Nonlinear Dynamics in Modified/Alternative Theories of Gravity

Hyun Lim

CCS-2 and CTA, Los Alamos National Laboratory

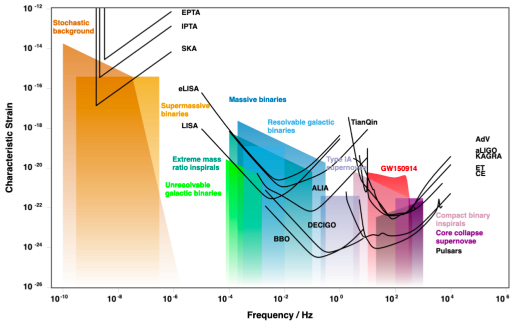
The 64th Workshop on Gravitational Waves and Numerical Relativity
March 31, 2022

Outline

- Introduction/Motivation : Why?
- Particular example : Quadratic gravity
  - Formulations : Well-posed initial value problem
  - Stability
- Discussion/Conclusion
- (Optional) High level introduction to researches at CTA/LANL (mostly compact merger/kilonova)

Introduction

- New era to investigate the universe using gravitational waves with EM counterparts
- Enable to test general relativity and beyond in strong field regime



Why test General Relativity?

$$G_{ab} = 8\pi T_{ab}$$

General relativity is successful but incomplete

- Cannot have mix of quantum/classical
- GR is not renormalizable
- GR+QM = new physics (ex BH information paradox problem)
- Alternatives/extensions of GR?

How to Perform the Tests?

How to Perform the Tests?

Empiricism : Ask nature

How to Perform the Tests?

Empiricism : Ask nature

Currently, we have

- Weak-field precision tests
- Lots of theories $\simeq$ GR
- Need to explore strong-field : Strong curvature, non-linear, dynamics

How to Perform the Tests?

Empiricism : Ask nature

Currently, we have

- Weak-field precision tests
- Lots of theories $\simeq$ GR
- Need to explore strong-field : Strong curvature, non-linear, dynamics

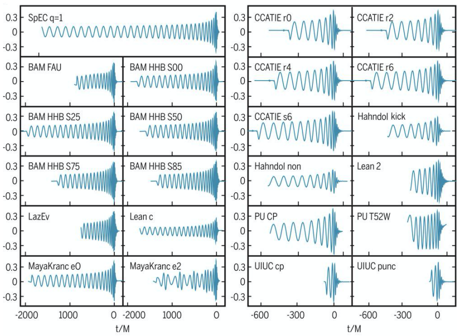
Now we can do precision tests of GR in the strong field using GW

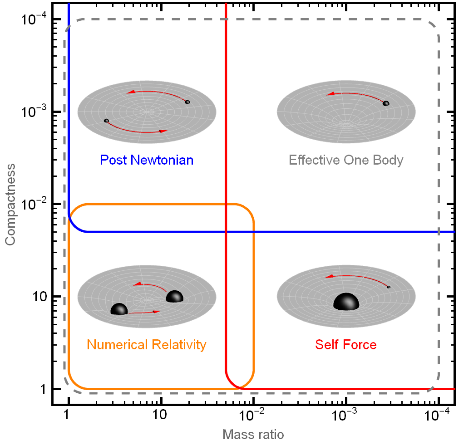
- Study BH dynamics with different theories (including binary BH merger)

Approaches to Studying Modified/Alternative Gravity Theories

- Study exact solutions to a particular modified gravity theory
  - dynamical Chern-Simon, EdGB, massive gravity, quadratic gravity, etc
- Study effective correction to GR (Effective field theory approach)
  - Assume a set of symmetries, and then write down all terms order by order in some set of small expansion parameters consistent with those symmetries

Approaches to Studying Modified/Alternative Gravity Theories





Understanding merger phase of BBH evolution is required to fully leverage the power of GWs to discover/constrain potential modifications of GR $\Rightarrow$ need numerical relativity for modified gravity.

Numerical Relativity for Modified/Alternative Theories of Gravity

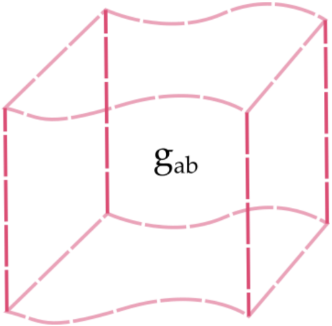
- Many different theories were suggested : scalar field theory, quasilinear theories, tensor theories, scalar-tensor theories, bimetric theories, massive gravity, non-metric theories...

$$S = \int d^4x \sqrt{|g|}[R + ...]$$

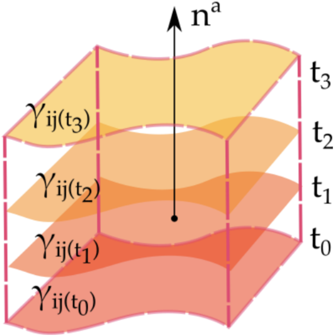
- Numerical relativity has produced first proof-of-principle simulations beyond GR[1]:
  - Scalar-tensor theories
  - Einstein-Maxwell-Dilaton models
  - cubic Horndeski theories
  - dynamical Chern-Simon theory
  - scalar Gauss-Bonnet
  - higher curvature effective theories
  - generic quadratic gravity

[1] Foucart et al. Snowmass2021 Cosmic Frontier White Paper Template

How to Study Dynamics?



Analytic relativists view of spacetime



Numerical relativists view of spacetime

$$g_{ab} = \gamma_{ab} - n_a n_b$$

How to Study Dynamics?

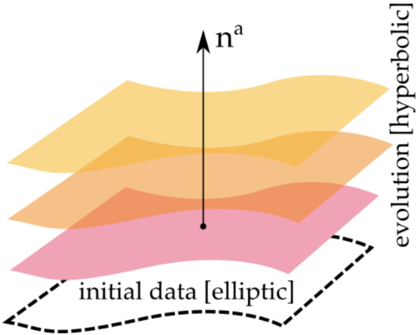
Consider GR again $G_{ab} = 8\pi T_{ab}$. Now we can decompose into:

$$\partial_t \gamma_{ij} = \ldots$$
$$\partial_t K_{ij} = \ldots$$

with

$$R + K^2 - K^{ij}K_{ij} = 16\pi\rho$$
$$D_j K_i^j - D_i K = 8\pi S_i$$

This formulation is also called Arnowitt-Deser-Misner (ADM) equations



Well-posedness

Hadamard (1902)

A problem is well-posed iff

- A solution exists
- The solution is unique
- The solution depends on continuously on initial and boundary data

Well-posedness

Example 1)

$$\partial_t^2 u - \partial_x^2 u = 0, \qquad x \in [0,1]$$
$$\text{ID} : u = 0, \;\; \partial_t u = \frac{\sin(2\pi n x)}{(2\pi n)^P}, \quad P \geq 1$$
$$\text{BC} : u = 0 \;\; \text{at} \;\; x = 0, 1$$

Well-posedness

Example 1)

$$\partial_t^2 u - \partial_x^2 u = 0, \qquad x \in [0,1]$$

$$\mathrm{ID}: u = 0,\ \ \partial_t u = \frac{\sin(2\pi n x)}{(2\pi n)^P}, \quad P \geq 1$$

$$\mathrm{BC}: u = 0 \ \text{ at } \ x = 0, 1$$

Solution:

$$u(x,t) = \frac{\sin(2\pi n x)\sin(2\pi n t)}{(2\pi n)^{P+1}}$$

For $n \to \infty$, ID $\to 0$ and $u(x,t) \to 0$ $\quad \Rightarrow$ **well-posed**

Well-posedness

Example 2)

$$\partial_t^2 u + \partial_x^2 u = 0, \qquad x \in [0,1] \quad \text{(Just change sign)}$$

$$\text{ID} : u = 0, \;\; \partial_t u = \frac{\sin(2\pi n x)}{(2\pi n)^P}, \quad P \geq 1$$

$$\text{BC} : u = 0 \;\; \text{at} \;\; x = 0, 1$$

Well-posedness

Example 2)

$$\partial_t^2 u + \partial_x^2 u = 0, \qquad x \in [0,1]$$
$$\text{ID} : u = 0, \;\; \partial_t u = \frac{\sin(2\pi n x)}{(2\pi n)^P}, \quad P \geq 1$$
$$\text{BC} : u = 0 \;\text{ at }\; x = 0, 1$$

Solution:

$$u(x,t) = \frac{\sin(2\pi n x)\sinh(2\pi n t)}{(2\pi n)^{P+1}}$$

For $n \to \infty$, ID $\to 0$ but $u(x,t) \to \infty$ $\qquad \Rightarrow$ **ill-posed**
In other word, small perturbation at $t = 0$ produces *arbitrarily large solution* at given *finite* time

Hyperbolicity of PDEs

Consider a first order system of evolution equations of the form

$$\partial_t u^a + A_b^{ia} \partial_i u^b = B^a(u)$$

where A_b^{ia} are spatial matrix that doesn't contain derivatives of u, and $B^a(u)$ is a source vector.
Pick an arbitrary spatial unit vector n_i. Then $n_i A_b^{ia}$ is the **characteristic matrix** in direction n_i
Let $e_a^{\hat{\alpha}}$ be the $\hat{\alpha}$the (left) eigenvector of characteristic matrix with eigenvalue $V_{(\hat{\alpha})}$:

$$e_a^{(\hat{\alpha})}(n_i A_b^{ia}) = V_{(\hat{\alpha})} \quad (\text{no sum on } \hat{\alpha})$$

then $V_{(\hat{\alpha})}$ are called **characteristic speeds**, and $u^{\hat{\alpha}} = u^a e_a^{(\hat{\alpha})}$ are called **characteristic field**

Hyperbolicity of PDEs

- A 1st order PDE system is **weakly hyperbolic** if all eigenvalues are real in any arbitrary direction n_i
  For weakly hyperbolic system, well-posedness depends on details of non-principal terms (ill-posed in many cases)
- A 1st order PDE system is **strongly hyperbolic** if all eigenvalues are real, and there is a complete set of linearly independent eigenvectors in any arbitrary direction n_i and independent of the solution

Hyperbolicity of PDEs

If a system is strongly hyperbolic:

- It is well posed
- At a boundary with outgoing normal n_i
  - Boundary conditions **must** be imposed on all characteristic field with $V_{(\hat{\alpha})} < 0$
  - Boundary conditions **must not** be imposed on all characteristic field with $V_{(\hat{\alpha})} > 0$ (failure to obey these could kill well-posedness)

A 1st order PDE system is **symmetric hyperbolic** if there exists a positive-definite symmetric matrix, S_{ab} (a symmetrizer), such that $S_{ab}A_c^{ib}$ is symmetric on a and c for all i.

Symmetric hyperbolic implies strongly hyperbolic

Hyperbolicity of Einstein's Equations

ADM is only weakly hyperbolic $\Rightarrow$ No well-posedness
Now, there are multiple formulations that satisfy well-posedness and use in numerical simulations

- Generalized Harmonics
- Baumgarte-Shaprio-Shibata-Nakamura (BSSN)
- Conformal and covariant Z4(CCZ4)/Z4

Numerical Relativity for Modified/Alternative Theories of Gravity

Challenges : Devising mathematically well-posed and numerically stable formulations of the evolution equations

Approaches:

- Recast as the evolution of a scalar field coupled to the usual equations of GR (scalar-tensor, Maxwell-dilaton, boson stars etc)
- Treating beyond-GR effects perturbatively e.g. dynamical (de)scalarization of black holes in scalar Gauss Bonnet gravity
- Find well-posed formulation : Horndeski class, quadratic gravity

Quadratic Gravity

The action for quadratic gravity is

$$S_{\text{QG}} = \int d^4x \sqrt{|g|} \left[\frac{1}{16\pi G} R + \alpha R_{ab} R^{ab} - \beta R^2 \right]$$

$$m_0^2 = \frac{1}{32\pi G(3\beta - \alpha)} , \qquad m_2^2 = \frac{1}{16\pi G\alpha}$$

- Perturbatively renormalizable (Stelle, Phy.Rev.D 1977)
- In the linearized theory, rise massive scalar mode, and massive spin 2 mode
- Massive spin-2 mode introduces Ostrogadski instability

Ostrogradski Instability

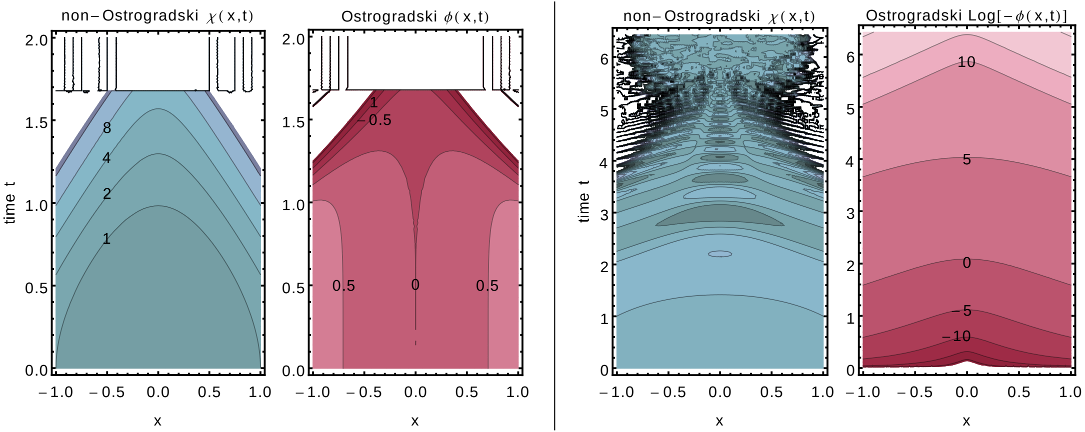
A feature of some solutions for certain theories having EOM with more than second order time derivatives exhibits unstable behavior during evolution. But, this does not imply that all solutions of higher derivative theories are unstable
Consider a scalar theory in 1+1 dimensions

$$\mathcal{L} = \frac{1}{2}(\Box\phi)^2 - \frac{1}{2}\chi\Box\chi + V(\phi,\chi)$$

where $V(\phi,\chi)$ is a priori arbitaray non-derivative potential. The EOM is

$$\Box(\Box\phi) = -\partial_\phi V(\phi,\chi)$$
$$\Box\chi = -\partial_\chi V(\phi,\chi)$$

Ostrogradski Instability



Time evolution of two coupled scalar fields determined by higher-derivative equation of motion for ϕ and one second-order equation of motion for χ, coupled by a potential $V(\phi,\chi) = \phi^2\,\chi^2$ (two left-hand panels) and $V(\phi,\chi) = \phi\,\chi^2$ (two right-hand panels).

Why Quadratic Gravity?

There are very few modified gravity theories that[2] :

- Are consistent with GR in regimes where it is well tested
- Predict observable in the dynamical, strong field regime relevant to BH mergers
- Posses a well-posed initial value problem

[2] Lehner and Pretorius, Ann.Rev.Astron.Astrophys. 52 (2014) 661-694

Why Quadratic Gravity?

There are very few modified gravity theories that:

- Are consistent with GR in regimes where it is well tested
  - $\Rightarrow$ QG can be constrained (not highly), solar system, CEMZ etc
- Predict observable in the dynamical, strong field regime relevant to BH mergers
  - $\Rightarrow$ QG have exotic BH solutions such as non-Schwarzschild solution for spherical symmetry. Could show big deviation from GR due to higher order curvature
- Posses a well-posed initial value problem
  - $\Rightarrow$ Motivation for this work. (Noakes, J. Math. Phy. 1983) proved well-posed IVP in QG. We built well-posed IVP in spherical symmetry (also 3+1 case)

Quadratic Gravity

$$S_{\mathsf{QG}} = \int d^4x \sqrt{|g|} \left[\frac{1}{16\pi G} R + \alpha R_{ab} R^{ab} - \beta R^2 \right]$$

$$m_0^2 = \frac{1}{32\pi G(3\beta - \alpha)} \,, \qquad m_2^2 = \frac{1}{16\pi G\alpha}$$

Would like to study dynamics in QG

Well-posed IVP

Noakes founds well-posed IVP (Noakes J.Math.Phys 1983):
Second order quasi-linear hyperbolic system (Choquet-Bruhat Acta Math. 1952)

$$g^{ab}(x,t,u_i)\frac{\partial^2 u_q}{\partial x^a \partial x^b} = f_q(u_i, \partial u_i)$$

In a given quasilinear, diagonal, second order hyperbolic system with constraints on initial data, it posses well-posed initial value formulation

Well-posed IVP for QG

EOM of QG is

$$H_{ab} = \frac{1}{16\pi} G_{ab} + E_{ab} = \frac{1}{2} T_{ab}$$

where G_{ab} is usual Einstein tensor and E_{ab} is quadratic order counter part

$$E_{ab} = (\alpha - 2\beta)\nabla_a \nabla_b R - \alpha \Box R_{ab} - \left(\frac{1}{2}\alpha - 2\beta\right) g_{ab} \Box R + 2\alpha R^{cd} R_{acbd}$$
$$- 2\beta R R_{ab} - \frac{1}{2} g_{ab} (\alpha R_{cd} R^{cd} - \beta R^2)$$

Well-posed IVP for QG

Introduce traceless part of Ricci tensor such that

$$R_{ab}(g) = \widetilde{\mathcal{R}}_{ab} + \frac{1}{4} g_{ab} \mathcal{R} \,,$$

$$\Box\, \mathcal{R} = m_0^2 \mathcal{R} + 2 T^c{}_c \,,$$

$$\begin{aligned}
\Box\, \widetilde{\mathcal{R}}_{ab} = {} & m_2^2 \widetilde{\mathcal{R}}_{ab} + 2 T^{\text{(TL)}}_{ab} \\
& - \frac{1}{3} \left(\frac{m_2^2}{m_0^2} - 1 \right) \left(\nabla_a \nabla_b \mathcal{R} - \frac{1}{4} g_{ab} m_0^2 \mathcal{R} \right) \\
& + 2 \widetilde{\mathcal{R}}^{cd} C_{acbd} - \frac{1}{3} \left(\frac{m_2^2}{m_0^2} + 1 \right) \mathcal{R} \widetilde{\mathcal{R}}_{ab} \\
& - 2 \widetilde{\mathcal{R}}_a{}^c \widetilde{\mathcal{R}}_{bc} + w \frac{1}{2} g_{ab} \widetilde{\mathcal{R}}^{cd} \widetilde{\mathcal{R}}_{cd}
\end{aligned}$$

Well-posed IVP for QG

Employ generalized harmonic coordinates $\Box x^a = F^a$. In terms of harmonic coordinate, EOM for QG can be written

$$g^{cd} g_{ab,cd} = -2\widetilde{\mathcal{R}}_{ab} - \frac{1}{2} g_{ab}\mathcal{R} + \mathcal{O}^1_{ab}(\partial g) \ ,$$

$$g^{cd}\mathcal{R}_{,cd} = m_0^2 \mathcal{R} \ ,$$

$$g^{cd}\widetilde{\mathcal{R}}_{ab,cd} = \mathcal{O}^2_{ab}(\partial\partial\mathcal{R}, \partial\widetilde{\mathcal{R}}, \partial\partial g)$$

Well-posed IVP for QG

Introducing additional variable $V_a \equiv \mathcal{R}_{,a}$ and $h_{abc} \equiv g_{ab,c}$ then

$$g^{mn} V_{a,mn} = \mathcal{O}_a(\partial V, h) \; ,$$
$$g^{mn} h_{abc,mn} = \mathcal{O}_{abc}(\partial h) \; ,$$
$$g^{mn} \widetilde{\mathcal{R}}_{ab,mn} = \mathcal{O}^2_{ab}(\partial V, \partial h, \partial \widetilde{\mathcal{R}})$$

These are well-posed form of the evolution equations

Reduction to Spherical Symmetry

Choosing coordinates in which spherical symmetry is explicit makes it impossible to maintain the harmonic gauge condition $\Box x^a = 0$
Remaining in Cartesian coordinates and applying **Cartoon** method

$$\xi_1^\mu = x(\boldsymbol{\partial}_y)^\mu - y(\boldsymbol{\partial}_x)^\mu \ ,$$
$$\xi_1^\mu = y(\boldsymbol{\partial}_z)^\mu - z(\boldsymbol{\partial}_y)^\mu \ ,$$
$$\xi_1^\mu = z(\boldsymbol{\partial}_x)^\mu - x(\boldsymbol{\partial}_z)^\mu \ .$$

where ξ_i^μ are Killing vector fields associated with spherical symmetry

Reduction to Spherical Symmetry

We can apply usual coordinate transformation for tensors

$$\Pi_{\bar{a}\bar{b}} = \frac{\partial X^a}{\partial \overline{X}^{\bar{a}}} \frac{\partial X^b}{\partial \overline{X}^{\bar{b}}} \Pi_{ab}$$

where $\overline{X} = (t, r, \theta, \phi)$ is spherical coordinate and $X = (t, x, y, z)$ is Cartesian coordinate.
We can have a new set of evolution equation

$$\partial_t^2 \mathbf{u} = \mathcal{O}\left(\mathbf{u},\, \mathbf{v},\, \partial_t \mathbf{u}\right) \ ,$$
$$\partial_t^2 \mathbf{v} = \mathcal{O}\left(\mathbf{u},\, \mathbf{v},\, \partial_t \mathbf{u},\, \partial_t \mathbf{v},\, \partial_t^2 \mathbf{u}\right)$$

with

$$\mathbf{u} = (\mathcal{R}, g_{tt}, g_{tx}, g_{xx}, g_{yy}) \text{ and } \mathbf{v} = (\widetilde{\mathcal{R}}_{tt}, \widetilde{\mathcal{R}}_{tx}, \widetilde{\mathcal{R}}_{xx})$$

Reduction to Spherical Symmetry

We finally introduces additional auxiliary variables to make quasi-linear 1st order form (in analogy to the 4D case) such that

$$\dot{\mathbf{u}} = (\dot{\mathcal{R}},\, \dot{g}_{tt},\, \dot{g}_{tx},\, \dot{g}_{xx},\, \dot{g}_{yy}) \equiv \partial_t \mathbf{u}$$
$$\ddot{\mathbf{u}} \equiv \partial_t \dot{\mathbf{u}} \quad \text{and} \quad \dot{\mathbf{v}} \equiv \partial_t \mathbf{v}$$

Thus, the evolution equations are

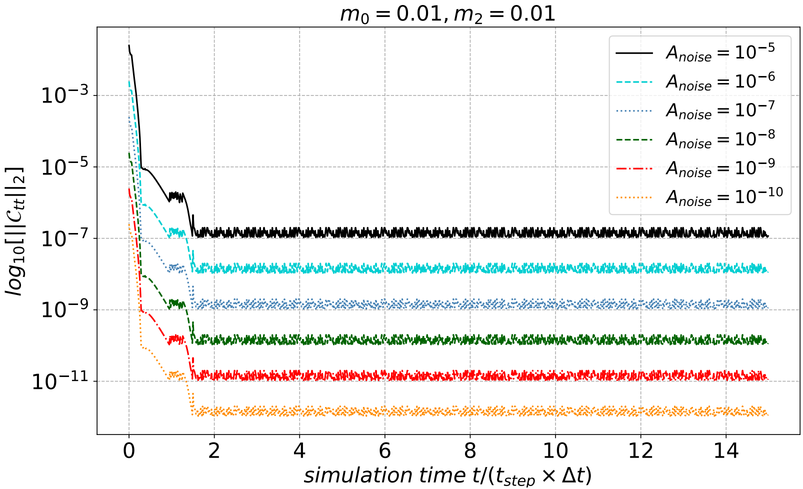
$$\begin{aligned}
\partial_t \ddot{\mathbf{u}} &= \mathcal{O}\left(\mathbf{u},\, \mathbf{v},\, \dot{\mathbf{u}},\, \ddot{\mathbf{u}},\, \dot{\mathbf{v}}\right)\ , \\
\partial_t \dot{\mathbf{v}} &= \mathcal{O}\left(\mathbf{u},\, \mathbf{v},\, \dot{\mathbf{u}},\, \ddot{\mathbf{u}},\, \dot{\mathbf{v}}\right)\ , \\
\partial_t \dot{\mathbf{u}} &\equiv \ddot{\mathbf{u}}\ , \\
\partial_t \mathbf{u} &\equiv \dot{\mathbf{u}}\ , \\
\partial_t \mathbf{v} &\equiv \dot{\mathbf{v}}\ , \\
\ddot{\mathbf{u}} &= \mathcal{O}\left(\mathbf{u},\, \mathbf{v},\, \dot{\mathbf{u}}\right)
\end{aligned}$$

Numerical Setup

- All explicit symbolic expressions were obtained via Mathematica with xAct package (Can be found in `https://github.com/aaron-hd/QG-sphSymm-ancillary`
- Applying fourth order finite differences for spatial derivatives and a fourth order Runge-Kutta method to evolve in time
- Simulations were performed with unigrid
- Flat spacetime and Schwarzschild black holes were used as initial data
- Adding random noises into initial data to check robust stability
- Monitoring constraint evolution

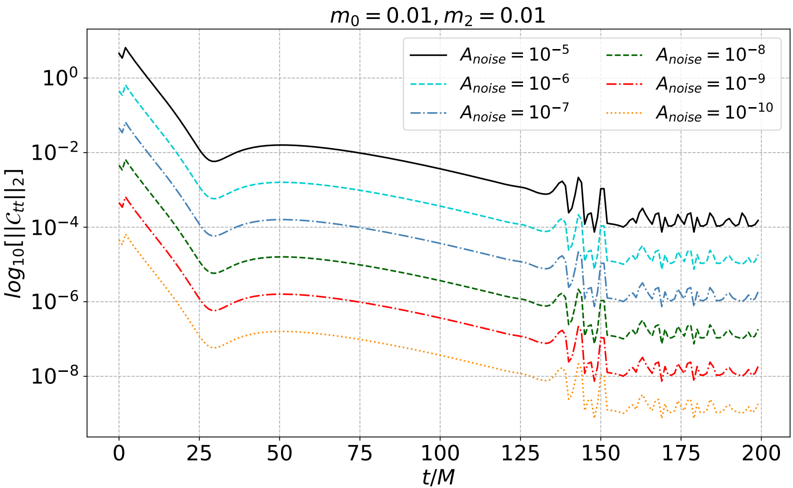
Results : Flat Spacetime

Checking evolution: add random noise into ID



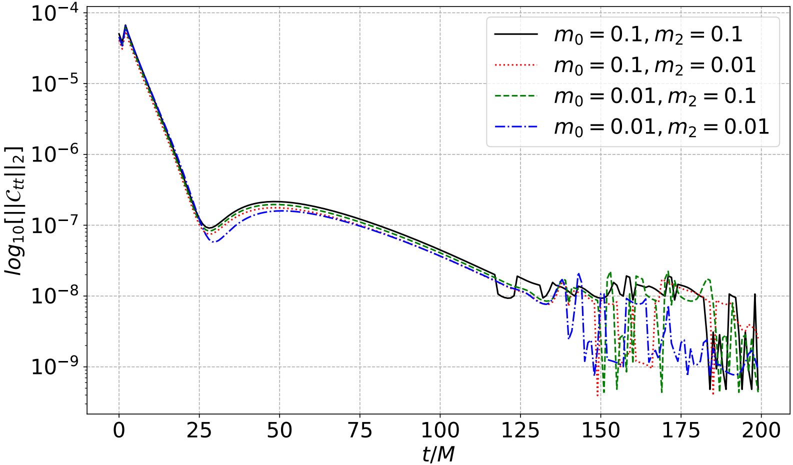
Results : Schwarzschild BH

Checking evolution: add random noise into ID



Results : Schwarzschild BH

Varying QG mass coupling parameter



Convergence Tests

We ”solved” our system numerically. Is this consistent with continuum PDE system?
By convergence, we mean:

- the difference between numerical approximation provided by a numerical scheme and the exact solution of the continuum PDE system tends to zero as the resolution is increased
- When the numerical scheme approximates the correct PDE system, it is called **consistent**, and the degree to which this is achieved is its **accuracy**

Convergence Tests

$$C_{\text{self}} = \log_d \frac{||\mathbf{u}_{h_c} - \perp_{h_c}^{h_c/d} \mathbf{u}_{h_c/d}||_{h_c}}{||\perp_{h_c}^{h_c/d} \mathbf{u}_{h_c/d} - \perp_{h_c}^{h_c/d^2} \mathbf{u}_{h_c/d^2}||_{h_c}}$$

where $\mathbf{u}$ is the state vector of PDE system, h_c is the grid spacing, d is order of numerical scheme, and $\perp_{h_c}^{h_c/d}$ denotes the projection operator from h_c/d grid onto h_c grid.
If we know the exact solution, we can evaluate exact convergence radio

$$C_{\text{self}} = \log_d \frac{||\mathbf{u}_{h_c} - \mathbf{u}_{\text{exact}}||_{h_c}}{||\perp_{h_c}^{h_c/d} \mathbf{u}_{h_c/d} - \mathbf{u}_{\text{exact}}||_{h_c}}$$

Why norm is the matter?

The notion of stability (or say well-posedness) for a fully first order system based on discrete L_2 norm might not be suitable for certain system.
A system of equation would be stable (well-posed) if the norm of the solution is bounded by the norm of the initial data in terms of constants independent of the initial data such that

$$||u(x^i, t)||_2 \leq ae^{bt}||f(x^i)||_2$$

where a, b are the same constants for all initial data $f(x^i) = u(x^i, 0)$

Why norm is the matter?

Suppose a wave equation in 1+1 equation in first order in time and second order in space

$$\partial_t \phi(x,t) = \Pi(x,t)$$
$$\partial_t \Pi(x,t) = \partial_x^2 \phi(x,t)$$

Consider, for simplicity, the case of solutions of periodicity L with initial data $\phi(x,0) = \sin(\omega x), \Pi(x,0) = 0$ where $\omega = 2\pi n/L$ and n is an integer is

$$\phi(x,t) = \sin(\omega x)\cos(\omega t)$$
$$\Pi(x,t) = -\omega \sin(\omega x)\sin(\omega t)$$

Why norm is the matter?

$$||f(x^i)||_2 = \frac{1}{L}\int_0^L \phi(x,0)^2 + \Pi(x,0)^2 dx = \frac{1}{2} \text{ for all } \omega$$
$$||u(x^i,t)||_2 = \frac{1}{L}\int_0^L \phi(x,t)^2 + \Pi(x,t)^2 dx = \frac{1}{2}(\cos^2(\omega t) + \omega^2 \sin^2(\omega t))$$

so we have

$$||u(x^i,t)||_2 = (\cos^2(\omega t) + \omega^2 \sin^2(\omega t))||f(x^i)||_2$$

There are no constants a and b such that $(\cos^2(\omega t) + \omega^2 \sin^2(\omega t)) \leq ae^{bt}$ for all initial data (all ω)

Why norm is the matter?

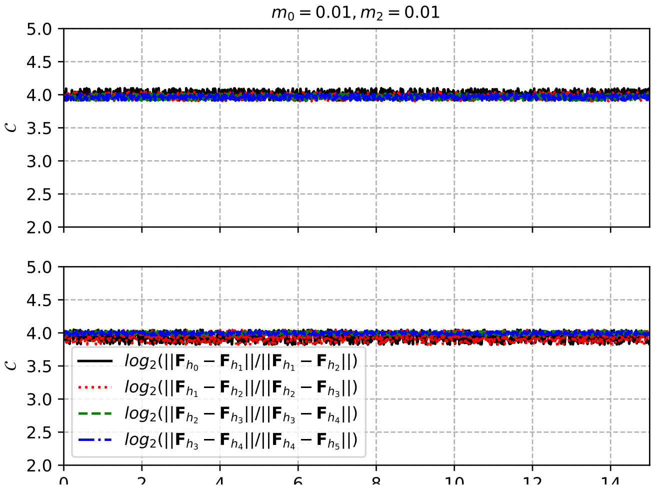
To overcome this issue, we can introduce a new variable $X = \partial_x \phi$ which allows the construction of a first order system i.e. well-posed in L_2.
The second order system can be shown to be well-posed in a norm containing derivatives such that

$$\frac{1}{L}\int_0^L \phi^2 + \pi^2 + (\partial_x \phi)^2 dx$$

This corresponds to the L_2 norm of the first order reduction

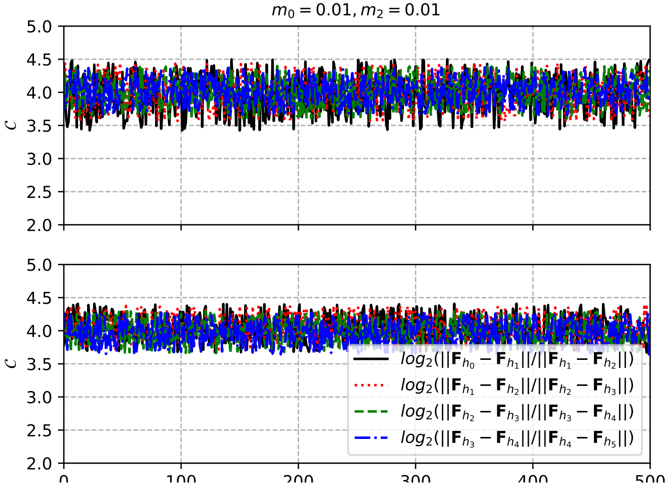
Convergence Tests

Self-convergence : Flat spacetime



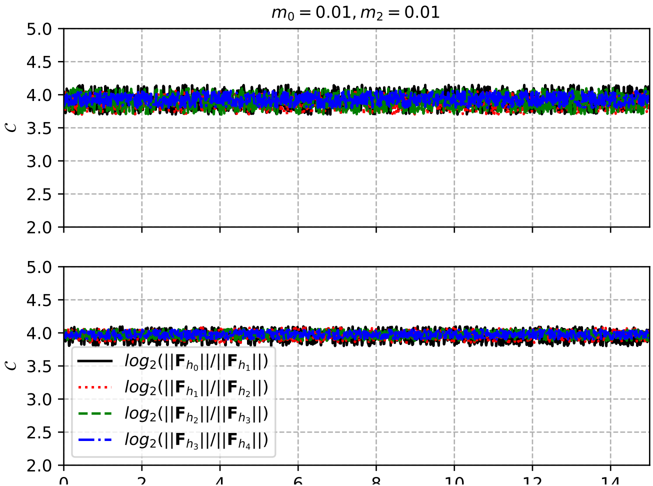
Convergence Tests

Self-convergence : Schwarzschild BH



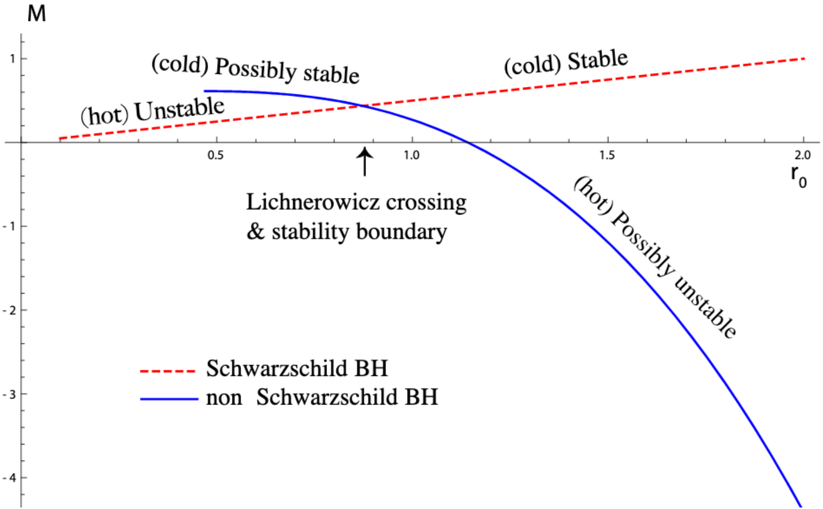
Convergence Tests

Exact-convergence



BH Stabilities

(Lü et al. Phy.Rev.D 2017) found unstable branch of BH



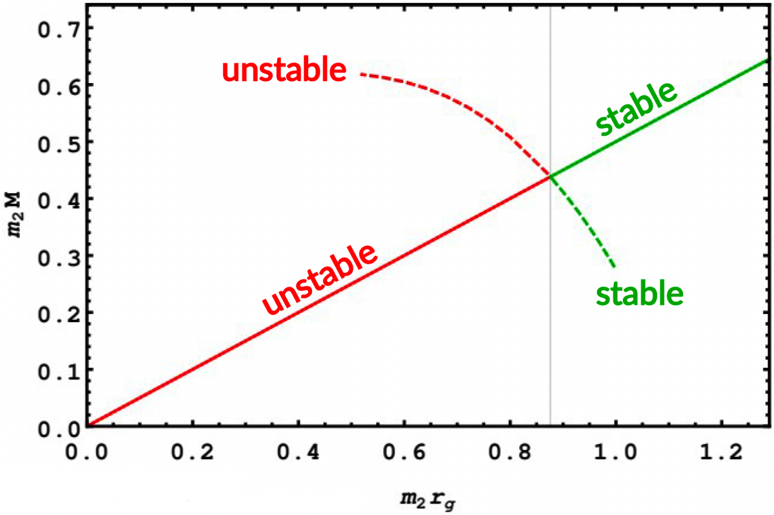
BH Stabilities

Dynamics: linear DOF

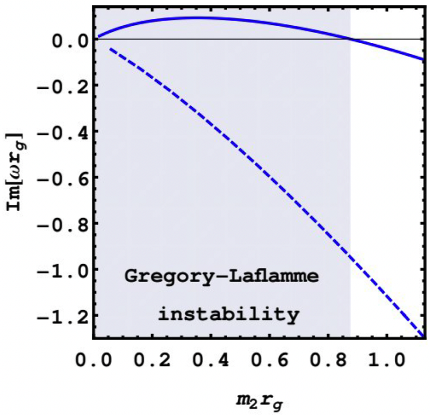
$$\mathcal{L}_{\text{EFT}} = \frac{1}{M_{pl}^2}\Bigg[\frac{1}{2}R \quad \text{massless spin-2 (graviton)} \\ + \frac{1}{12m_0^2}R^2 \quad \text{massive spin-0 (scalar)} \\ + \frac{1}{4m_2^2}C_{abcd}C^{abcd} \quad \text{massive spin-2 (ghost)}\Bigg]$$

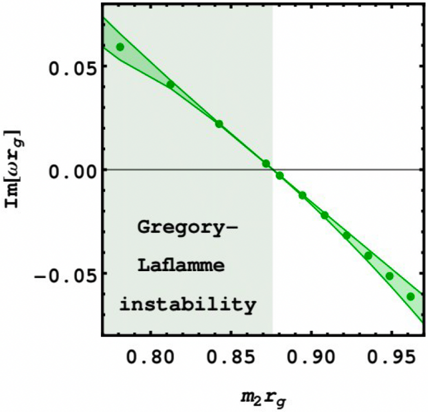
Then mode decompositioning (background) by using spherical harmonics, focus on axisymmetric monopole ($l = m = 0$).

BH Stabilities



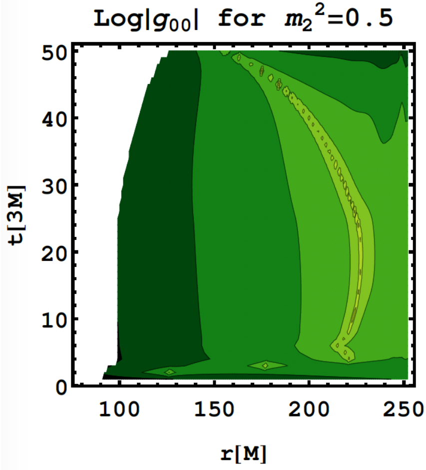
BH Stabilities

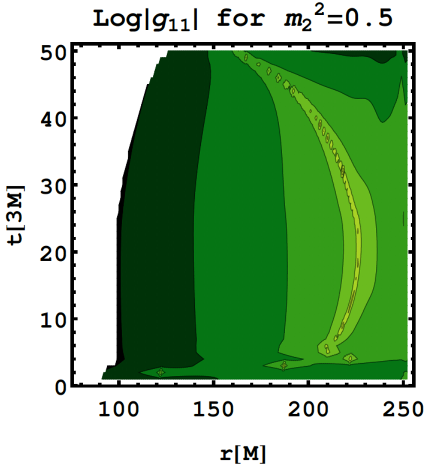




BH Stabilities

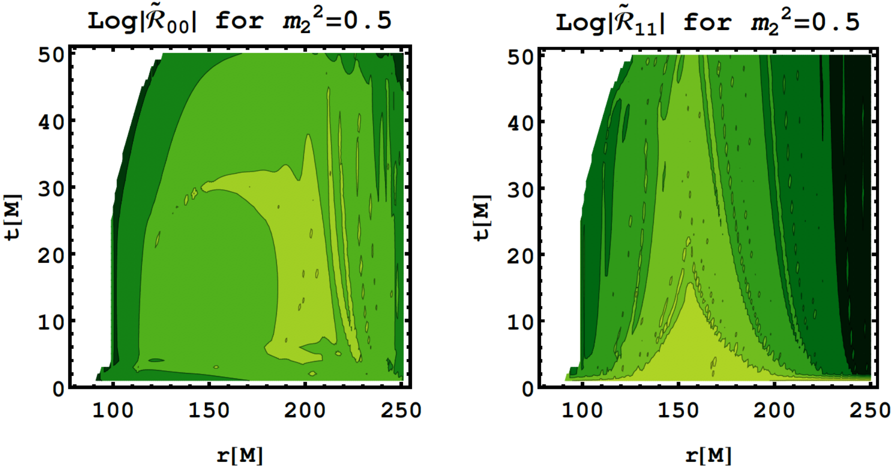
Metric component





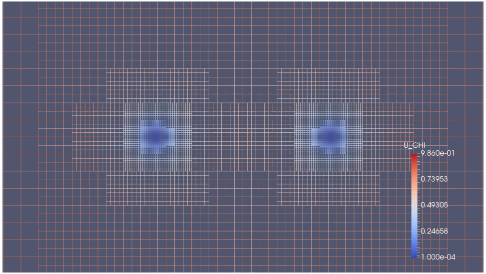
BH Stabilities

Ricci components



Discussion & Future Work

- Enable to test beyond GR theories using GWs
- First nonlinear stable numerical evolution of QG
- Explore (un)stable BH branches in QG
- Working towards to full 3+1 evolution and BBH merger (code is availabe as open source `https://github.com/lanl/Dendro-GRCA`)



10bbh-c2/bbh001145
BBH merger is performed with wavelet AMR code Dendro

Well-posed IVP in 3+1

Introduce

$$\widetilde{V}_{ab} \equiv -n^c \nabla_c \widetilde{\mathcal{R}}_{ab} ,$$
$$\hat{\mathcal{R}} \equiv -n^c \nabla_c \mathcal{R} .$$

for the fiducial Ricci variables.
For $\widetilde{\mathcal{R}}_{ab}$ and its 1st-order variable $\widetilde{V}_{ab}$, we have

$$0 = n^a n^b \widetilde{\mathcal{R}}_{ab} , \qquad 0 = n^a n^b \widetilde{V}_{ab} ,$$
$$\mathcal{A} = \gamma^{cd} \widetilde{\mathcal{R}}_{cd} , \qquad \mathcal{B} = \gamma^{cd} \widetilde{V}_{cd} ,$$
$$\mathcal{A}_{ab} = \gamma_a^c \gamma_b^d \widetilde{\mathcal{R}}_{cd} - \frac{1}{3} \gamma_{ab} \mathcal{A} , \qquad \mathcal{B}_{ab} = \gamma_a^c \gamma_b^d \widetilde{V}_{cd} - \frac{1}{3} \gamma_{ab} \mathcal{B} ,$$
$$\mathcal{C}_a = n^c \gamma_a^d \widetilde{\mathcal{R}}_{cd} , \qquad \mathcal{E}_a = n^c \gamma_a^d \widetilde{V}_{cd} ,$$

Well-posed IVP in 3+1

or equivalently by writing

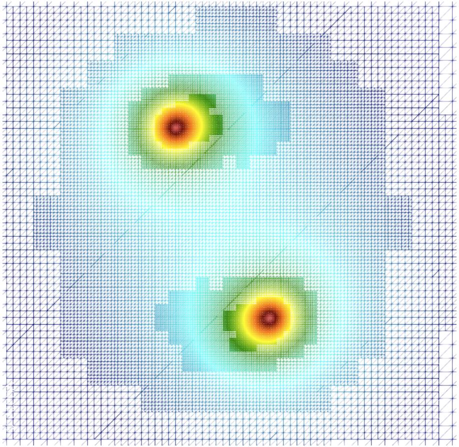
$$\widetilde{\mathcal{R}}_{ab} = \mathcal{A}_{ab} + \frac{1}{3}\gamma_{ab}\,\mathcal{A} - 2\,n_{(a}\mathcal{C}_{b)}\ , \qquad \widetilde{V}_{ab} = \mathcal{B}_{ab} + \frac{1}{3}\gamma_{ab}\,\mathcal{B} - 2\,n_{(a}\mathcal{E}_{b)}\ .$$

Using these variables, we obtain decomposed equations for metric, Ricci scalar, and Ricci tensors such that

$$\begin{aligned}
\text{Metric} &: n^a\nabla_a(\gamma_{ij}, K_{ij}) = \ldots \\
\text{Ricci Scalar} &: n^a\nabla_a(\mathcal{R}, \hat{\mathcal{R}}) = \ldots \\
\text{Ricci Tensors} &: n^a\nabla_a(\mathcal{A}, \mathcal{A}_{ij}, \mathcal{B}, \mathcal{B}_{ij}) = \ldots
\end{aligned}$$

Discussion & Future Work

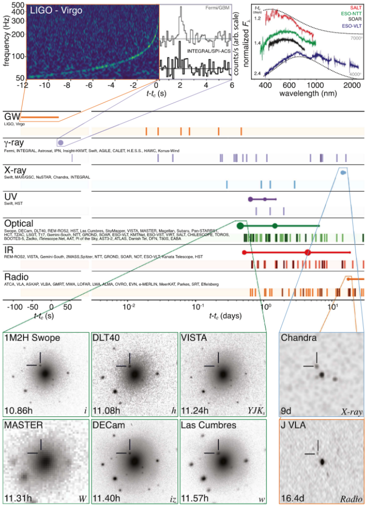
- Enable to test beyond GR theories using GWs
- First nonlinear stable numerical evolution of QG
- Explore (un)stable BH branches in QG
- Working towards to full 3+1 evolution and BBH merger (code is availabe as open source `https://github.com/lanl/Dendro-GRCA`)
- Please check Phy. Rev. D 104(8) 084075 for more details



Center for Theoretical Astrophysics at LANL

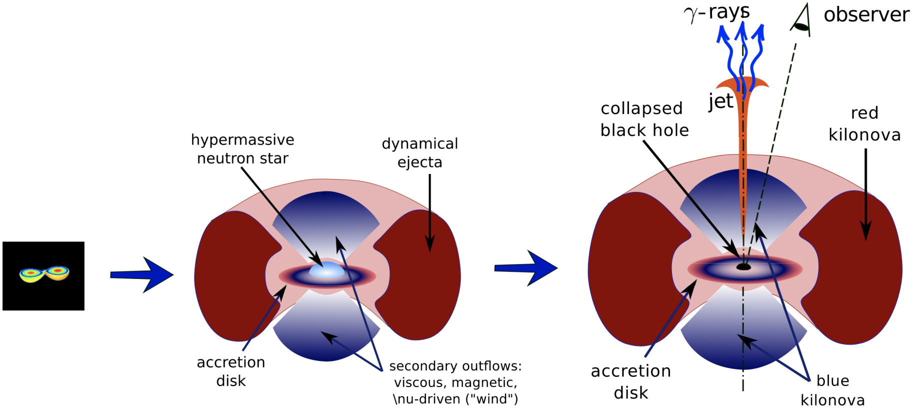
- The Center for Theoretical Astrophysics brings together a diverse set of scientists from across LANL to study a wide variety of topics of astrophysical research including AGN, Cosmology, Nuclear Astrophysics, Stellar Astrophysics, SNe, Compact Objects, Experimental/Observational Astrophysics...
- More details can be found in `https://ccsweb.lanl.gov/astro/index.html`

Binary Neutron Star Mergers



- BNS mergers provide rich astrophysical phenomena:
  - Produce the heavy elements observed in the galaxy (r-process nucleosynthesis)
  - BNS mergers were postulated to generate a SNe-like optical and infrared transients (kilonova)
  - Emit accross a wide range of EM spectrum from radio to gamma-rays
  - Source of GWs

Simulation and Modeling BNS/Kne

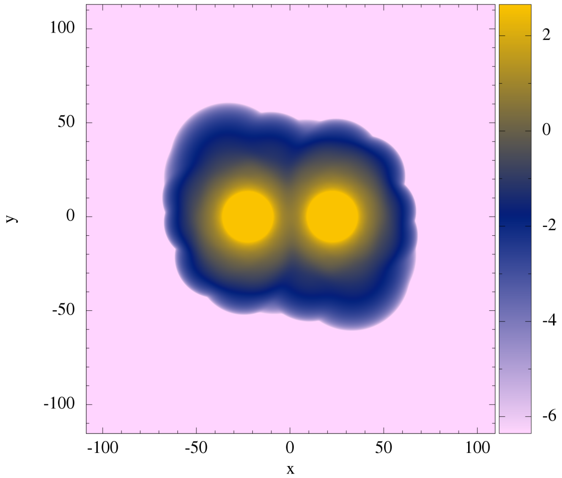


Simulation and Modeling BNS/Kne

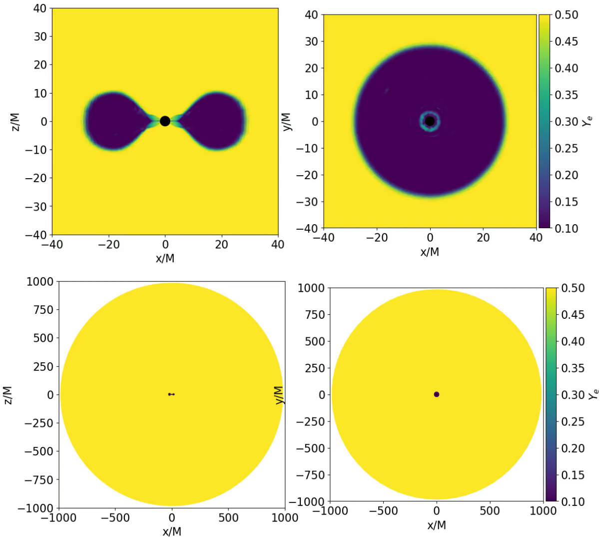
- Equipped with the new data to constrain the many model uncertainties we can begin to model GW170817 and other kilonova
  - GWs provide the masses of the NSs
  - EM is available over multiple wavelengths and time scales
- Simultaneously matching all of the available data requires sophisticated models of multiple physical process that must be coupled together
  - Dynamics of mergers to obtain the structure and mass of the ejected material
  - The r-process nucleosynthesis to obtain the heating and elemental abundances
  - Opacities and radiation transport to predict the observed EM counterparts

Merger (and post merger) Dynamics Modeling

- Model the gravity and hydrodynamics of the system to calculate the amount of material ejected from the merger
- Performed with smoothed particle hydrodynamics (SPH) methods with millions of fluid particles
  - Newtonian gravity (with fixed background metric) : FleCSPH
  - General Relativity : SPaRTA
- This merger is used to study post-merger dynamics with different tools

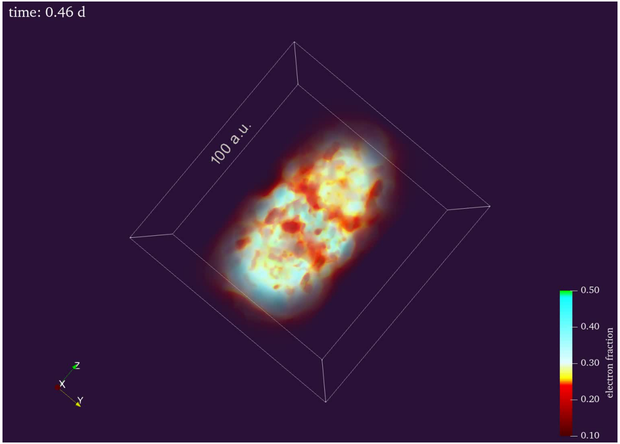


Merger (and post merger) Dynamics Modeling



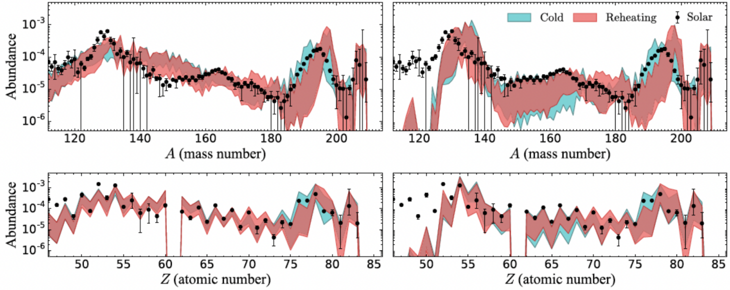
- Perform disk-wind system produced by GW170817 simulation using general relativistic radiation magnetohydrodynamics code νbhlight (`https://github.com/lanl/nubhlight`)

Merger (and post merger) Dynamics Modeling



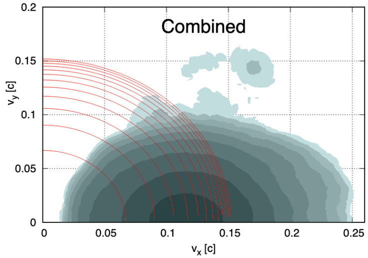
- Perform kilonova ejecta expansion produced by GW170817 simulation using SPH code FleCSPH (`https://github.com/laristra/flecsph`)

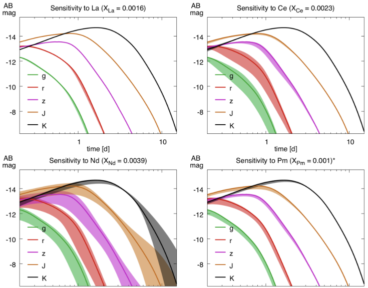
Nucleosynthesis Modeling



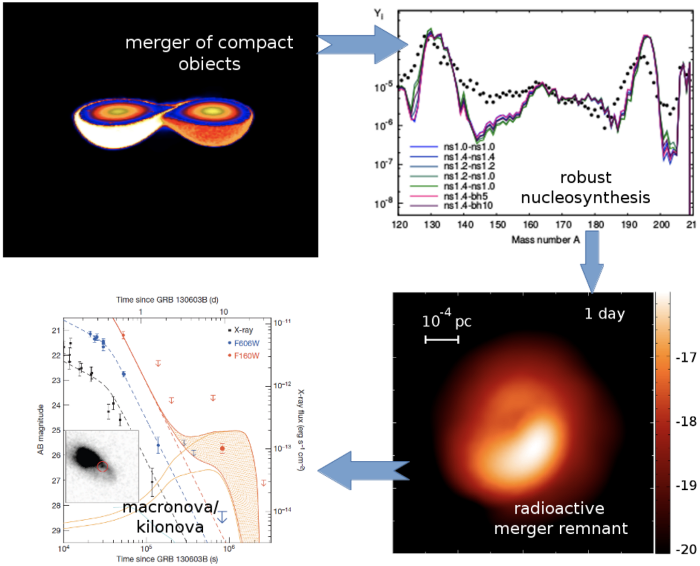
- The particles from hydrodynamics simulations each have an unique history of temperature and density
- These were used to the nucleosynthesis codes Skynet (`https://bitbucket.org/jlippuner/skynet`) and Prism
- Simultaneously solve the reactions of thousands of isotopes

Radiation Transport Modeling





- Input the structure and composition from the previous simulations
- Simulate millions of photon packets over a wide group of frequencies to obtain light curves and spectra
- SuperNu (`https://bitbucket.org/drrossum/supernu/`) (Implicit Monte Carlo and Discrete Diffusion Monte Carlo Radiation Transport Software)



Understanding Phenomena from BNS mergers requires large teams!

Conclusion

- New era to investigate the universe using gravitational waves with EM counterparts
- Enable to test general relativity and beyond in strong field regime
- A lot of questions are still unanswered and will soon be unraveled
- There are unknown unknowns!

thank you
danke
謝謝
ngiyabonga
teşekkür ederim
рахмат
Баярлалаа
спасибо
faafetai lava
mersi
kia ora
barka
welalin
tack
spas
dank je
misaotra
matondo
paldies
grazzi
mahalo
tapadh leat
gracias
vinaka
blagodaram
kiitos
dankie
nanni
nandri
dhanyavad
mauruuru
koszonom
hvala
gracie
bayarlalaa
bedankt
enkosi
dziękuję
хвала
asante
manana
obrigada
tenki
murakoze
djiere dieuf
tau
mochchakkeram
мамнун
дякую
chnorakaloutioun
gratias ago
gracies
sulpay
taiku
go raibh maith agat
sobodi
dėkuji
obrigado
sagolun
sukriya
kop khun krap
ありがとう
arigatô
takk
dakujem
trugarez
mesi
didi madloba
najis tuke
terima kasih
tanemirt
grazie
rahmet
diolch
dhanyavadagalu
shukriya
merce
мерси
kam sah hamnida
rahmat
xiexie
ευχαριστώ
감사합니다
merci

Symmetry Relations

For the present case of spherical symmetry, transformation of the symmetry identities $\Pi_{t\theta} = 0$, $\Pi_{t\phi} = 0$, $\Pi_{r\theta} = 0$, $\Pi_{r\phi} = 0$, $\Pi_{\theta\phi} = 0$, and $\Pi_{\theta\theta} \sin^2 \theta = \Pi_{\phi\phi}$ back to Cartesian coordinates implies the relations

$$\Pi_{ty} = \frac{y\,\Pi_{tx}}{x}\,, \qquad \Pi_{tz} = \frac{z\,\Pi_{tx}}{x}\,,$$

$$\Pi_{xy} = \frac{xy\,(\Pi_{xx} - \Pi_{yy})}{x^2 - y^2}\,, \quad \Pi_{xz} = \frac{xz\,(\Pi_{xx} - \Pi_{yy})}{x^2 - y^2}\,,$$

$$\Pi_{yz} = \frac{yz\,(\Pi_{xx} - \Pi_{yy})}{x^2 - y^2}\,,$$

$$\Pi_{zz} = \frac{(x^2 - z^2)\Pi_{yy} - (y^2 - z^2)\Pi_{xx}}{x^2 - y^2}\,.$$

Physical Constraints

In spherical symmetry, $21 - 5(\text{auxiliary}) - 12(\text{physical}) = 4$ independent pieces of initial data, i.e., two degrees of freedom.
The Hamiltonian and shift constraint reduce to

$$\mathcal{C}_{tt} \equiv G_{tt} - \widetilde{\mathcal{R}}_{tt} + \frac{1}{4} g_{tt}\mathcal{R} = 0 \ ,$$
$$\mathcal{C}_{tx} \equiv G_{tx} - \widetilde{\mathcal{R}}_{tx} + \frac{1}{4} g_{tx}\mathcal{R} \ .$$

Decomposition (background)

- Spherical harmonics $Y_{lm}(\theta, \phi)$
- Axisymmetric perturbations : $m = 0$
- Focus on the monopole : $l = 0$

$$h_{ab}^{\text{polar}} = e^{-i\omega t} \begin{pmatrix} AH_0 & H_1 & 0 & 0 \\ H_1 & H_2/B & 0 & 0 \\ 0 & 0 & r^2\mathcal{K} & 0 \\ 0 & 0 & 0 & r^2 \sin^2\theta \mathcal{K} \end{pmatrix} Y_{00}$$

$$\psi_{ab}^{\text{polar}} = e^{-i\omega t} \begin{pmatrix} AF_0 & F_1 & 0 & 0 \\ F_1 & F_2/B & 0 & 0 \\ 0 & 0 & r^2\mathcal{M} & 0 \\ 0 & 0 & 0 & \sin^2\theta \mathcal{M} \end{pmatrix} Y_{00}$$

Decomposition (background)

Master equation

$$\frac{d^2}{dr_*^2}\psi(r) + \psi(r)[\omega^2 - V(r)] = 0$$

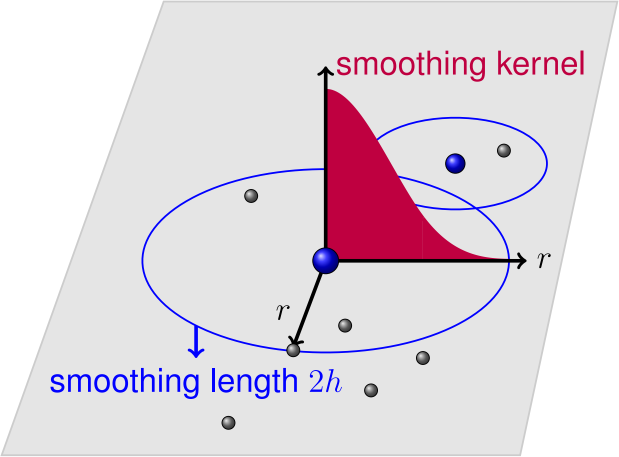
Boundary conditions:

- Purely ingoing waves at the horizon
- Outgoing waves at asymptotic infinity define QNMs
- Ingoing waves at asymptotic infinity define bound states

Smoothed Particle Hydrodynamics (SPH)

- Solves hydrodynamical equations
- Numerical *mesh-free* method
- Discretizes fluid in elements called particles
- Fundamental SPH equation for density:

$$\rho(\vec{r}) \approx \sum_b m_b W(|\vec{r} - \vec{r}_b|, h)$$



Simulation Best Fits to Data for GW170817

- Wind mass : 0.03 $\sim$ 0.1$M_{\odot}$
- Wind velocity : 0.08c
- Wind kinetic energy : 2×10^{50}erg

- Dynamical ejecta mass : 0.002 $\sim$ 0.003$M_{\odot}$
- Dynamical ejecta velocity : 0.2 $\sim$ 0.3c
- Dynamical ejecta kinetic energy : 6×10^{50}erg

- Viewing angle $<$ 40 degrees