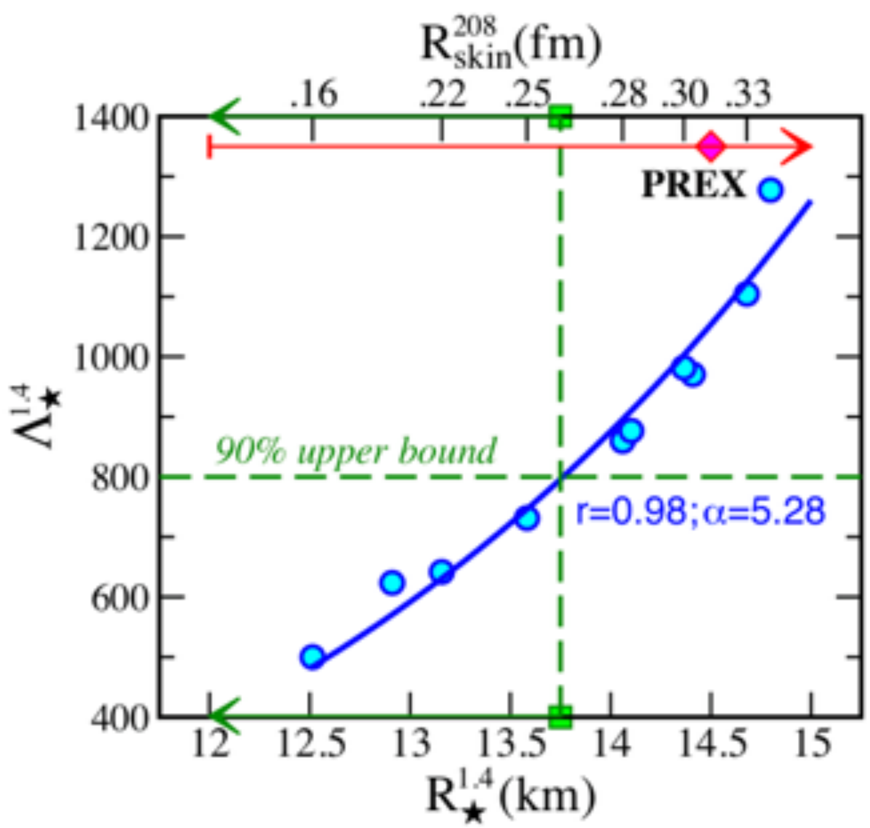
$$\tilde{\Lambda}(M_{chirp} = 1.188 M_\odot) \geq 400$$





Radice et al. , ApJL, 852, L29 (2018)

Recent Studies from Other groups (1)



[A] F.J. Fattoyev, J. Piekarewicz, and C.J. Horowitz, arXiv:1711.06615v2

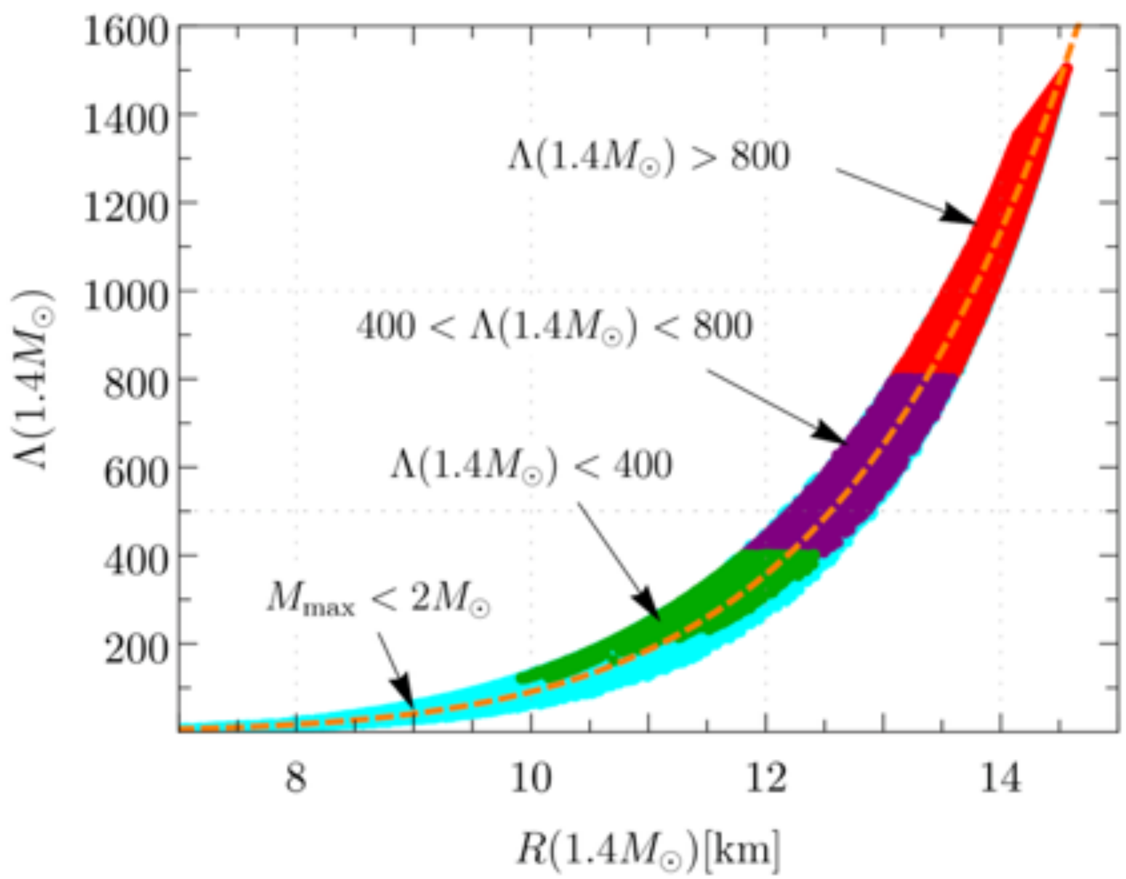
- RMF models
- Correlating neutron skin of ^{208}Pb , Λ(1.4M$\odot$) and R(1.4M$\odot$)
- 490 < Λ(1.4M$\odot$) < 800
- 12.55 km < R(1.4M$\odot$) < 13.76 km

EOS	R	β	k_2	λ	L
APR	11.55	0.179	0.0721	1.48	62
MDI ($x = 0$)	11.85	0.174	0.0707	1.65	62
MDI ($x = -1$)	13.59	0.152	0.0831	3.85	107
DBHF+Bonn B	12.64	0.163	0.0946	3.06	69
FPS	10.84	0.191	0.0664	1.00	35
SLY4	11.72	0.176	0.0762	1.68	47

[B] P.G. Krastev, and B.-A. Li, arXiv:1801.04620v1

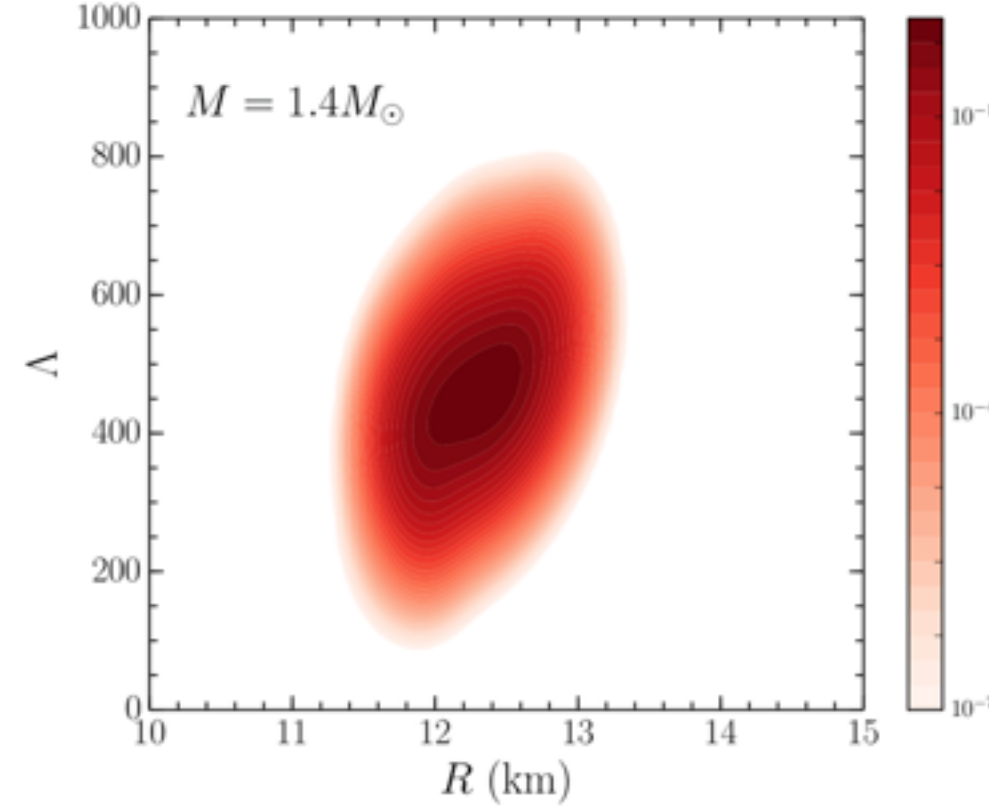
- MDI EoS
  - SNM part and symmetry energy constrained by heavy-ion reaction data up to 4.5 ρ0 and 1.2 ρ0, respectively
- 341 < Λ(1.4M$\odot$) < 782
- 11.5 km < R(1.4M$\odot$) < 13.6 km

Recent Studies from Other groups (2)



[C] E. Annala, T. Gorda, A. Kurkela, and A. Vuorinen, arXiv:1711.02644v2

- Low density EoS from EFT
- High density EoS from pQCD
- Polytropic interpolation in the intermediate density
- $120 < \Lambda(1.4M_{\odot}) < 800$
- 9.9 km < $R(1.4M_{\odot})$ < 13.6 km
- $\Lambda(1.4M_{\odot}) = 2.88 * 10^{-6} (R/\mathrm{km})^{7.5}$

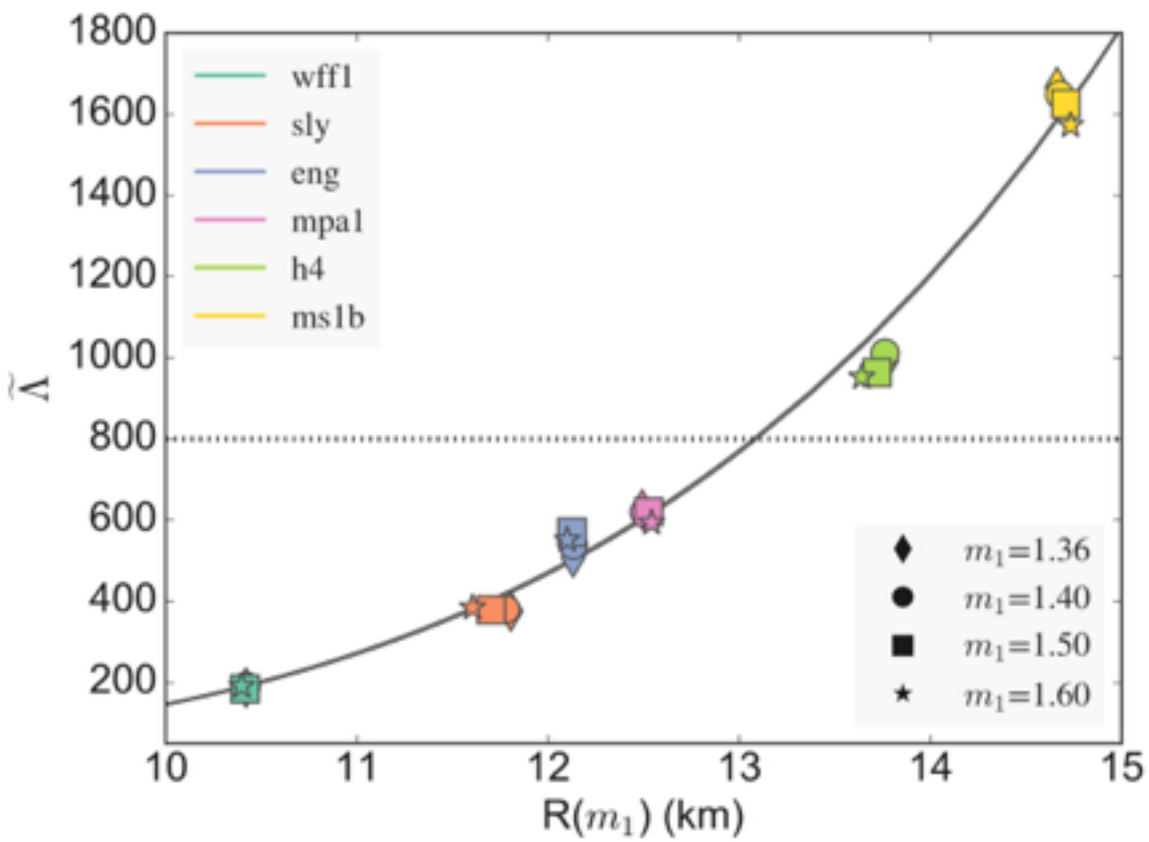


[D] Y. Lim and J. Holt, arXiv:1803.02803

- Prediction with uncertainties inherent EFT
- ~ 73000 energy density functionals
- $350 < \Lambda(1.4M_{\odot}) < 540$
- 11.65 km < $R(1.4M_{\odot})$ < 12.84 km

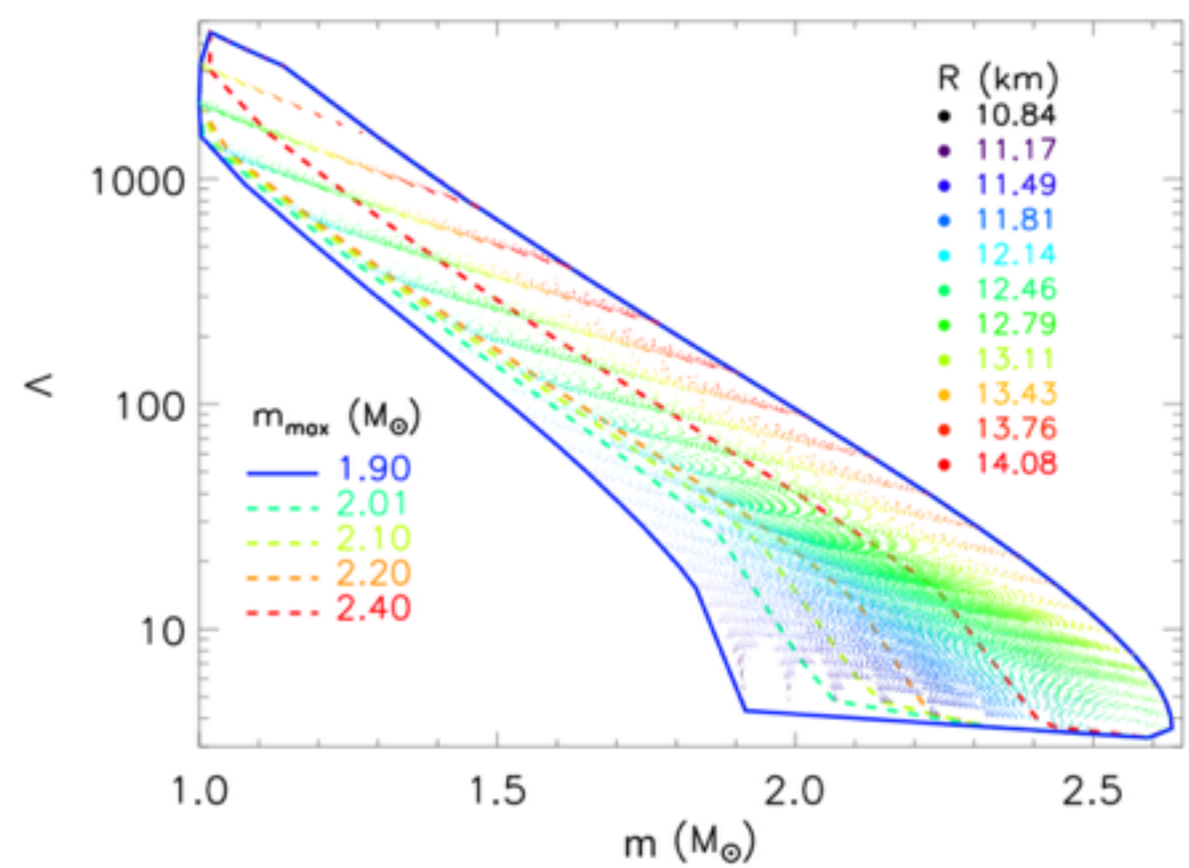
Mass-Radius as a function of Λ

Raithel, Özel, & Psaltis (ApJL,857,L23(2018))



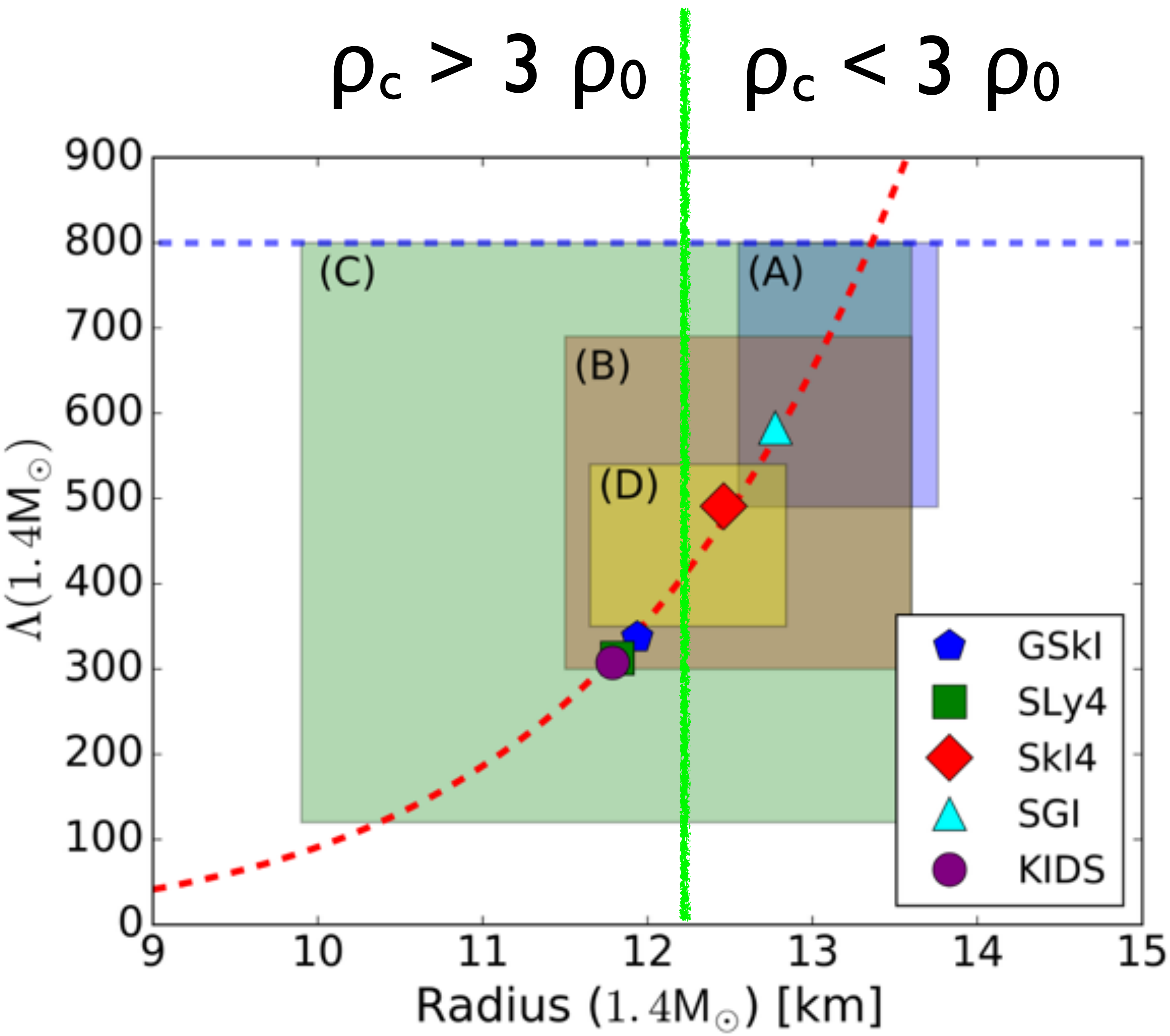
$$C = a_0 + a_1 \ln\Lambda + a_2 (\ln\Lambda)^2 \,,$$

De, et al., arxiv:1804.08583



$$\hat{R} \simeq R_{1.4} \simeq (11.2 \pm 0.2) \frac{\mathcal{M}}{M_\odot} \left(\frac{\tilde{\Lambda}}{800} \right)^{1/6} \text{km.}$$

Comparison with recent works

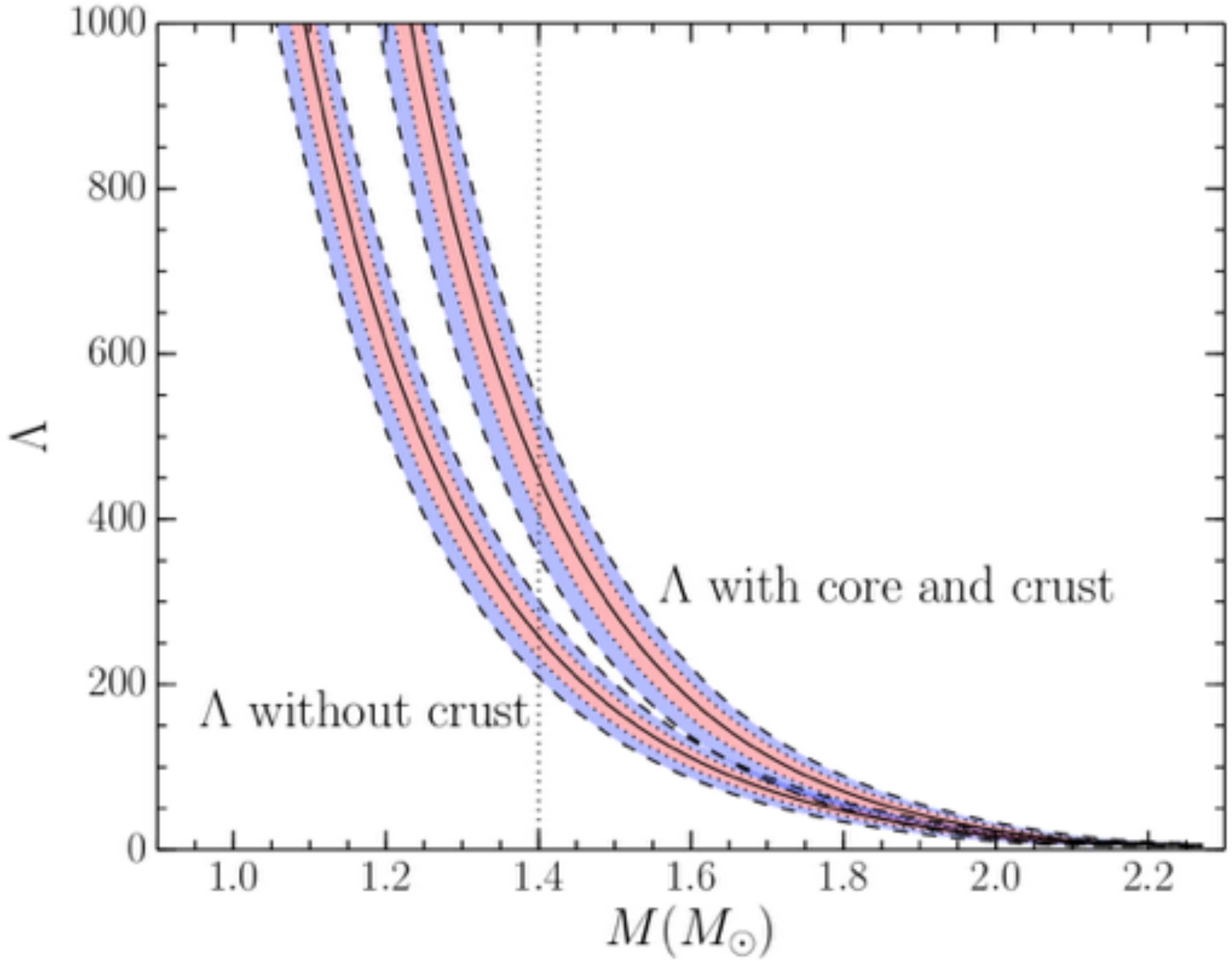


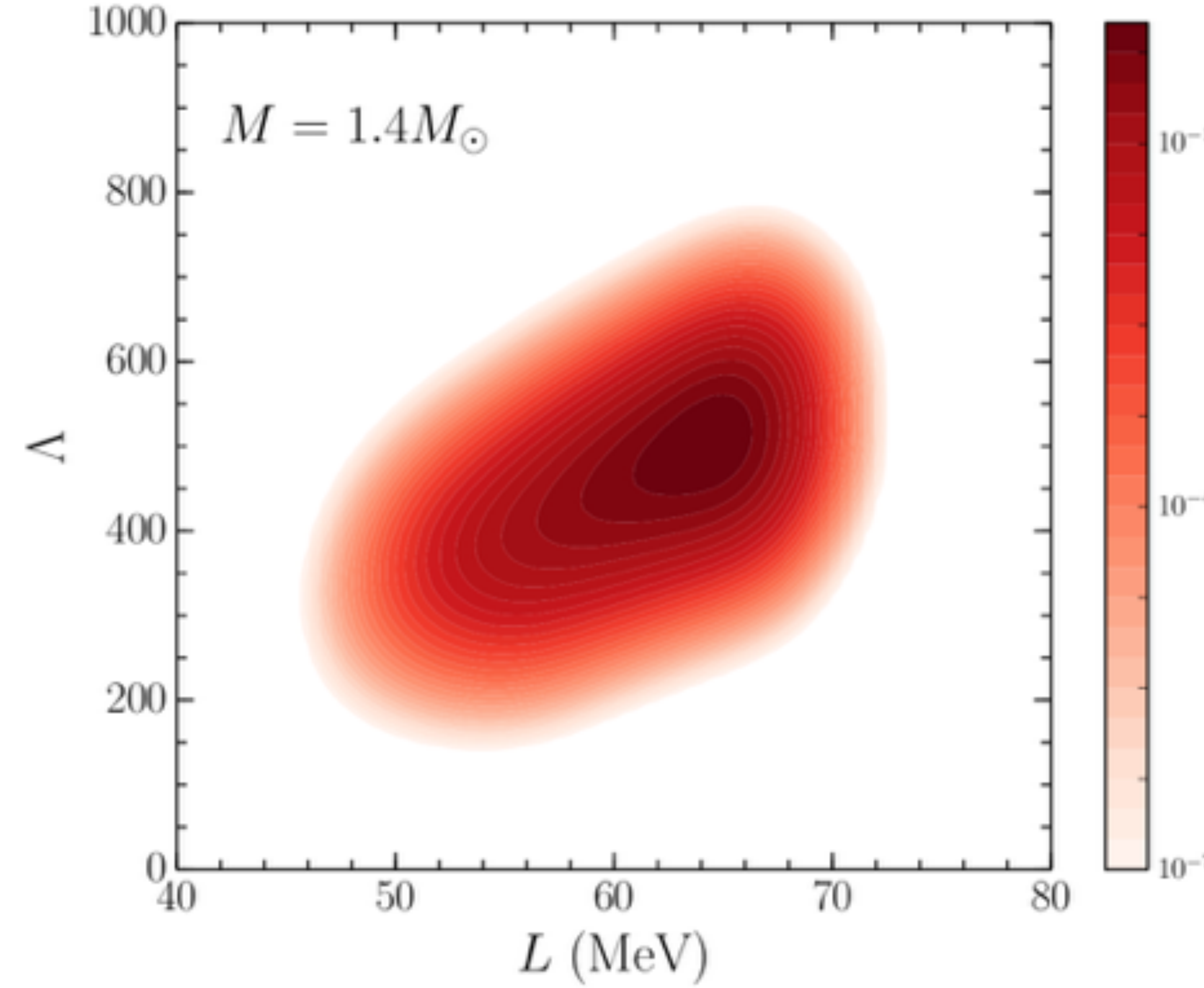
[A] F.J. Fattoyev, J. Piekarewicz, and C.J. Horowitz, arXiv:1711.06615v2
[B] P.G. Krastev, and B.-A. Li, arXiv:1801.04620v1
[C] E. Annala, T. Gorda, A. Kurkela, and A. Vuorinen, arXiv:1711.02644v2
[D] Y.Lim and J. Holt, arXiv:1803.02803

Kim et al., arxiv:1805.00219

Red line: $\Lambda(1.4M_\odot) = 2.88 * 10^{-6}\,(R/km)^{7.5}$ (fitting function in [C])

Crust EoS, Symmetry Energy





Y.Lim and J. Holt, arXiv:1803.02803

Prospects of GW observation for searching NS and its properties

Overview of Advanced LIGO 2nd Obs.

1. Period: Nov. 30, 2016 ~ Aug. 25, 2017

   break: Dec. 22, 2016 ~ Jan. 4, 2017, May 8 ~ 26.

2. Coincident observing data of LHO-LLO : ~81 days (as of Jun. 23, 2017) - 8 candidates

3. Averaged range

   - 1.4 $M_\odot$ - 1.4 $M_\odot$: ~70 Mpc
   - 10 $M_\odot$ - 10 $M_\odot$: ~ 300 Mpc
   - 30 $M_\odot$ - 30 $M_\odot$: ~700 Mpc

4. **Detection**

   - BBH : GW170104, GW170608, GW170814
   - BNS : GW170817

Overview of Advanced LIGO 2nd Obs.

1. Period: Nov. 30, 2016 ~ Aug. 25, 20

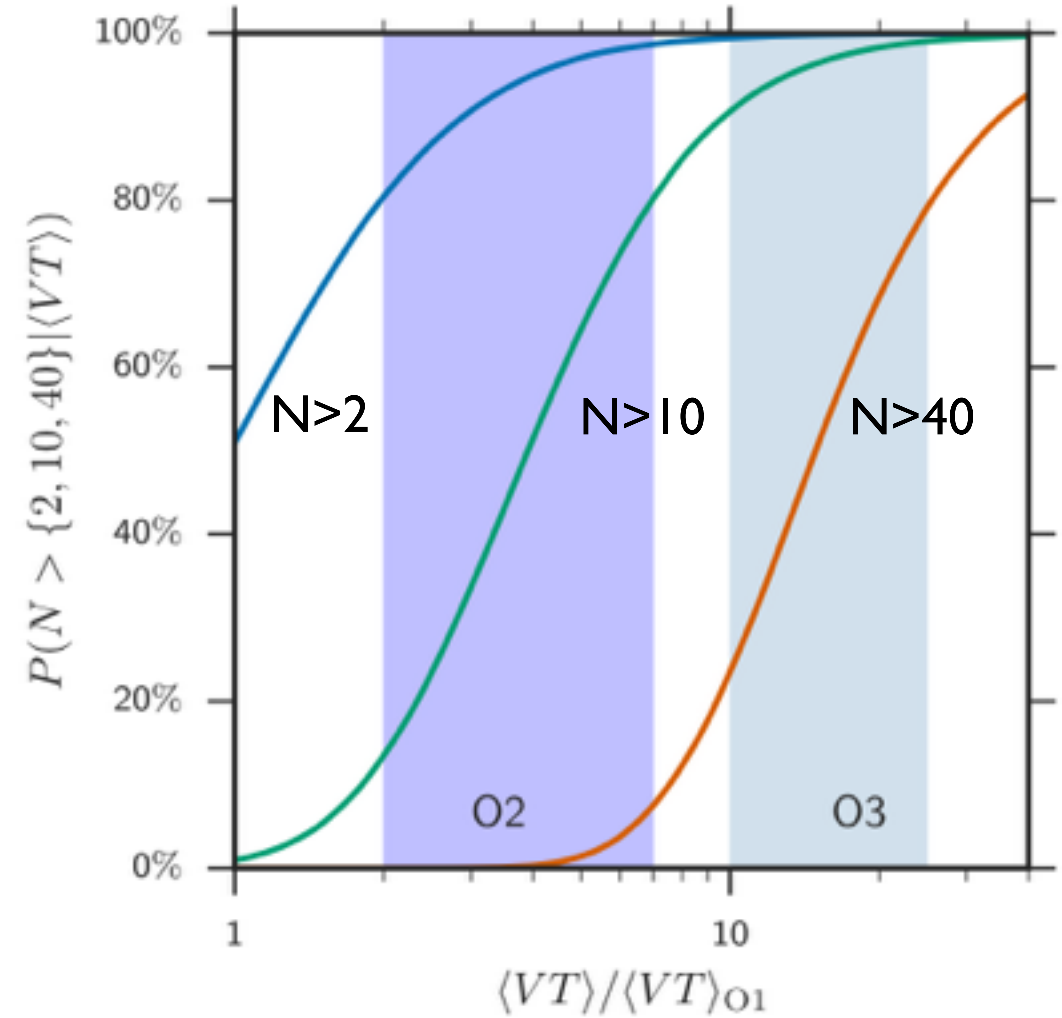
   break: Dec. 22, 2016 ~ Jan. 4, 2017,

2. Coincident observing data of LHO
   2017) - 8 candidates

3. Averaged range

   - 1.4 $M_\odot$ - 1.4 $M_\odot$: ~70 Mpc
   - 10 $M_\odot$ - 10 $M_\odot$: ~ 300 Mpc
   - 30 $M_\odot$ - 30 $M_\odot$: ~700 Mpc

PhysRevX.6.041015



4. **Detection**

   - BBH : GW170104, GW170608, GW170814
   - BNS : GW170817

Prospects of the Observing Runs

"Prospects for Observing and Localizing Gravitational-Wave Transients with Advanced LIGO, Advanced Virgo and KAGRA", arXiv:1304.0670v4, LIGO-P1200087-v45, in preparation to submit Living Rev. Relativity

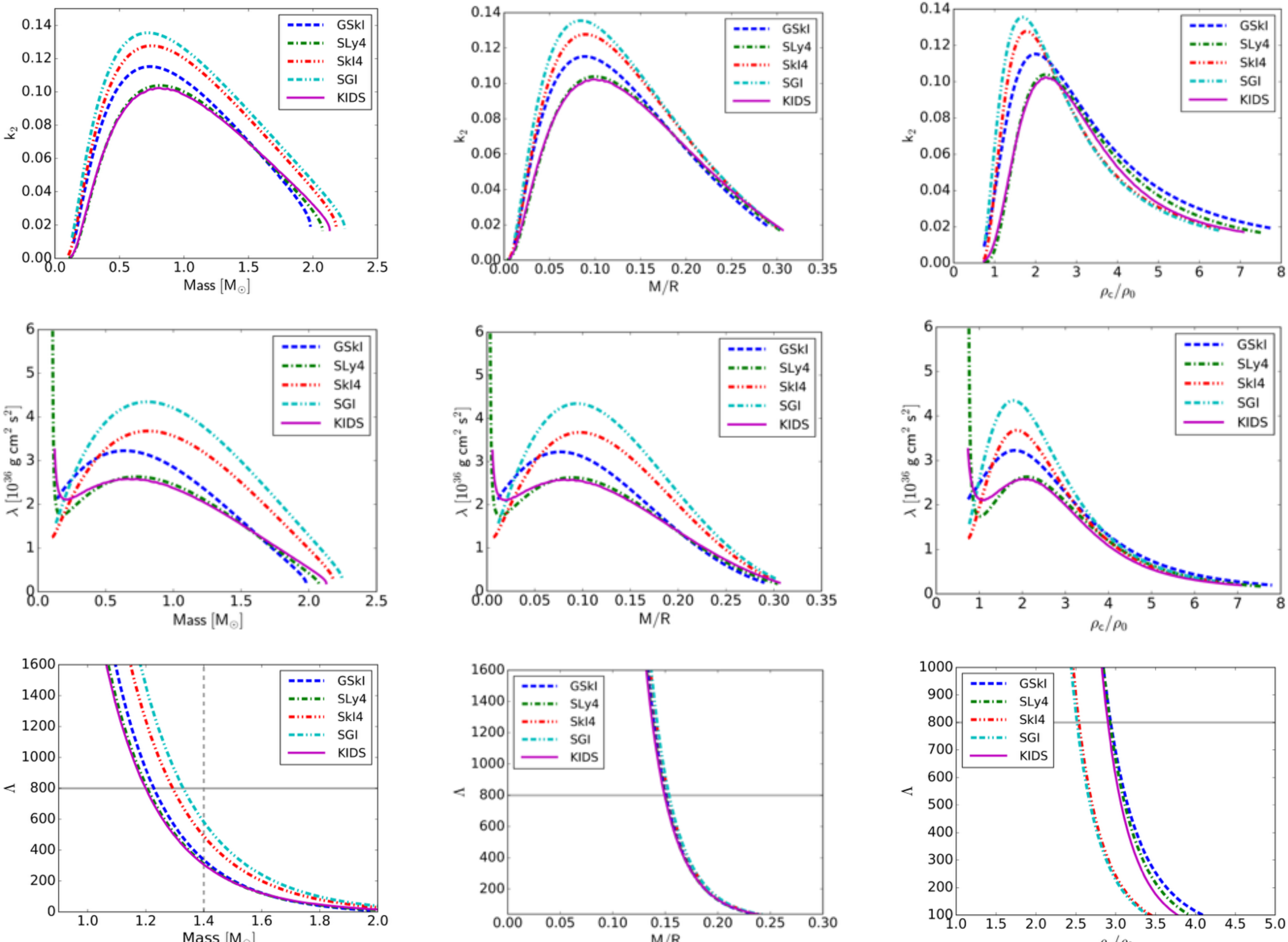
Epoch			2015 – 2016	2016 – 2017	2018 – 2019	2020+	2024+
Planned run duration			4 months	9 months	12 months	(per year)	(per year)
Expected burst range/Mpc		LIGO	40 – 60	60 – 75	75 – 90	105	105
		Virgo	—	20 – 40	40 – 50	40 – 70	80
		KAGRA	—	—	—	—	100
Expected BNS range/Mpc		LIGO	40 – 80	80 – 120	120 – 170	190	190
		Virgo	—	20 – 65	65 – 85	65 – 115	125
		KAGRA	—	—	—	—	140
Achieved BNS range/Mpc		LIGO	60 – 80	60 – 100	—	—	—
		Virgo	—	25 – 30	—	—	—
		KAGRA					
Estimated BNS detections			0.05 – 1	0.2 – 4.5	1 – 50	4 – 80	11 – 180
Actual BNS detections			0	1	—	—	—
90% CR	% within	5 deg^2	< 1	1 – 5	1 – 4	3 – 7	23 – 30
		20 deg^2	< 1	7 – 14	12 – 21	14 – 22	65 – 73
	median/deg^2		460 – 530	230 – 320	120 – 180	110 – 180	9 – 12
Searched area	% within	5 deg^2	4 – 6	15 – 21	20 – 26	23 – 29	62 – 67
		20 deg^2	14 – 17	33 – 41	42 – 50	44 – 52	87 – 90

We expect to observe more BNS and/or NS-BH

Summary

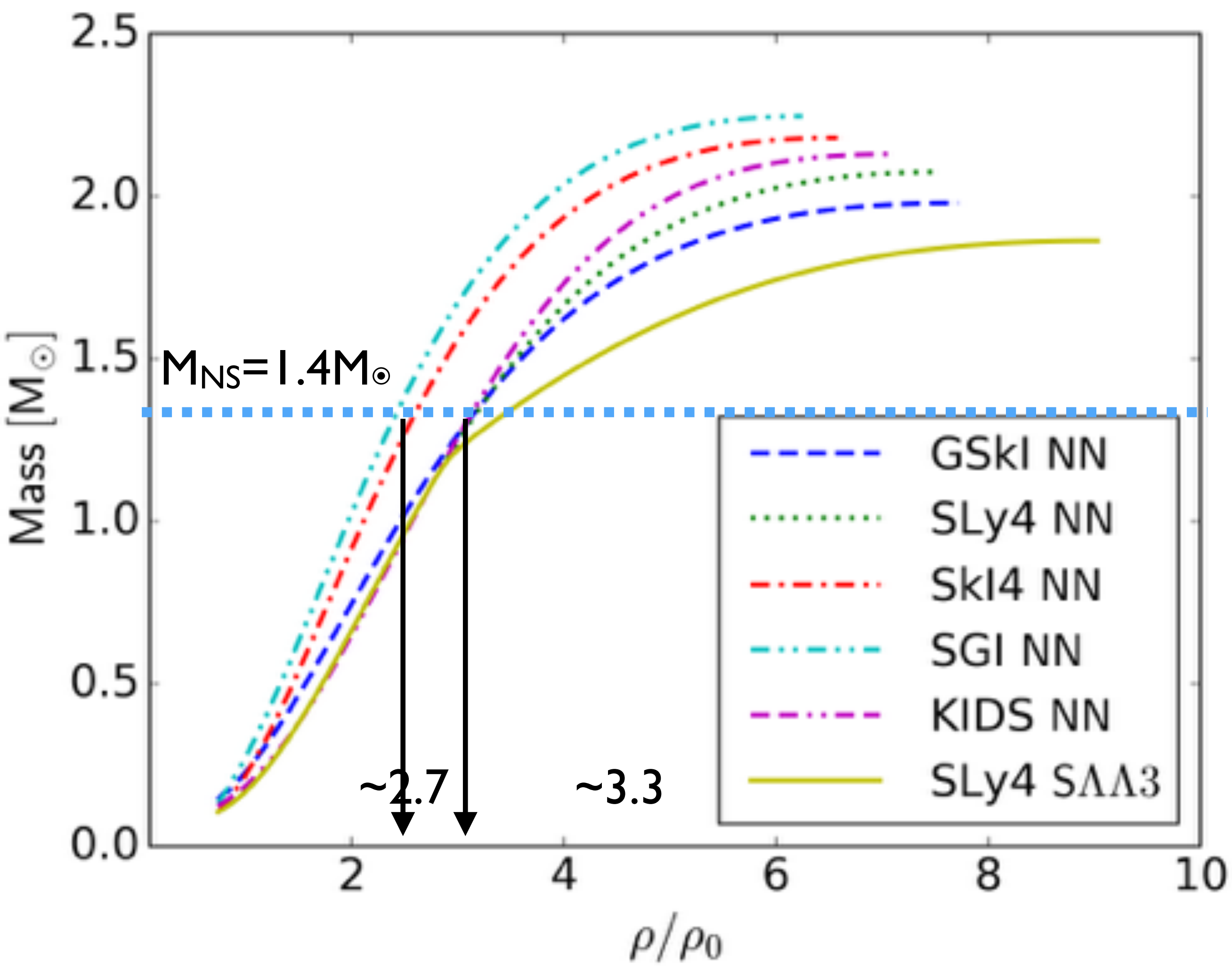
1. Tidal deformability of a neutron star can be observed by gravitational-wave detection.
    - The most dominant tidal coefficient is l=2 electric-type coefficient λ_2.
    - The weighted Λ in BNS was estimated by observation of GW170817
2. Tidal deformability is a new constraint on nuclear equation of states provided by GW observation.
    - more compact NS EoS is preferred.
3. Further investigation on Λ and NS EoS will be conducted by using MC simulation and Bayesian analysis.

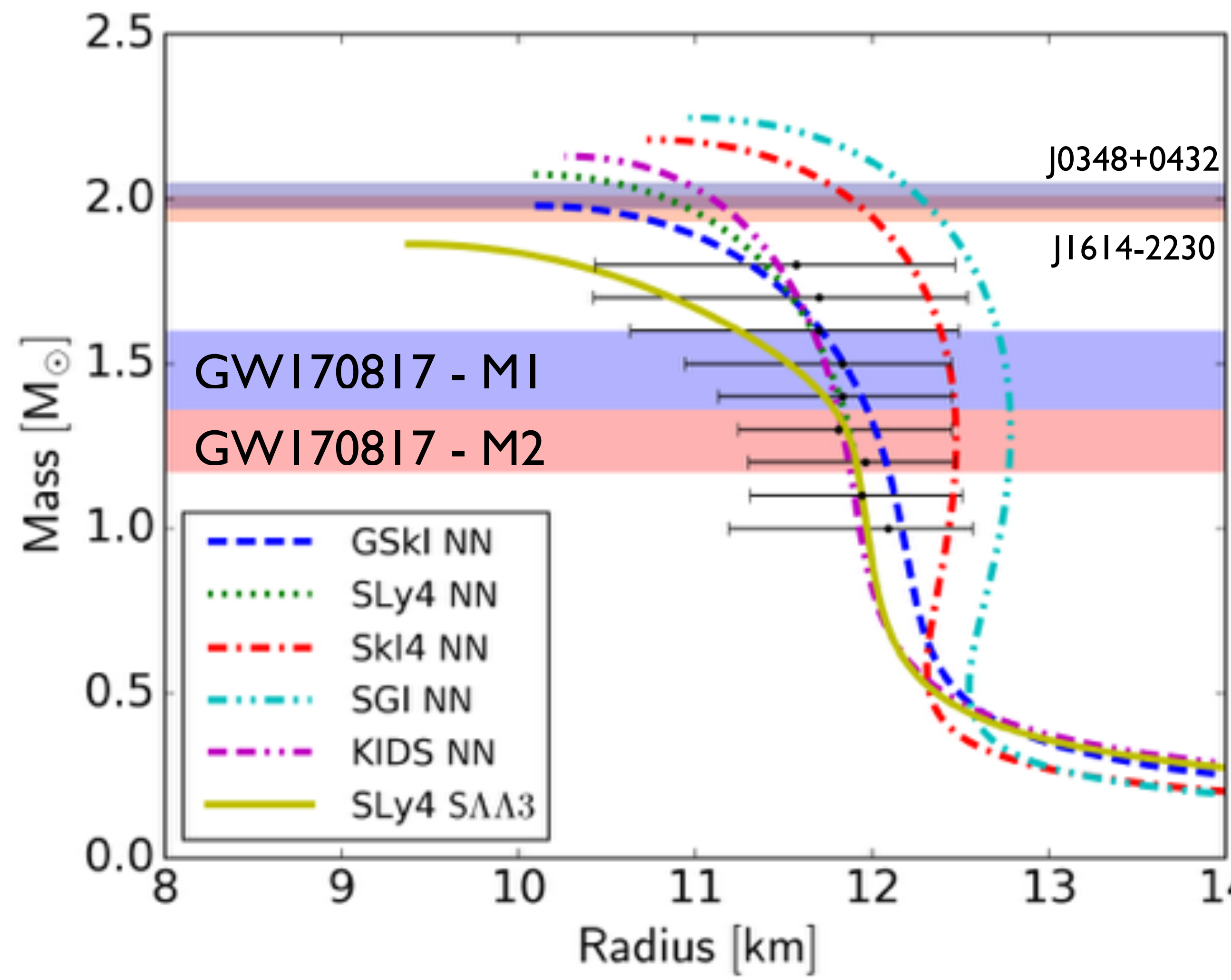
Extra Slides



GSkI
SLy4
SkI4
SGI
KIDS
k_2
Mass [$M_\odot$]
M/R
ρ_c/ρ_0
$\lambda\ [10^{36}\ \mathrm{g\ cm^2\ s^2}]$
Λ

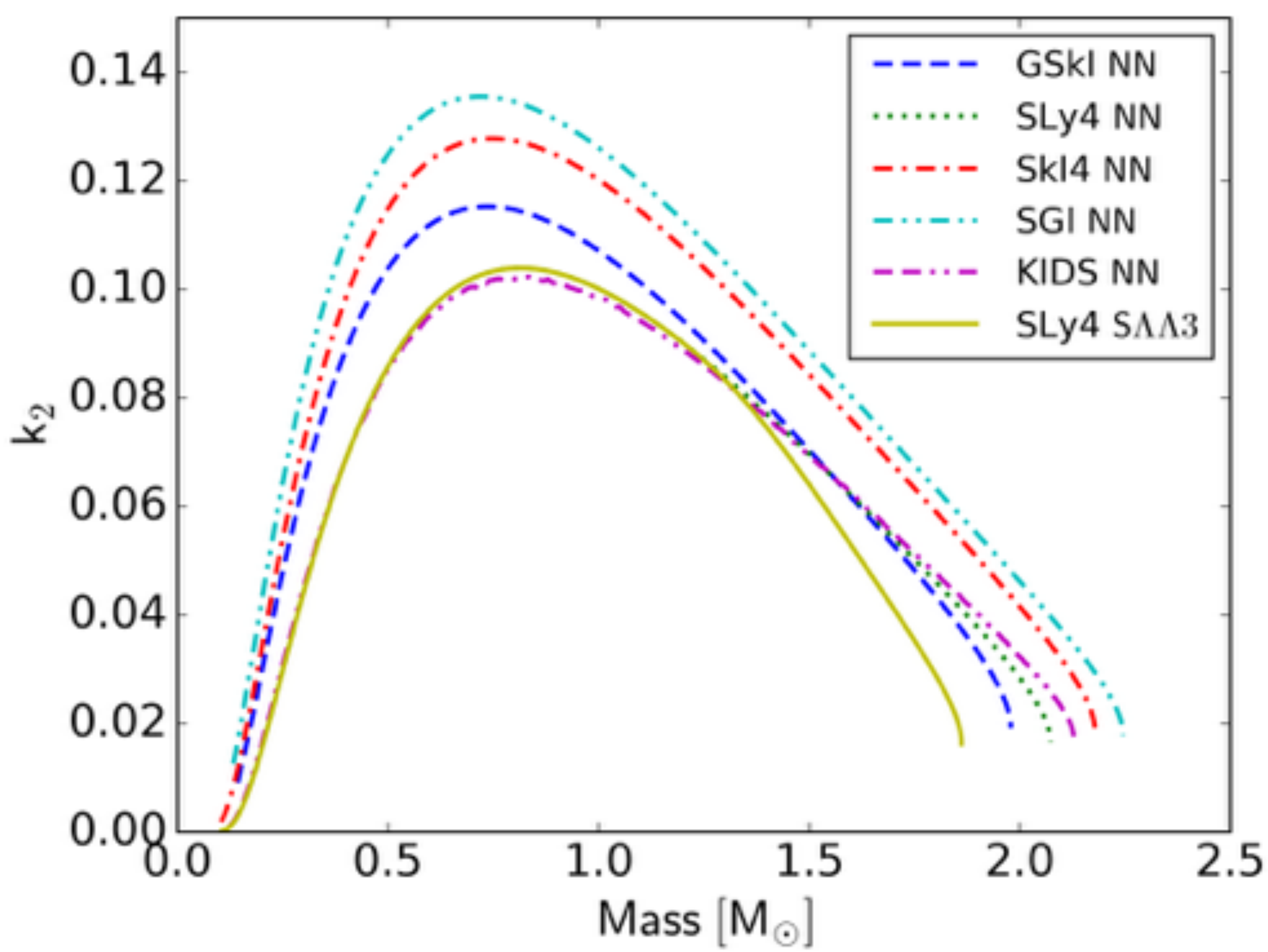
Mass-Radius relations

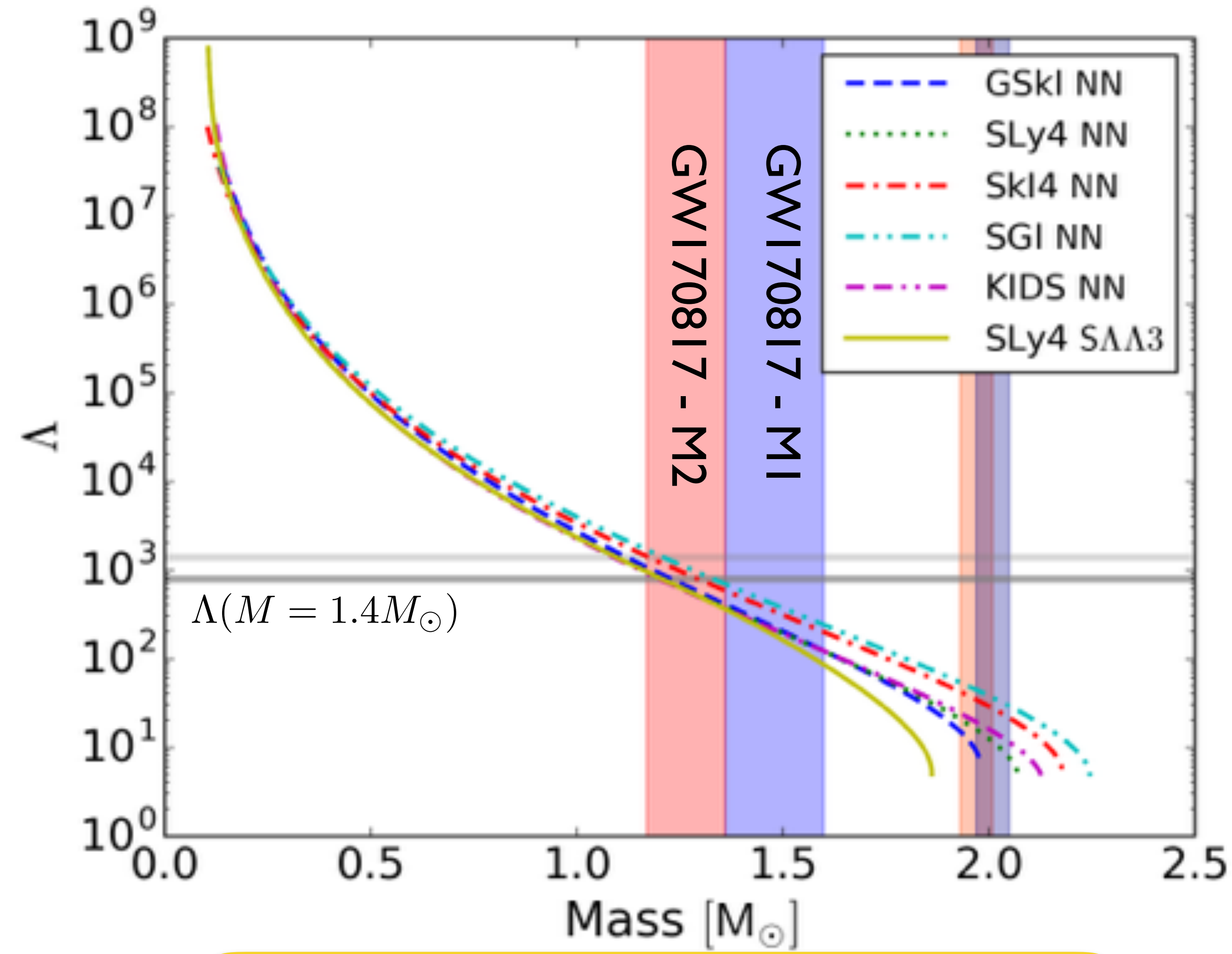




GW170817
- Mchirp = 1.188 $M_\odot$
- low spin prior : M1 = 1.36 ~ 1.60 $M_\odot$, M2 = 1.17 ~ 1.36 $M_\odot$
- high spin prior : M1 = 1.36 ~ 2.26 $M_\odot$, M2 = 0.86 ~ 1.36 $M_\odot$

Tidal deformability of a NS

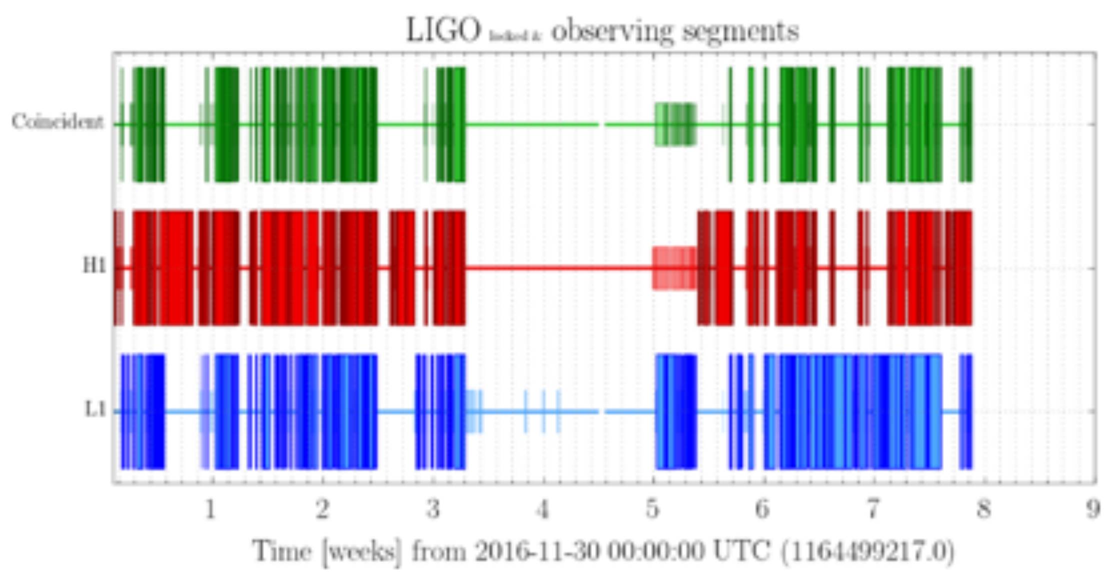


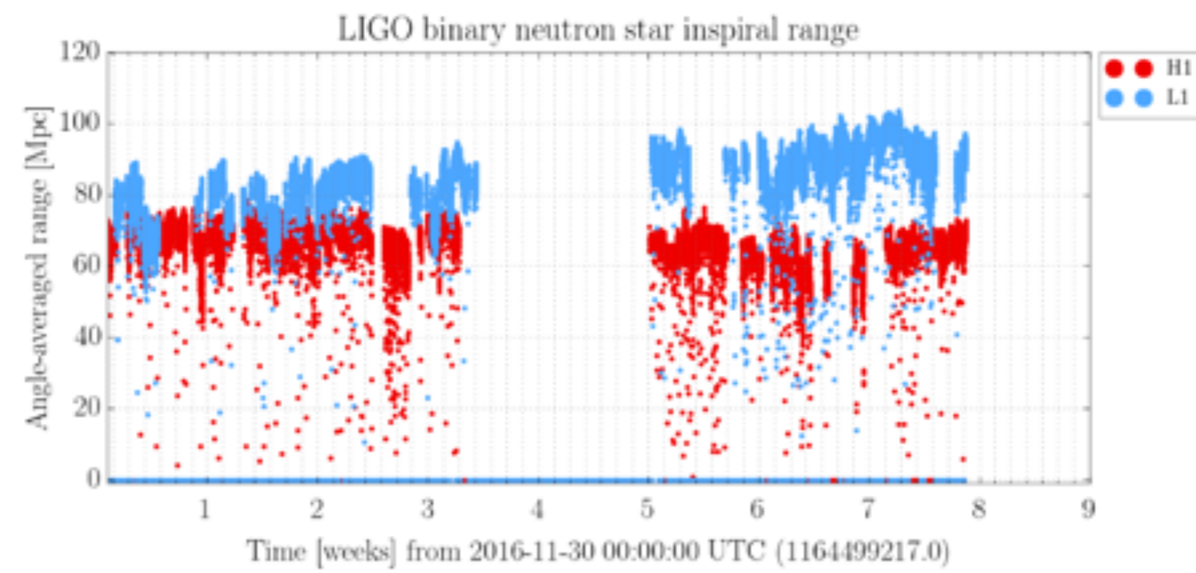


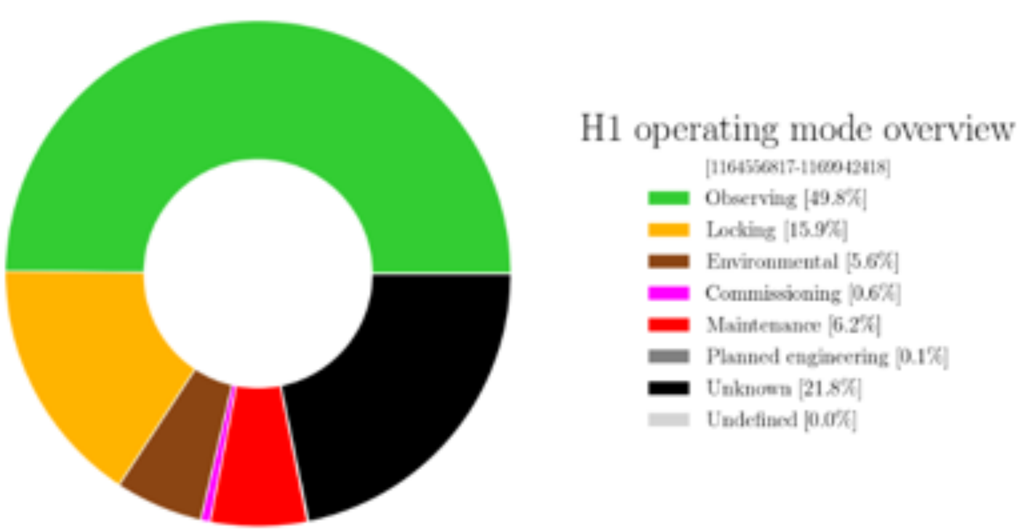
GW170817, Mchirp = 1.188 M⊙
- low spin prior : Lambda(1.4 M⊙) < 800
- high spin prior : Lambda(1.4 M⊙) < 1400

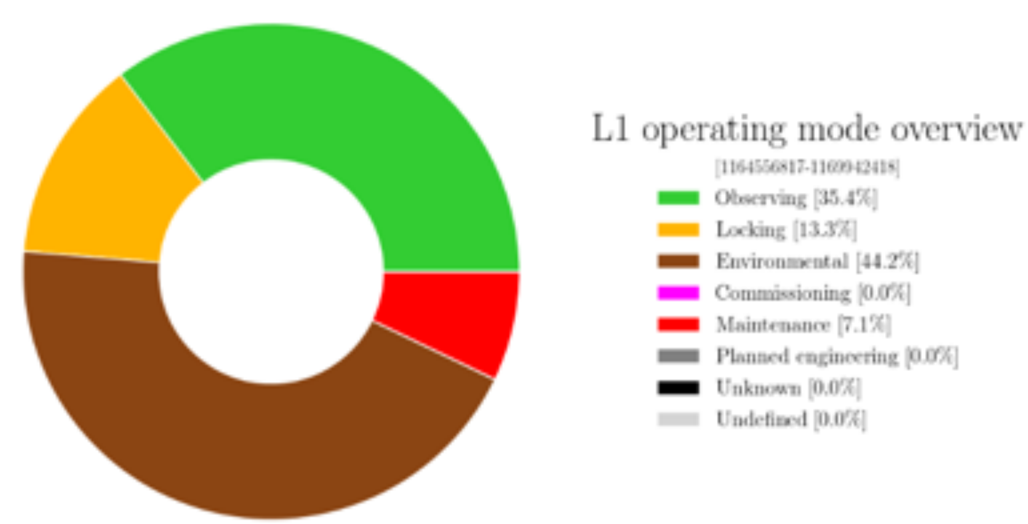
Kim et al. in preparation

O2 Summary


LIGO observing segments
Coincident
H1
L1
1 2 3 4 5 6 7 8 9
Time [weeks] from 2016-11-30 00:00:00 UTC (1164499217.0)


LIGO binary neutron star inspiral range
Angle-averaged range [Mpc]
0 20 40 60 80 100 120
H1
L1
1 2 3 4 5 6 7 8 9
Time [weeks] from 2016-11-30 00:00:00 UTC (1164499217.0)


H1 operating mode overview
[1164556817-1169942418]
Observing [49.8%]
Locking [15.9%]
Environmental [5.6%]
Commissioning [0.6%]
Maintenance [6.2%]
Planned engineering [0.1%]
Unknown [21.8%]
Undefined [0.0%]


L1 operating mode overview
[1164556817-1169942418]
Observing [35.4%]
Locking [13.3%]
Environmental [44.2%]
Commissioning [0.0%]
Maintenance [7.1%]
Planned engineering [0.0%]
Unknown [0.0%]
Undefined [0.0%]